Chapter 2
Polynomial, Power, and Rational Functions

Section 2.1 Linear and Quadratic Functions and Modeling

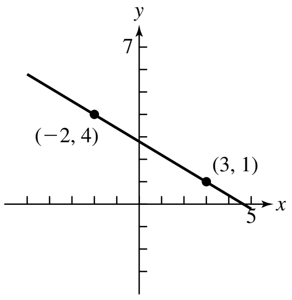
Exploration 1

1. $-\$2000$ per year
2. The equation will have the form $v(t) = mt + b$. The value of the building after 0 year is $v(0) = m(0) + b = b = 50{,}000$.

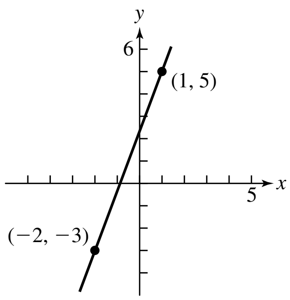
   The slope m is the rate of change, which is -2000 (dollars per year). So an equation for the value of the building (in dollars) as a function of the time (in years) is $v(t) = -2000t + 50{,}000$.
3. $v(0) = 50{,}000$ and $v(16) = -2000(16) + 50{,}000 = 18{,}000$ dollars
4. The equation $v(t) = 39{,}000$ becomes
$$\begin{aligned} -2000t + 50{,}000 &= 39{,}000 \\ -2000t &= -11{,}000 \\ t &= 5.5 \text{ years} \end{aligned}$$

Quick Review 2.1

1. $y = 8x + 3.6$
2. $y = -1.8x - 2$
3. $y - 4 = -\frac{3}{5}(x + 2)$, or $y = -0.6x + 2.8$



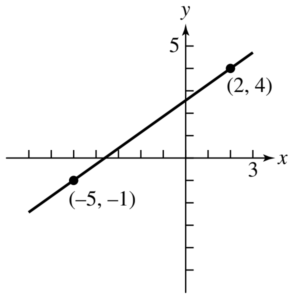
4. $y - 5 = \frac{8}{3}(x - 1)$, or $y = \frac{8}{3}x + \frac{7}{3}$



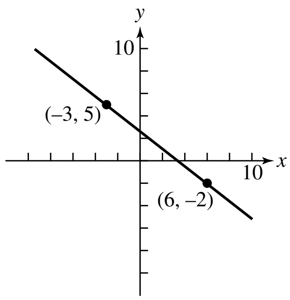
5. $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9$
   $= x^2 + 6x + 9$
6. $(x - 4)^2 = (x - 4)(x - 4) = x^2 - 4x - 4x + 16$
   $= x^2 - 8x + 16$
7. $3(x - 6)^2 = 3(x - 6)(x - 6) = (3x - 18)(x - 6)$
   $= 3x^2 - 18x - 18x + 108 = 3x^2 - 36x + 108$
8. $-3(x + 7)^2 = -3(x + 7)(x + 7)$
   $= (-3x - 21)(x + 7) = -3x^2 - 21x - 21x - 147$
   $= -3x^2 - 42x - 147$
9. $2x^2 - 4x + 2 = 2(x^2 - 2x + 1) = 2(x - 1)(x - 1)$
   $= 2(x - 1)^2$
10. $3x^2 + 12x + 12 = 3(x^2 + 4x + 4) = 3(x + 2)(x + 2)$
    $= 3(x + 2)^2$

Section 2.1 Exercises

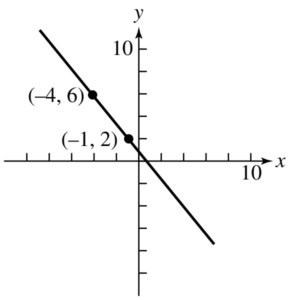
1. Not a polynomial function because of the exponent -5
2. Polynomial of degree 1 with leading coefficient 2
3. Polynomial of degree 5 with leading coefficient 2
4. Polynomial of degree 0 with leading coefficient 13
5. Not a polynomial function because of cube root
6. Polynomial of degree 2 with leading coefficient -5
7. $m = \frac{5}{7}$ so $y - 4 = \frac{5}{7}(x - 2) \Rightarrow f(x) = \frac{5}{7}x + \frac{18}{7}$



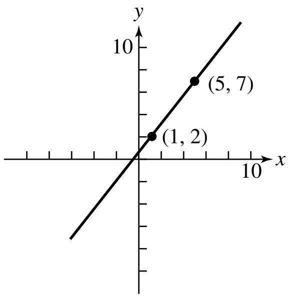
8. $m = -\frac{7}{9}$ so $y - 5 = -\frac{7}{9}(x + 3) \Rightarrow f(x) = -\frac{7}{9}x + \frac{8}{3}$



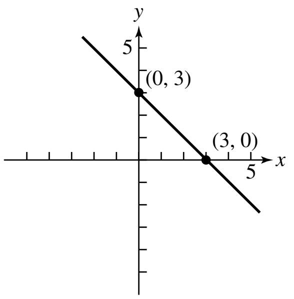
9. $m = -\frac{4}{3}$ so $y - 6 = -\frac{4}{3}(x + 4) \Rightarrow f(x) = -\frac{4}{3}x + \frac{2}{3}$



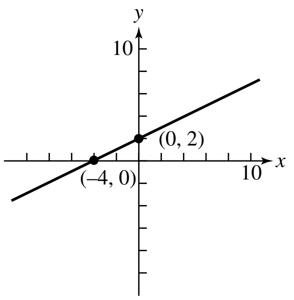
10. $m = \frac{5}{4}$ so $y - 2 = \frac{5}{4}(x - 1) \Rightarrow f(x) = \frac{5}{4}x + \frac{3}{4}$



11. $m = -1$ so $y - 3 = -1(x - 0) \Rightarrow f(x) = -x + 3$



12. $m = \frac{1}{2}$ so $y - 2 = \frac{1}{2}(x - 0) \Rightarrow f(x) = \frac{1}{2}x + 2$



13. (a)—the vertex is at $(-1, -3)$, in Quadrant III, eliminating all but (a) and (d). Since $f(0) = -1$, it must be (a).

14. (d)—the vertex is at $(-2, -7)$, in Quadrant III, eliminating all but (a) and (d). Since $f(0) = 5$, it must be (d).

