

More Practice Proving Trigonometric Identities

1. $\csc x = \frac{\cot x}{\cos x}$

$$\left| \begin{array}{l} \frac{\cos x}{\sin x} = \frac{1}{\cos x} \\ \frac{1}{\sin x} \\ \csc x \end{array} \right|$$

2. $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$$\left| \begin{array}{l} \cos^2 x + \sin^2 x \\ 1 \end{array} \right|$$

3. $\csc^2 x \tan^2 x - 1 = \tan^2 x$

$$\left| \begin{array}{l} \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1 \\ \frac{1}{\cos^2 x} - 1 \\ \sec^2 x - 1 \\ \tan^2 x \end{array} \right|$$

4. $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$

$$\left| \begin{array}{l} \frac{1}{\cos x} \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x} \\ \frac{1 - \sin^2 x}{\cos^2 x} \\ \frac{\cos^2 x}{\cos^2 x} \end{array} \right|$$

5. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$$\left| \begin{array}{l} (\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x) \\ (\csc^2 x + \cot^2 x) \left(\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \right) \\ (\csc^2 x + \cot^2 x) \left(\frac{1 - \cos^2 x}{\sin^2 x} \right) \\ (\csc^2 x + \cot^2 x) (1) \end{array} \right|$$

6. $(1 - \tan x)^2 = \sec^2 x - 2 \tan x$

$$\left| \begin{array}{l} 1 - 2 \tan x + \tan^2 x \\ 1 + \tan^2 x - 2 \tan x \\ \sec^2 x - 2 \tan x \end{array} \right|$$

7. $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

$$\left| \begin{array}{l} \cos^2 x (1 + \tan^2 x) \\ \cos^2 x + \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} \right) \\ \cos^2 x + \sin^2 x \end{array} \right|$$

8. $2 \sin^2 x - 1 = 1 - 2 \cos^2 x$

$$\left| \begin{array}{l} 2(1 - \cos^2 x) - 1 \\ 2 - 2 \cos^2 x - 1 \\ 1 - 2 \cos^2 x \end{array} \right|$$

9. $\csc x + \cot x = \frac{1 + \cos x}{\sin x}$

$$\left| \begin{array}{l} \frac{1}{\sin x} + \frac{\cos x}{\sin x} \\ \frac{1 + \cos x}{\sin x} \end{array} \right|$$

10. $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$

$$\left| \begin{array}{l} (1 - \sin^2 x) - \sin^2 x \\ 1 - 2 \sin^2 x \end{array} \right|$$

$$\frac{1}{\cos x} + \frac{1}{\sin x}$$

11. $\frac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$

$$\left| \begin{array}{l} \left(\frac{1}{\cos^2 x} \right) \left(\frac{\cos^2 x}{\sin^2 x} \right) \\ \frac{1}{\sin^2 x} \\ \csc^2 x \end{array} \right|$$

12. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

$$\left| \begin{array}{l} \sin x \left(\frac{\sin x}{\cos x} \right) + \sin x \left(\frac{\cos x}{\sin x} \right) + \cos x \left(\frac{\sin x}{\cos x} \right) + \cos x \left(\frac{\cos x}{\sin x} \right) \\ \left(\frac{\sin^2 x}{\cos x} + \cos x \right) + \sin x + \frac{\cos^2 x}{\sin x} \\ \left(\frac{\sin^2 x + \cos^2 x}{\cos x} \right) + \left(\frac{\sin^2 x + \cos^2 x}{\sin x} \right) \\ \frac{1}{\cos x} + \frac{1}{\sin x} \end{array} \right|$$