Projectile Motion

We can use parametric equations to describe the motion of an object.
We can specify not only where the object travels, its location (x,y) but also when it gets there, the time t.

Equations of the path of a projectile fired at an inclination, θ to the horizontal, with an initial speed v_0, from a height h above the horizontal are:

$x = v_0 \cos(\theta)\, t$ **$y = -1/2\, g t^2 + v_0 \sin(\theta)\, t + h$**

where **t** is the time and **g** is the constant acceleration due to gravity (approximately $32 ft/s^2$ or $9.8\ m/s^2$).

Example:
Suppose that Josh hit a golf ball with an initial velocity of 150 ft per second at an angle of 30 degrees to the horizontal.

a. Find parametric equations that describe the position of the ball as a function of time.

$X = 150(\cos 30)t$ $y = -\frac{1}{2}(32)t^2 + 150(\sin 30)t$

$x = 150\left(\frac{\sqrt{3}}{2}\right)t$ $y = -16t^2 + 150\left(\frac{1}{2}\right)t$

$x = (75\sqrt{3})t$ $y = -16t^2 + 75t$

b. How long is the golf ball in the air? (Let y=0)

$0 = -16t^2 + 75t$

$0 = (-16t + 75)t$

$t = 0$ or $-16t + 75 = 0$

$-16t = -75$

$t = \frac{75}{16}$ or 4.6875 sec.

c. When is the ball at its maximum height? (Use t = -b/2a)

$y = -16t^2 + 75t$

$t = \frac{-(75)}{2(-16)} = \frac{75}{32} \approx 2.34375$

d. What is the maximum height of the ball? $y = -16(2.34375)^2 + 75(2.34375)$

$y \approx 87.89$ ft

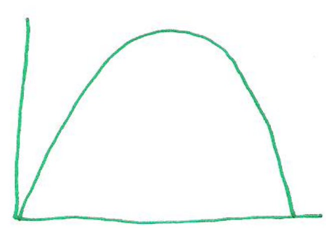
e. Determine the distance that the ball traveled before hitting the ground.

$x = (75\sqrt{3})(4.6875)$

$x \approx 608.9$ ft

f. Simulate the motion of the golf ball on the graphing calculator, in parametric mode. Sketch the graph, record the window, and check your answers on the graph.

$0 < t < 5$



$(0, 700) \times (0, 100)$

1.a. horizontal $= 60(\cos 38) \approx 47.3$ ft/sec
vertical $= 60(\sin 38) \approx 36.9$ ft/sec

b. $x = 60(\cos 38)t$
$y = -16t^2 + (60\sin 38)t$

c. $x = 60(\cos 38)(2) = 94.56$ ft
$y = 60(\sin 38)(2) - 16(2)^2 = 9.88$ ft

d. $y = -16t^2 + (60\sin 38)t = 0$
$t(-16t + (60\sin 38)) = 0$
$t = 0$ or $-16t + 60\sin 38 = 0$
$-16t = -60\sin 38$
$t = \frac{-60\sin 38}{-16}$
$t = 0$ sec or $t \approx 2.309$ sec

check $y = -16(2.309)^2 + 60(\sin 38)(2.309) = 0$

horizontal $x = (60\cos 38)(2.309) \approx \underline{109.16 \text{ ft}}$

2. $x = 80(\cos 40)t$ $\quad$ $y = -16t^2 + 80(\sin 40)t + 2$

a. $0 = -16t^2 + 80(\sin 40)t + 2$

$t = \frac{-80(\sin 40) \pm \sqrt{(80\sin 40)^2 - 4(-16)(2)}}{2(-16)}$
$\frac{-51.423 \pm \sqrt{2772.326}}{-32}$
$= 3.25$ sec

$x = 80(\cos 40)(3.25)$
$= 199.3$ ft

b, $0 = -16t^2 + 80(\sin 45)t + 2$

$t = \frac{-80(\sin 45) \pm \sqrt{(80\sin 45)^2 - 4(-16)(2)}}{-32}$

$t \approx 3.57$ sec

$x = 80(\cos 45)(3.57)$

$x = 201.98$

2c. $0 = -16t^2 + 80(\sin 50)t + 2$ $\quad t = \frac{-80(\sin 50) \pm \sqrt{(80\sin 50)^2 - 4(-16)(2)}}{2(-16)}$

$x = 80(\cos 50)(3.86)$ $\quad t = 3.86$ sec

$x \approx 198.63$ ft

2d. $190 = 80(\cos 40)t \quad t = 3.100$

$y = -16(3.1)^2 + 80(\sin 40)(3.1) + 2 = 7.6$ ft

40° does not clear

$190 = 80(\cos 45)t \quad t = 3.359$

$y = -16(3.359)^2 + 80(\sin 45)(3.359) + 2 = 11.5$ ft

45° does clear

$190 = 80(\cos 50)t \quad t = 3.695$

$y = -16(3.695)^2 + 80(\sin 50)(3.695) + 2 = 10.003$

50° does clear, barely (Might bounce on the fence)

3a. $x = 200(\cos 0)t = 200t$
$y = -4.9t^2 + 200(\sin 0)t + 40 = -4.9t^2 + 40$

3b. $y = -4.9(2.5)^2 + 40$
a drop of 30.625 m to a height of 9.375 m

3c. $0 = -4.9t^2 + 40 \quad t \approx 2.857$ sec
$x = 200(2.857)$
≈ 571.4 meters

4a. $x = 75(\cos 80)t$
$y = -4.9t^2 + 75(\sin 80)t$

4b. $x = 75(\cos 80)(1)$ $\approx$ 13 meters away
$y = -4.9(1)^2 + 75(\sin 80)(1)$ $\approx$ 69 meters high

4c. $0 = -4.9(t)^2 + 75(\sin 80)t$
$0 = t(-4.9t + 75\sin 80)$
$t = 0$ or $-4.9t + 75\sin 80 = 0$
$\underline{t \approx 15 \text{ sec}}$
$x = 75(\cos 80)(15)$
$x = \underline{195.4 \text{ meters}}$

4d. $y = -4.9t^2 + 75(\sin 80)t$ $\quad t = \frac{-b}{2a}$ $\quad t = \frac{-75(\sin 80)}{2(-4.9)}$
$t \approx 7.537$

$y = -4.9(7.537)^2 + 75(\sin 80)(7.537)$

$y \approx 278.3$ meters

5a. $x = 80(\cos 45)t$
$y = -16t^2 + 80(\sin 45)t + 5$

use table $t = 1$ $(56.569, 45.569)$
$t = 2$ $(113.14, 54.137)$
$t = 3$ $(169.71, 30.706)$

5b. $0 = -16t^2 + 80(\sin 45)t + 5$ $\quad t = \frac{-80(\sin 45) \pm \sqrt{(80\sin 45)^2 - 4(-16)(5)}}{2(-16)}$

$t = 3.62$

$x = 80(\cos 45)(3.62)$

$x \approx 204.9$ ft