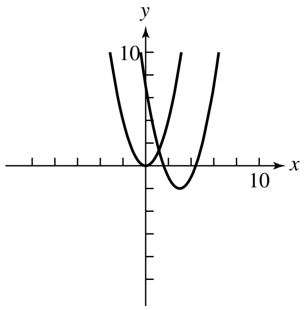
15. (b)—the vertex is in Quadrant I, at $(1, 4)$, meaning it must be either (b) or (f). Since $f(0) = 1$, it cannot be (f): if the vertex in (f) is $(1, 4)$, then the intersection with the y-axis would be about $(0, 3)$. It must be (b).

16. (f)—the vertex is in Quadrant I, at $(1, 12)$, meaning it must be either (b) or (f). Since $f(0) = 10$, it cannot be (b): if the vertex in (b) is $(1, 12)$, then the intersection with the y-axis occurs considerably lower than $(0, 10)$. It must be (f).

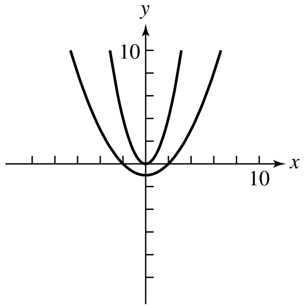
17. (e)—the vertex is at $(1, -3)$ in Quadrant IV, so it must be (e).

18. (c)—the vertex is at $(-1, 12)$ in Quadrant II and the parabola opens down, so it must be (c).

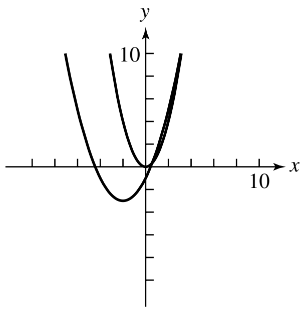
19. Translate the graph of $f(x) = x^2$ 3 units right to obtain the graph of $h(x) = (x - 3)^2$, and translate this graph 2 units down to obtain the graph of $g(x) = (x - 3)^2 - 2$.



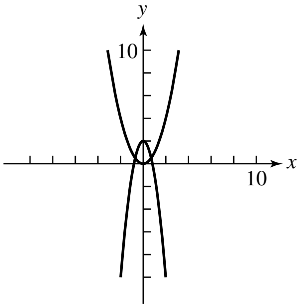
20. Vertically shrink the graph of $f(x) = x^2$ by a factor of $\frac{1}{4}$ to obtain the graph of $g(x) = \frac{1}{4}x^2$, and translate this graph 1 unit down to obtain the graph of $h(x) = \frac{1}{4}x^2 - 1$.



21. Translate the graph of $f(x) = x^2$ 2 units left to obtain the graph of $h(x) = (x + 2)^2$, vertically shrink this graph by a factor of $\frac{1}{2}$ to obtain the graph of $k(x) = \frac{1}{2}(x + 2)^2$, and translate this graph 3 units down to obtain the graph of $g(x) = \frac{1}{2}(x + 2)^2 - 3$.



22. Vertically stretch the graph of $f(x) = x^2$ by a factor of 3 to obtain the graph of $g(x) = 3x^2$, reflect this graph across the x-axis to obtain the graph of $k(x) = -3x^2$, and translate this graph up 2 units to obtain the graph of $h(x) = -3x^2 + 2.1$



For #23–32, with an equation of the form $f(x) = a(x - h)^2 + k$, the vertex is (h, k) and the axis is $x = h$.

23. Vertex: $(1, 5)$; axis: $x = 1$

24. Vertex: $(-2, -1)$; axis: $x = -2$

25. Vertex: $(1, -7)$; axis: $x = 1$

26. Vertex: $(\sqrt{3}, 4)$; axis: $x = \sqrt{3}$

27. $f(x) = 3\left(x^2 + \frac{5}{3}x\right) - 4$

$= 3\left(x^2 + 2\cdot\frac{5}{6}x + \frac{25}{36}\right) - 4 - \frac{25}{12} = 3\left(x + \frac{5}{6}\right)^2 - \frac{73}{12}$

Vertex: $\left(-\frac{5}{6}, -\frac{73}{12}\right)$; axis: $x = -\frac{5}{6}$

28. $f(x) = -2\left(x^2 - \frac{7}{2}x\right) - 3$

$= -2\left(x^2 - 2\cdot\frac{7}{4}x + \frac{49}{16}\right) - 3 + \frac{49}{8}$

$= -2\left(x - \frac{7}{4}\right)^2 + \frac{25}{8}$

Vertex: $\left(\frac{7}{4}, \frac{25}{8}\right)$; axis: $x = \frac{7}{4}$

29. $f(x) = -(x^2 - 8x) + 3$

$= -(x^2 - 2\cdot 4x + 16) + 3 + 16 = -(x - 4)^2 + 19$

Vertex: $(4, 19)$; axis: $x = 4$

30. $f(x) = 4\left(x^2 - \frac{1}{2}x\right) + 6$

$= 4\left(x^2 - 2\cdot\frac{1}{4}x + \frac{1}{16}\right) + 6 - \frac{1}{4} = 4\left(x - \frac{1}{4}\right)^2 + \frac{23}{4}$

Vertex: $\left(\frac{1}{4}, \frac{23}{4}\right)$; axis: $x = \frac{1}{4}$

31. $g(x) = 5\left(x^2 - \frac{6}{5}x\right) + 4$

$= 5\left(x^2 - 2\cdot\frac{3}{5}x + \frac{9}{25}\right) + 4 - \frac{9}{5} = 5\left(x - \frac{3}{5}\right)^2 + \frac{11}{5}$

Vertex: $\left(\frac{3}{5}, \frac{11}{5}\right)$; axis: $x = \frac{3}{5}$

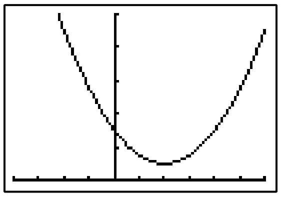
32. $h(x) = -2\left(x^2 + \frac{7}{2}x\right) - 4$

$= -2\left(x^2 + 2\cdot\frac{7}{4}x + \frac{49}{16}\right) - 4 + \frac{49}{8}$

$= -2\left(x + \frac{7}{4}\right)^2 + \frac{17}{8}$

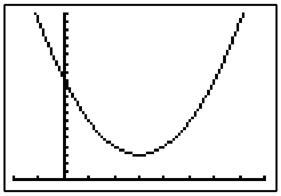
Vertex: $\left(-\frac{7}{4}, \frac{17}{8}\right)$; axis: $x = -\frac{7}{4}$

33. $f(x) = (x^2 - 4x + 4) + 6 - 4 = (x - 2)^2 + 2$.
Vertex: $(2, 2)$; axis: $x = 2$; opens upward; does not intersect x–axis.



