Review Session 6.4-6.6

① Solve the differential equation

$$xy + y' = 100x$$

$$xy + \frac{dy}{dx} = 100x$$

$$\frac{dy}{dx} = 100x - xy$$

$$\frac{dy}{dx} = x(100-y)$$

$$\int \frac{1}{100-y}\,dy = \int x\,dx$$

$$-\ln|100-y| = \frac{x^2}{2} + C$$

$$e^{\ln|100-y|} = e^{-\frac{x^2}{2} + C}$$

$$|100-y| = Ce^{-x^2/2}$$

$$100-y = \pm Ce^{-x^2/2}$$

$$-y = -100 + Ce^{-x^2/2}$$

$$\boxed{y = 100 + Ce^{-x^2/2}}$$

② Write and solve the differential equation that models the statement:

The rate of change of P with respect to t is proportional to 25-t.

$$\frac{dP}{dt} = k(25-t)$$

$$\int dP = \int k(25-t)\,dt$$

$$P = k\int (25-t)\,dt$$

$$\boxed{P = k\left(25t - \frac{t^2}{2}\right) + C}$$

③ The rate of change of N is proportional to N. When $t=0$ $N=250$, and when $t=1$, $N=400$. What is the value of N when $t=4$?

$\frac{dN}{dt} = kN$

$\frac{dN}{N} = k\,dt$

$\ln|N| = kt + C$

$\ln 250 = C$

$e^{\ln|N|} = e^{kt + \ln 250}$

$|N| = 250e^{kt}$

$N = 250e^{kt}$

$400 = 250e^{k}$

$k = .470\ldots$

$N = 250e^{.470t}$

$N = 250e^{.470(4)}$

$\boxed{N = 1638.4}$

Given $y' = \frac{\cos x}{3y^2}$

④ Approximate the first 4 x and y values using Euler's method with $dx = 0.1$ and $y(\pi) = 5$. Round all values to 4 decimal places but use stored values in calculations.

x	y - calculations	y (Euler)
π	$y_n = y_{n-1} + f(x_{n-1}, y_{n-1})\,dx$	5
$\pi + .1$	$5 + (\cos\pi / 3(5)^2)(.1)$	4.9987
$\pi + .2$	$4.9987 + (\cos(\pi + .1)/3(4.9987)^2)(.1)$	4.9973
$\pi + .3$		4.9960
$\pi + .4$		4.9948

Find the function for which y is the differential equation.

$\frac{dy}{dx} = \frac{\cos x}{3y^2}$

$\int 3y^2\,dy = \int \cos x\,dx$

$y^3 = \sin x + C$

$5^3 = \sin\pi + C$

$125 = C$

$y^3 = \sin x + 125$

$y = \sqrt[3]{\sin x + 125}$

Determine the actual value and the error in Euler's method for $x = \pi + .4$

$y(\pi + .4) = 4.9948$ $\quad .0012$

ERROR = ACTUAL − PREDICTED