AP Calculus Exam Review

**Show all steps leading to your answers, including any intermediate results using a graphing utility. **

1. Write an equation for the line through P(5, -3) that is

   A. Parallel to the line $4x - 5y = 2$ and
   B. Perpendicular to the line $4x - 5y = 2$

2. Let $f(x) = 2 - |1 + x|$

   A. Draw the graph of $y = f(x)$.
   B. Find the domain of f.
   C. Find the range of f.

3. Solve the equation $1.08^x = 5$ algebraically.

4. Let $g(x) = 2 \sec(3x - \pi) + 1$. Determine

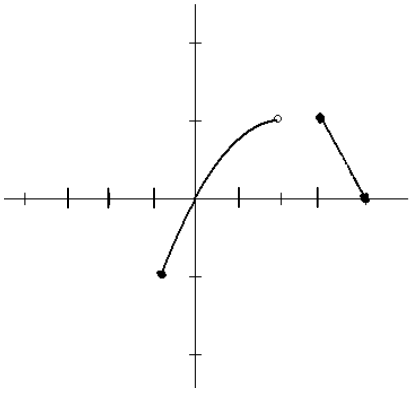
   A. the period.
   B. the domain
   C. the range
   D. draw the graph of g(x)

5. Which of the following statements are true about the function f whose graph is shown below.

   I. $\lim_{x \to -1^+} f(x) = -1$

   II. $f(2) = \lim_{x \to 2} f(x)$

   III. $\lim_{x \to 1^-} f(x) = .5$



A. I and II B. I and III C. II and III D. I only E. I, II, and III

6. Determine $\lim_{x \to 0} \left(\frac{\sin x}{2x} + 3x + 4 \right)$

7. Let f(x) = $\begin{cases} 5 - x^2, & x \le 2 \\ \frac{x+3}{x-2}, & x > 2 \end{cases}$

Find the limit of f(x) as

A. $x \to -\infty$ B. $x \to 2^-$ C. $x \to 2^+$ D. $x \to \infty$

8. . Let f(x) = $\begin{cases} \dfrac{3x}{x+4}, & x \le 4, x \ne -4 \\ \sqrt{x-3}, & x > 4 \end{cases}$

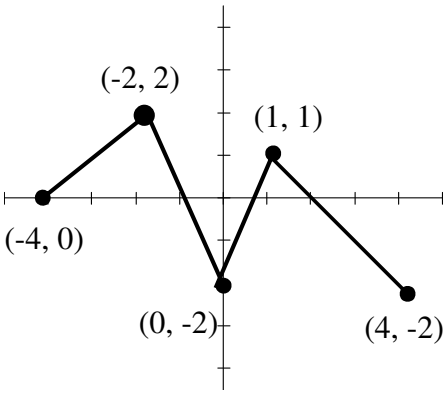
Find the points of discontinuity of the function f. Identify each type of discontinuity.

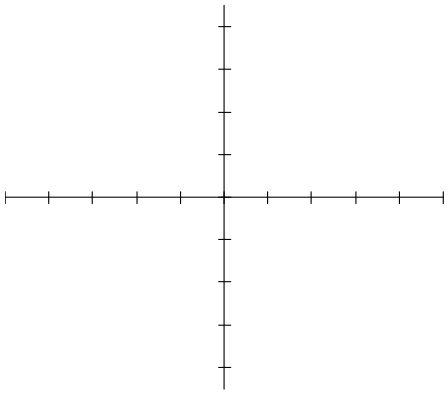
9. Let f(x) = $\dfrac{x^2+3x-4}{x^2-4x+3}$. Give a formula for the extended function that is continuous at x = 1.

10. An object is dropped from a 75 ft. cliff. Its height in feet above the beach after t sec is given by $h(t) = 75 - 16\,t^2$.

A. Find the average velocity during the interval from t = 1 to t = 2.
B. Find the instantaneous velocity at t = 2.

11. The graph of y = g(x) is shown here is made of line segments joined end to end. Graph the function's derivative.





12. Let f(x) = $x^2 - |x-4|$ Which of the following describes the behavior of f at x = 4?

A. differentiable B. corner C. cusp D. vertical tangent E. discontinuity

13. Find $\dfrac{dy}{dx}$, where y = $\dfrac{(x+2)(x+4)}{x-1}$.

14. Find the first and second derivatives of f(x) when f(x) = $x^4 - 5x^3 + 9x - 15$.

15. Find $\dfrac{dy}{dx}$ if y = sec $(3x^2)$

16. A particle moves along a line so that its position at any time $t \ge 0$ is given by the function $s(t) = 2t^3 - 1.5t + 6$, where s is measured in meters and t is measured in minutes.