[−4, 6] by [0, 20]

34. $g(x) = (x^2 - 6x + 9) + 12 - 9 = (x - 3)^2 + 3$.
Vertex: $(3, 3)$; axis: $x = 3$; opens upward; does not intersect x–axis.

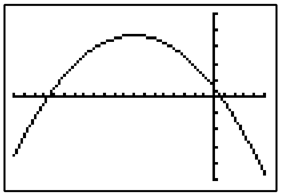


[−2, 8] by [0, 20]

35. $f(x) = -(x^2 + 16x) + 10$

$= -(x^2 + 16x + 64) + 10 + 64 = -(x + 8)^2 + 74$.

Vertex: $(-8, 74)$; axis: $x = -8$; opens downward; intersects x–axis at about -16.602 and $0.602 (-8 \pm \sqrt{74})$.

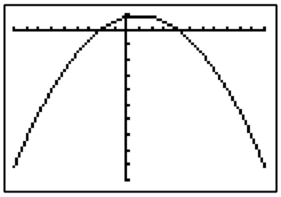


[−20, 5] by [−100, 100]

36. $h(x) = -(x^2 - 2x) + 8 = -(x^2 - 2x + 1) + 8 + 1$

$= -(x - 1)^2 + 9$

Vertex: $(1, 9)$; axis: $x = 1$; opens downward; intersects x–axis at -2 and 4.

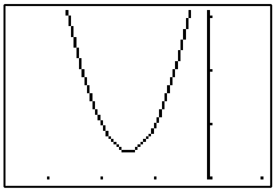


[−9, 11] by [−100, 10]

37. $f(x) = 2(x^2 + 3x) + 7$

$$= 2\left(x^2 + 3x + \frac{9}{4}\right) + 7 - \frac{9}{2} = 2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}$$

Vertex: $\left(-\frac{3}{2}, \frac{5}{2}\right)$; axis: $x = -\frac{3}{2}$; opens upward; does not intersect the x-axis; vertically stretched by 2.



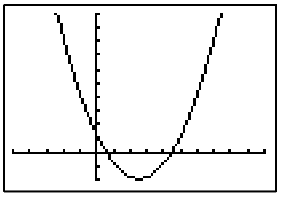
[−3.7, 1] by [2, 5.1]

38. $g(x) = 5(x^2 - 5x) + 12$

$$= 5\left(x^2 - 5x + \frac{25}{4}\right) + 12 - \frac{125}{4}$$

$$= 5\left(x - \frac{5}{2}\right)^2 - \frac{77}{4}$$

Vertex: $\left(\frac{5}{2}, -\frac{77}{4}\right)$; axis: $x = \frac{5}{2}$; opens upward; intersects x–axis at about 0.538 and 4.462 $\left(\text{or } \frac{5}{2} \pm \frac{1}{10}\sqrt{385}\right)$; vertically stretched by 5.



[−5, 10] by [−20, 100]

For #39–44, use the form $y = a(x - h)^2 + k$, taking the vertex (h, k) from the graph or other given information.

39. $h = -1$ and $k = -3$, so $y = a(x + 1)^2 - 3$. Now substitute $x = 1$, $y = 5$ to obtain $5 = 4a - 3$, so $a = 2$: $y = 2(x + 1)^2 - 3$.

40. $h = 2$ and $k = -7$, so $y = a(x - 2)^2 - 7$. Now substitute $x = 0$, $y = 5$ to obtain $5 = 4a - 7$, so $a = 3$: $y = 3(x - 2)^2 - 7$.

41. $h = 1$ and $k = 11$, so $y = a(x - 1)^2 + 11$. Now substitute $x = 4$, $y = -7$ to obtain $-7 = 9a + 11$, so $a = -2$: $y = -2(x - 1)^2 + 11$.

42. $h = -1$ and $k = 5$, so $y = a(x + 1)^2 + 5$. Now substitute $x = 2$, $y = -13$ to obtain $-13 = 9a + 5$, so $a = -2$: $y = -2(x + 1)^2 + 5$.

43. $h = 1$ and $k = 3$, so $y = a(x - 1)^2 + 3$. Now substitute $x = 0$, $y = 5$ to obtain $5 = a + 3$, so $a = 2$: $y = 2(x - 1)^2 + 3$.

44. $h = -2$ and $k = -5$, so $y = a(x + 2)^2 - 5$. Now substitute $x = -4$, $y = -27$ to obtain $-27 = 4a - 5$, so $a = -\frac{11}{2}$: $y = -\frac{11}{2}(x + 2)^2 - 5$.

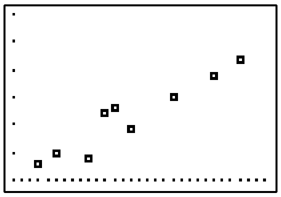
45. Strong positive

46. Strong negative

47. Weak positive

48. No correlation

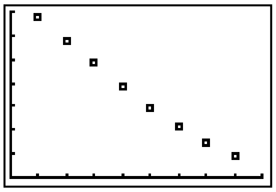
49. (a)



[15, 45] by [20, 50]

(b) Strong positive

50. (a)



[0, 90] by [0, 70]

(b) Strong negative

51. $m = -\frac{2350}{5} = -470$ and $b = 2350$, so $v(t) = -470t + 2350$. At $t = 3$, $v(3) = (-470)(3) + 2350 = \$940$.

52. Let x be the number of dolls produced each week and y be the average weekly costs. Then $m = 4.70$, and $b = 350$, so $y = 4.70x + 350$, or $500 = 4.70x + 350$: $x = 32$; 32 dolls are produced each week.

53. (a) $y \approx 0.541x + 4.072$. The slope, $m \approx 0.541$, represents the average annual increase in hourly compensation for production workers, about \$0.54 per year.

(b) Setting $x = 40$ in the regression equation leads to $y \approx \$25.71$.

54. If the length is x, then the width is $50 - x$, so $A(x) = x(50 - x)$; maximum of 625 ft^2 when $x = 25$ (the dimensions are 25 ft × 25 ft).

55. (a) [0, 100] by [0, 1000] is one possibility.

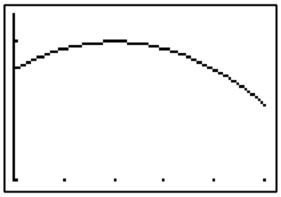
(b) When $x \approx 107.335$ or $x \approx 372.665$ — either 107, 335 units or 372, 665 units.

56. The area of the picture and the frame, if the width of the picture is x ft, is $A(x) = (x + 2)(x + 5)$ ft^2. This equals 208 when $x = 11$, so the painting is 11 ft × 14 ft.

57. If the strip is x feet wide, the area of the strip is $A(x) = (25 + 2x)(40 + 2x) - 1000$ ft^2. This equals 504 ft^2 when $x = 3.5$ ft.

58. (a) $R(x) = (800 + 20x)(300 - 5x)$.

(b) [0, 25] by [200,000, 260,000] is one possibility (shown).

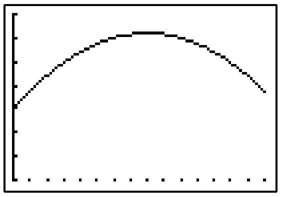


[0, 25] by [200,000, 260,000]

(c) The maximum income — \$250,000 — is achieved when $x = 10$, corresponding to rent of \$250 per month.

59. **(a)** $R(x) = (26{,}000 - 1000x)(0.50 + 0.05x)$.

(b) Many choices of Xmax and Ymin are reasonable. Shown is [0, 15] by [10,000, 17,000].



[0, 15] by [10,000, 17,000]

(c) The maximum revenue — $16,200 — is achieved when $x = 8$; that is, charging 90 cents per can.

60. Total sales would be $S(x) = (30 + x)(50 - x)$ thousand dollars, when x additional salespeople are hired. The maximum occurs when $x = 10$ (halfway between the two zeros, at -30 and 50).

61. **(a)** $g \approx 32$ ft/sec². $s_0 = 83$ ft and $v_0 = 92$ ft/sec. So the models are height $= s(t) = -16t^2 + 92t + 83$ and vertical velocity $= v(t) = -32t + 92$. The maximum height occurs at the vertex of $s(t)$.

$$h = -\frac{b}{2a} = -\frac{92}{2(-16)} = 2.875, \text{ and}$$

$k = s(2.875) = 215.25$. The maximum height of the baseball is about 215 ft above the field.

(b) The amount of time the ball is in the air is a zero of $s(t)$. Using the quadratic formula, we obtain

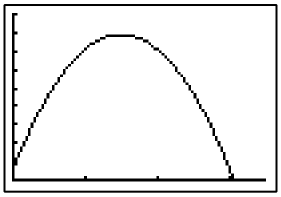
$$t = \frac{-92 \pm \sqrt{92^2 - 4(-16)(83)}}{2(-16)}$$
$$= \frac{-92 \pm \sqrt{13{,}776}}{-32} \approx -0.79 \text{ or } 6.54. \text{ Time is not}$$

negative, so the ball is in the air about 6.54 seconds.

(c) To determine the ball's vertical velocity when it hits the ground, use $v(t) = -32t + 92$, and solve for $t = 6.54$. $v(6.54) = -32(6.54) + 92 \approx -117$ ft/sec when it hits the ground.

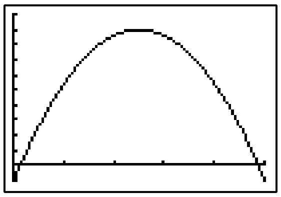
62. **(a)** $h = -16t^2 + 48t + 3.5$.

(b) The graph is shown in the window [0, 3.5] by [0, 45]. The maximum height is 39.5 ft, 1.5 sec after it is thrown.



[0, 3.5] by [0, 45]

63. **(a)** $h = -16t^2 + 80t - 10$. The graph is shown in the window [0, 5] by [−10, 100].

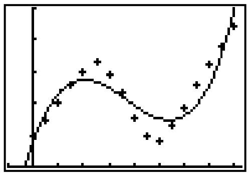


[0, 5] by [−10, 100]

(b) The maximum height is 90 ft, 2.5 sec after it is shot.

64. The exact answer is $32\sqrt{3}$, or about 55.426 ft/sec. In addition to the guess-and-check strategy suggested, this can be found algebraically by noting that the vertex of the parabola $y = ax^2 + bx + c$ has y coordinate $c - \frac{b^2}{4a} = \frac{b^2}{64}$ (note $a = -16$ and $c = 0$), and setting this equal to 48.

65. The quadratic regression is $y \approx 0.449x^2 + 0.934x + 114.658$. Plot this curve together with the curve $y = 450$, and then find the intersection to find when the number of patent applications will reach 450,000. Note that we use $y = 450$ because the data were given as a number of thousands. The intersection occurs at $x \approx 26.3$, so the number of applications will reach 450,000 approximately 26 years after 1980—in 2006.



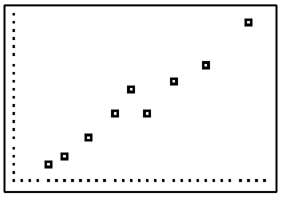
[1, 8.25] by [0, 5]

66. **(a)** $m = \frac{6 \text{ ft}}{100 \text{ ft}} = 0.06$

(b) $r \approx 4167$ ft, or about 0.79 mile.

(c) 2217.6 ft

67. **(a)**

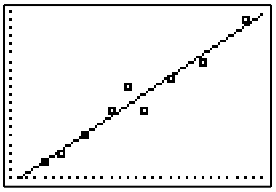


[15, 45] by [20, 40]

(b) $y \approx 0.68x + 9.01$

(c) On average, the children gained 0.68 pound per month.

(d)



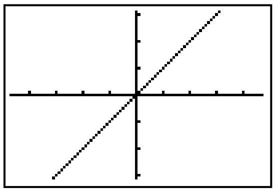
[15, 45] by [20, 40]

(e) ≈ 29.41 lbs

68. **(a)** The linear regression is $y \approx 548.30x + 21{,}027.56$, where x represents the number of years since 1940.

(b) 2010 is 70 years after 1940, so substitute 70 into the equation to predict the median U.S. family income in 2010.
$y = 548.30(70) + 21{,}027.56 \approx \$59{,}400$.