A. Find the displacement during the first 5 minutes.
B. Find the average velocity during the first 5 minutes.
C. Find the instantaneous velocity when t = 5 minutes
D. Find the acceleration of the particle when t = 5 minutes.
E. At what value or values of t does the particle change directions?

17. The monthly profit (in dollars) of a certain artist is given by $P(x) = -15x^3 + 150x^2 + 300x - 400$, where x is the number of articles sold.

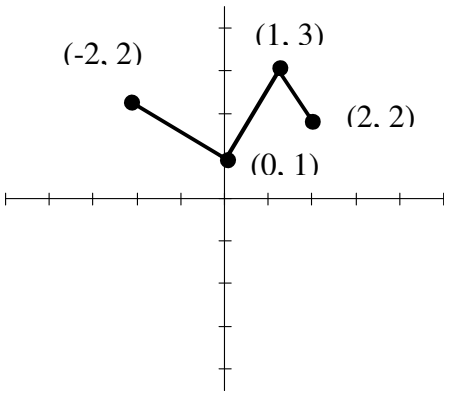
    A. Graph P(x) and P'(x).
    B. What is the marginal profit when 6 pieces are sold?
    C. What is the profit when the marginal profit is greatest?
    D. What is the maximum profit possible? (Assume x is an integer.)

18. Use implicit differentiation to find $\frac{dy}{dx}$ if $\cos xy = 2x^2 - 3y$.

19. Let $y = x^{\cos x}$. Use logarithmic differentiation to find y' .

20. Find y' if $y = e^{2x} - \ln x^2$

21. Find the extreme values and where they occur.



22. For $y = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3$, find the exact intervals on which the function is

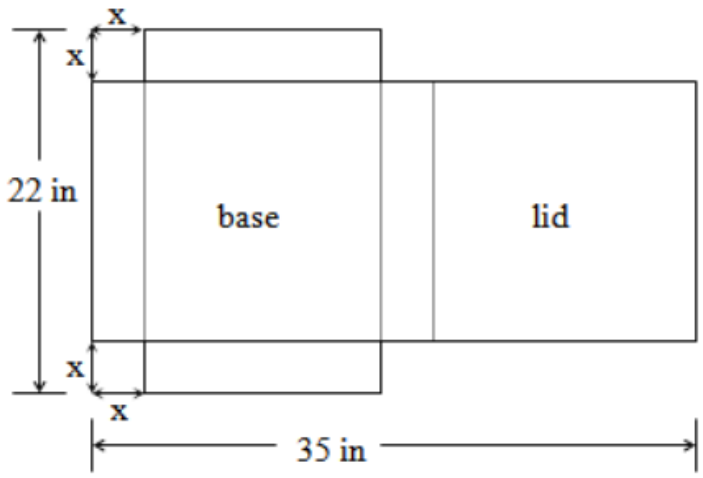
    A. increasing
    B. decreasing
    C. concave up
    D. concave down
    E. Find any local extreme values
    F. Find any inflection points.

23. A car's odometer reads exactly 59,024 miles at 8:00 am and 59,094 miles at 10:00 am Assuming the car's position and velocity functions are differentiable, what theorem can be used to show that the car was traveling at exactly 35 mph at some time between 8:00 am and 10:00 am?

I. Intermediate Value Theorem for Derivatives
II. Extreme Value Theorem
III. Mean Value Theorem

A. I only B. II only C. III only D. I and III E. II and III

24. A piece of cardboard measures 22 in by 35 in. Two equal squares are removed from the corners of the 22 in side as shown in the figure. Two equal rectangles are removed from the other corners so that the tabs can be folded to form a rectangular box with a lid.



A. Write a formula for V(x) for the volume of the box.
B. Find the domain of V for the problem situation
C. Find the maximum volume and the value of x that gives it.

Memorize all the derivative formulas.