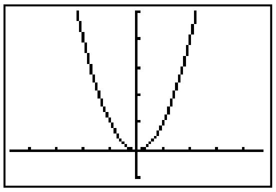
69. The Identity Function $f(x) = x$



[–4.7, 4.7] by [–3.1, 3.1]

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: Increasing for all x
Symmetry: Symmetric about the origin
Boundedness: Not bounded
Local extrema: None
Horizontal asymptotes: None
Vertical asymptotes: None
End behavior: $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

70. The Squaring Function $f(x) = x^2$



[–4.7, 4.7] by [–1, 5]

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: Increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$.
Symmetry: Symmetric about the y-axis
Boundedness: Bounded below, but not above
Local extrema: Local minimum of 0 at $x = 0$
Horizontal asymptotes: None
Vertical asymptotes: None
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$

71. False. For $f(x) = 3x^2 + 2x - 3$, the initial value is $f(0) = -3$.

72. True. By completing the square, we can rewrite $f(x)$ so that $f(x) = \left(x^2 - x + \frac{1}{4}\right) + 1 - \frac{1}{4}$
$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$. Since $f(x) \geq \frac{3}{4}$, $f(x) > 0$ for all x.

73. $m = \frac{1-3}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$. The answer is E.

74. $f(x) = mx + b$

$$3 = -\frac{1}{3}(-2) + b$$

$$3 = \frac{2}{3} + b$$

$$b = 3 - \frac{2}{3} = \frac{7}{3}$$

The answer is C.

For #75–76, $f(x) = 2(x + 3)^2 - 5$ corresponds to $f(x) = a(x - h)^2 + k$ with $a = 2$ and $(h, k) = (-3, -5)$.

75. The axis of symmetry runs vertically through the vertex: $x = -3$. The answer is B.

76. The vertex is $(h, k) = (-3, -5)$. The answer is E.

77. **(a)** Graphs (i), (iii), and (v) are linear functions. They can all be represented by an equation $y = ax + b$, where $a \neq 0$.

(b) In addition to graphs (i), (iii), and (v), graphs (iv) and (vi) are also functions, the difference is that (iv) and (vi) are *constant* functions, represented by $y = b$, $b \neq 0$.

(c) (ii) is not a function because a single value x (i.e., $x = -2$) results in a multiple number of y-values. In fact, there are infinitely many y-values that are valid for the equation $x = -2$.

78. **(a)** $\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4$

(b) $\frac{f(5) - f(2)}{5 - 2} = \frac{25 - 4}{3} = 7$

(c) $\frac{f(c) - f(a)}{c - a} = \frac{c^2 - a^2}{c - a} = \frac{(c - a)(c + a)}{c - a} = c + a$

(d) $\frac{g(3) - g(1)}{3 - 1} = \frac{11 - 5}{2} = 3$

(e) $\frac{g(4) - g(1)}{4 - 1} = \frac{14 - 5}{3} = 3$

(f) $\frac{g(c) - g(a)}{c - a} = \frac{(3c + 2) - (3a + 2)}{c - a}$
$= \frac{3c - 3a}{c - a} = 3$

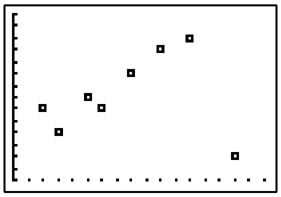
(g) $\frac{h(c) - h(a)}{c - a} = \frac{(7c - 3) - (7a - 3)}{c - a}$
$= \frac{7c - 7a}{c - a} = 7$

(h) $\frac{k(c) - k(a)}{c - a} = \frac{(mc + b) - (ma + b)}{c - a}$
$= \frac{mc - ma}{c - a} = m$

(i) $\frac{l(c) - l(a)}{c - a} = \frac{c^3 - a^3}{c - a} = \frac{-2b}{2a} = \frac{-b}{a} = -\frac{b}{a}$
$= \frac{(c - a)(c^2 + ac + a^2)}{(c - a)} = c^2 + ac + a^2$

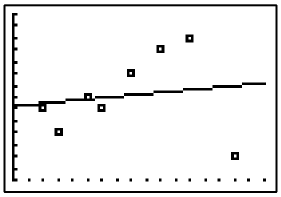
79. Answers will vary. One possibility: When using the least-squares method, mathematicians try to minimize the residual $y_i - (ax_i + b)$, i.e., place the "predicted" y-values as close as possible to the actual y-values. If mathematicians reversed the ordered pairs, the objective would change to minimizing the residual $x_i - (cy_i + d)$, i.e., placing the "predicted" x-values as close as possible to the actual x-values. In order to obtain an exact inverse, the x- and y-values for each xy pair would have to be almost exactly the same distance from the regression line—which is statistically impossible in practice.

80. (a)



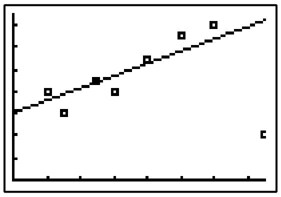
[0, 17] by [2, 16]

(b) $y \approx 0.115x + 8.245$



[0, 17] by [2, 16]

(c) $y \approx 0.556x + 6.093$



[0, 15] by [0, 15]

(d) The median–median line appears to be the better fit, because it approximates more of the data values more closely.

81. (a) If $ax^2 + bx^2 + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ by the quadratic formula. Thus, $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$ and

$$x_1 + x_2 = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a} = -\frac{b}{a}.$$

(b) Similarly,

$$x_1 \cdot x_2 = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}.$$

82. $f(x) = (x - a)(x - b) = x^2 - bx - ax + ab$ $= x^2 + (-a - b)x + ab$. If we use the vertex form of a quadratic function, we have $h = -\left(\frac{-a - b}{2}\right)$ $= \frac{a + b}{2}$. The axis is $x = h = \frac{a + b}{2}$.

83. Multiply out $f(x)$ to get $x^2 - (a + b)x + ab$. Complete the square to get $\left(x - \frac{a + b}{2}\right)^2 + ab - \frac{(a + b)^2}{4}$. The vertex is then (h, k) where $h = \frac{a + b}{2}$ and $k = ab - \frac{(a + b)^2}{4} = -\frac{(a - b)^2}{4}$.

84. x_1 and x_2 are given by the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; then $x_1 + x_2 = -\frac{b}{a}$, and the line of symmetry is $x = -\frac{b}{2a}$, which is exactly equal to $\frac{x_1 + x_2}{2}$.

85. The Constant Rate of Change Theorem states that a function defined on all real numbers is a linear function if and only if it has a constant nonzero average rate of change between any two points on its graph. To prove this, suppose $f(x) = mx + b$ with m and b constants and $m \neq 0$. Let x_1 and x_2 be real numbers with $x_1 \neq x_2$. Then the average rate of change is

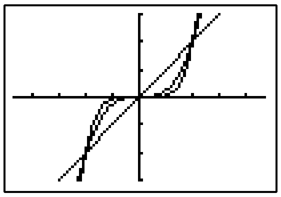
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1} =$$
$$\frac{mx_2 - mx_1}{x_2 - x_1} = \frac{m(x_2 - x_1)}{x_2 - x_1} = m, \text{ a nonzero constant.}$$

Now suppose that m and x_1 are constants, with $m \neq 0$. Let x be a real number such that $x \neq x_1$, and let f be a function defined on all real numbers such that $\frac{f(x) - f(x_1)}{x - x_1} = m$. Then $f(x) - f(x_1) = m(x - x_1) = mx - mx_1$, and $f(x) = mx + (f(x_1) - mx_1)$. $f(x_1) - mx_1$ is a constant; call it b. Then $f(x_1) - mx_1 = b$; so, $f(x_1) = b + mx_1$ and $f(x) = b + mx$ for all $x \neq x_1$. Thus, f is a linear function.

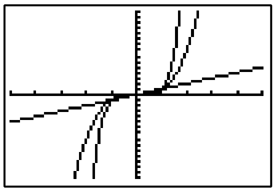
■ Section 2.2 Power Functions with Modeling

Exploration 1

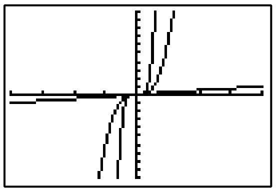
1.



[−2.35, 2.35] by [−1.5, 1.5]



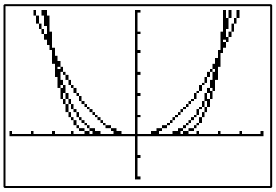
[−5, 5] by [−15, 15]



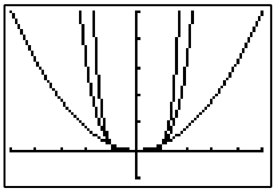
$[-20, 20]$ by $[-200, 200]$

The pairs $(0, 0)$, $(1, 1)$, and $(-1, -1)$ are common to all three graphs. The graphs are similar in that if $x < 0$, $f(x)$, $g(x)$, and $h(x) < 0$ and if $x > 0$, $f(x)$, $g(x)$, and $h(x) > 0$. They are different in that if $|x| < 1$, $f(x)$, $g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1$, $f(x)$, $g(x)$, and $h(x) \to \infty$ at dramatically different rates.

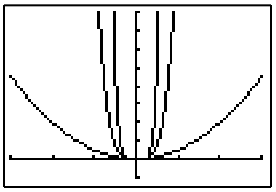
2.



$[-1.5, 1.5]$ by $[-0.5, 1.5]$



$[-5, 5]$ by $[-5, 25]$



$[-15, 15]$ by $[-50, 400]$

The pairs $(0, 0)$, $(1, 1)$, and $(-1, 1)$ are common to all three graphs. The graphs are similar in that for $x \neq 0$, $f(x)$, $g(x)$, and $h(x) > 0$. They are diffferent in that if $|x| < 1$, $f(x)$, $g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1$, $f(x)$, $g(x)$, and $h(x) \to \infty$ at dramatically different rates.

Quick Review 2.2

1. $\sqrt[3]{x^2}$

2. $\sqrt{p^5}$

3. $\frac{1}{d^2}$

4. $\frac{1}{x^7}$

5. $\frac{1}{\sqrt[5]{q^4}}$

6. $\frac{1}{\sqrt{m^3}}$

7. $3x^{3/2}$

8. $2x^{5/3}$

9. $\approx 1.71x^{-4/3}$

10. $\approx 0.71x^{-1/2}$

Section 2.2 Exercises

1. power $= 5$, constant $= -\frac{1}{2}$

2. power $= \frac{5}{3}$, constant $= 9$

3. not a power function

4. power $= 0$, constant $= 13$

5. power $= 1$, constant $= c^2$

6. power $= 5$, constant $= \frac{k}{2}$

7. power $= 2$, constant $= \frac{g}{2}$

8. power $= 3$, constant $= \frac{4\pi}{3}$

9. power $= -2$, constant $= k$

10. power $= 1$, constant $= m$

11. degree $= 0$, coefficient $= -4$

12. not a monomial function; negative exponent

13. degree $= 7$, coefficient $= -6$

14. not a monomial function; variable in exponent

15. degree $= 2$, coefficient $= 4\pi$

16. degree $= 1$, coefficient $= l$

17. $A = ks^2$

18. $V = kr^2$

19. $I = V/R$

20. $V = kT$

21. $E = mc^2$

22. $p = \sqrt{2gd}$

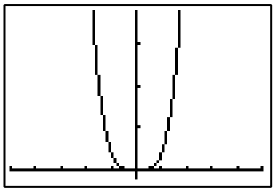
23. The weight w of an object varies directly with its mass m, with the constant of variation g.

24. The circumference C of a circle is proportional to its diameter D, with the constant of variation π.

25. The refractive index n of a medium is inversely proportional to v, the velocity of light in the medium, with constant of variation c, the constant velocity of light in free space.

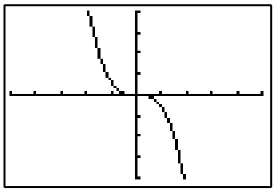
26. The distance d traveled by a free-falling object dropped from rest varies directly with the square of its speed p, with the constant of variation $\frac{1}{2g}$.

27. power $= 4$, constant $= 2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuous
Decreasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Even. Symmetric with respect to y-axis.
Bounded below, but not above
Local minimum at $x = 0$.
Asymptotes: None
End Behavior: $\lim_{x\to-\infty} 2x^4 = \infty, \lim_{x\to\infty} 2x^4 = \infty$



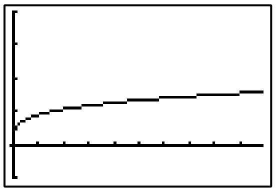
$[-5, 5]$ by $[-1, 49]$

28. power $= 3$, constant $= -3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuous
Decreasing for all x
Odd. Symmetric with respect to origin
Not bounded above or below
No local extrema
Asymptotes: None
End Behavior: $\lim_{x\to-\infty} -3x^3 = \infty, \lim_{x\to\infty} -3x^3 = -\infty$



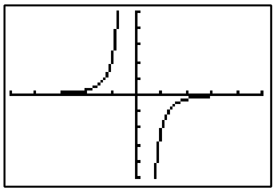
$[-5, 5]$ by $[-20, 20]$

29. power $= \frac{1}{4}$, constant $= \frac{1}{2}$
Domain: $[0, \infty)$
Range: $[0, \infty)$
Continuous
Increasing on $[0, \infty)$
Bounded below
Neither even nor odd
Local minimum at $(0, 0)$
Asymptotes: None
End Behavior: $\lim_{x\to\infty} \frac{1}{2} \sqrt[4]{x} = \infty$



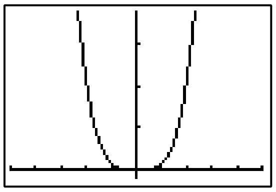
$[-1, 99]$ by $[-1, 4]$

30. power $= -3$, constant $= -2$
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Discontinuous at $x = 0$
Increasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Odd. Symmetric with respect to origin
Not bounded above or below
No local extrema
Asymptotes at $x = 0$ and $y = 0$
End Behavior: $\lim_{x\to-\infty} -2x^{-3} = 0, \lim_{x\to\infty} -2x^{-3} = 0.$



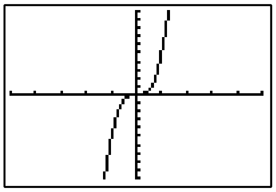
$[-5, 5]$ by $[-5, 5]$

31. Start with $y = x^4$ and shrink vertically by $\frac{2}{3}$. Since $f(-x) = \frac{2}{3}(-x)^4 = \frac{2}{3}x^4$, f is even.



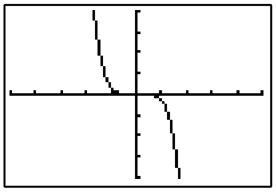
$[-5, 5]$ by $[-1, 19]$

32. Start with $y = x^3$ and stretch vertically by 5. Since $f(-x) = 5(-x)^3 = -5x^3 = -f(x)$, f is odd.



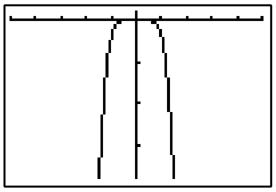
$[-5, 5]$ by $[-20, 20]$

33. Start with $y = x^5$, then stretch vertically by 1.5 and reflect over the x-axis. Since $f(-x) = -1.5(-x)^5 = 1.5x^5$ $= -f(x)$, f is odd.



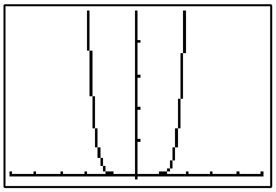
$[-5, 5]$ by $[-20, 20]$

34. Start with $y = x^6$, then stretch vertically by 2 and reflect over the x-axis. Since $f(-x) = -2(-x)^6 = -2x^6$ $= f(x)$, f is even.



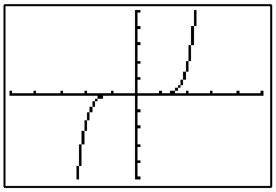
$[-5, 5]$ by $[-19, 1]$

35. Start with $y = x^8$, then shrink vertically by $\frac{1}{4}$. Since $f(-x) = \frac{1}{4}(-x)^8 = \frac{1}{4}x^8 = f(x)$, f is even.



$[-5, 5]$ by $[-1, 49]$

36. Start with $y = x^7$, then shrink vertically by $\frac{1}{8}$. Since $f(-x) = \frac{1}{8}(-x)^7 = -\frac{1}{8}x^7 = -f(x)$, f is odd.



$[-5, 5]$ by $[-50, 50]$

37. (g)

38. (a)

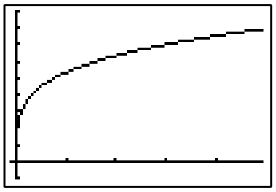
39. (d)

40. (g)

41. (h)

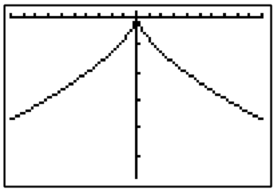
42. (d)

43. $k = 3, a = \frac{1}{4}$. In the first quadrant, the function is increasing and concave down. f is undefined for $x < 0$.



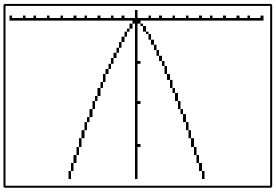
$[-1, 99]$ by $[-1, 10]$

44. $k = -4, a = \frac{2}{3}$. In the fourth quadrant, the function is decreasing and concave up. $f(-x) = -4(\sqrt[3]{(-x)^2}) = -4\sqrt[3]{x^2} = -4x^{2/3} = f(x)$, so f is even.



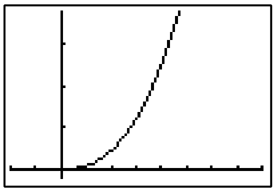
$[-10, 10]$ by $[-29, 1]$

45. $k = -2, a = \frac{4}{3}$. In the fourth quadrant, f is decreasing and concave down. $f(-x) = -2(\sqrt[3]{(-x)^4}) = -2(\sqrt[3]{x^4}) = -2x^{4/3} = f(x)$, so f is even.



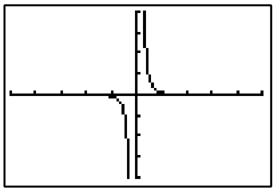
$[-10, 10]$ by $[-29, 1]$

46. $k = \frac{2}{5}, a = \frac{5}{2}$. In the first quadrant, f is increasing and concave up. f is undefined for $x < 0$.



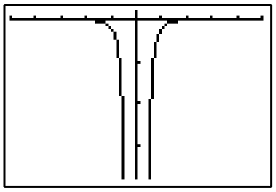
$[-2, 8]$ by $[-1, 19]$

47. $k = \frac{1}{2}, a = -3$. In the first quadrant, f is decreasing and concave up. $f(-x) = \frac{1}{2}(-x)^{-3} = \frac{1}{2(-x)^3} = -\frac{1}{2}x^{-3} = -f(x)$, so f is odd.



$[-5, 5]$ by $[-20, 20]$

48. $k = -1, a = -4$. In the fourth quadrant, f is increasing and concave down. $f(-x) = -(-x)^{-4} = -\frac{1}{(-x)^4} = -\frac{1}{x^4} = -x^{-4} = f(x)$, so f is even.



$[-5, 5]$ by $[-19, 1]$

49. $y = \frac{8}{x^2}$, power $= -2$, constant $= 8$

50. $y = -2\sqrt{x}$, power $= \frac{1}{2}$, constant $= -2$

51. $V = \dfrac{kT}{P}$, so $k = \dfrac{PV}{T} = \dfrac{(0.926 \text{ atm})(3.46 \text{ L})}{302°\text{K}}$
$= 0.0106 \dfrac{\text{atm–L}}{\text{K}}$

At $P = 1.452$ atm, $V = \dfrac{\left(\dfrac{0.0106 \text{ atm–L}}{\text{K}}\right)(302°\text{K})}{1.452 \text{ atm}}$
$= 2.21$ L

52. $V = kPT$, so $k = \dfrac{V}{PT} = \dfrac{(3.46 \text{ L})}{(0.926 \text{ atm})(302°\text{K})}$
$= 0.0124 \dfrac{\text{L}}{\text{atm–K}}$

At $T = 338°\text{K}$, $V = \left(0.0124 \dfrac{\text{L}}{\text{atm–K}}\right)(0.926 \text{ atm})$
$(338°\text{K}) = 3.87$ L

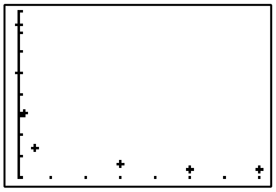
53. $n = \dfrac{c}{v}$, so $v = \dfrac{c}{n} = \dfrac{\left(\dfrac{3.00 \times 10^8 \text{ m}}{\text{sec}}\right)}{2.42} = 1.24 \times 10^8 \dfrac{\text{m}}{\text{sec}}$

54. $P = kv^3$, so $k = \dfrac{P}{v^3} = \dfrac{15 \text{ w}}{(10 \text{ mph})^3} = 1.5 \times 10^{-2}$

Wind Speed (mph)	Power (W)
10	15
20	120
40	960
80	7680

Since $P = kv^3$ is a cubic, power will increase significantly with only a small increase in wind speed.

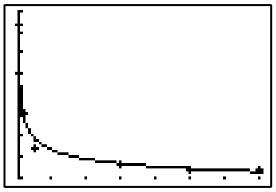
55. (a)



[−2, 71] by [50, 450]

(b) $r \approx 231.204 \cdot w^{-0.297}$

(c)

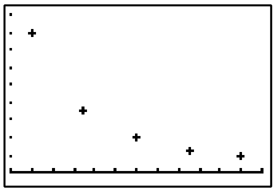


[−2, 71] by [50, 450]

(d) Approximately 37.67 beats/min, which is very close to Clark's observed value.

56. Given that n is an integer, $n \geq 1$:
If n is odd, then $f(-x) = (-x)^n = -(x^n) = -f(x)$ and so $f(x)$ is odd.
If n is even, then $f(-x) = (-x)^n = x^n = f(x)$ and so $f(x)$ is even.

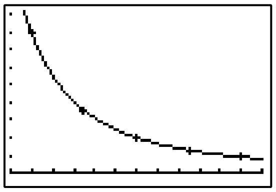
57. (a)



[0.8, 3.2] by [−0.3, 9.2]

(b) $y \approx 7.932 \cdot x^{-1.987}$; yes

(c)



[0.8, 3.2] by [−0.3, 9.2]

(d) Approximately $2.76 \dfrac{\text{W}}{\text{m}^2}$ and $0.697 \dfrac{\text{W}}{\text{m}^2}$, respectively.

58. True, because $f(-x) = (-x)^{-2/3} = [(-x)^2]^{-1/3}$
$= (x^2)^{-1/3} = x^{-2/3} = f(x)$.

59. False. $f(-x) = (-x)^{1/3} = -(x^{1/3}) = -f(x)$ and so the function is odd. It is symmetric about the origin, not the y-axis.

60. $f(4) = 2(4)^{-1/2} = \dfrac{2}{4^{1/2}} = \dfrac{2}{\sqrt{4}} = \dfrac{2}{2} = 1$.

The answer is A.

61. $f(0) = -3(0)^{-1/3} = -3 \cdot \dfrac{1}{0^{1/3}} = -3 \cdot \dfrac{1}{0}$ is undefined.

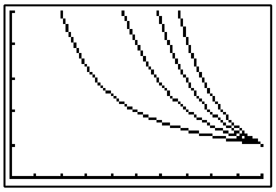
Also, $f(-1) = -3(-1)^{-1/3} = -3(-1) = 3$,
$f(1) = -3(1)^{-1/3} = -3(1) = -3$, and
$f(3) = -3(3)^{-1/3} \approx -2.08$. The answer is E.

62. $f(-x) = (-x)^{2/3} = [(-x)^2]^{1/3} = (x^2)^{1/3} = x^{2/3} = f(x)$
The function is even. The answer is B.

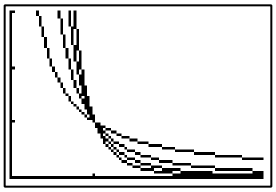
63. $f(x) = x^{3/2} = (x^{1/2})^3 = (\sqrt{x})^3$ is defined for $x \geq 0$.
The answer is B.

64. Answers will vary. In general, however, students will find
n even: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$, so f is undefined for $x < 0$.
m even, n odd: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = k \cdot \sqrt[n]{x^m} = f(x)$,
so f is even.
m odd, n odd: $f(x) = k \cdot x^{m/n} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = -k \cdot \sqrt[n]{x^m}$
$= -k \cdot x^{m/n} = -f(x)$, so f is odd.

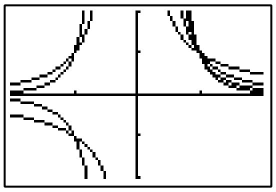
65. (a)



[0, 1] by [0, 5]



[0, 3] by [0, 3]

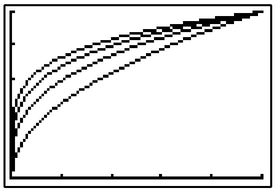


[−2, 2] by [−2, 2]

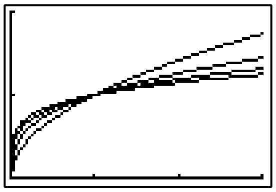
The graphs of $f(x) = x^{-1}$ and $h(x) = x^{-3}$ are similar and appear in the 1st and 3rd quadrants only. The graphs of $g(x) = x^{-2}$ and $k(x) = x^{-4}$ are similar and appear in the 1st and 2nd quadrants only. The pair (1, 1) is common to all four functions.

	f	g	h	k
Domain	$x \neq 0$	$x \neq 0$	$x \neq 0$	$x \neq 0$
Range	$y \neq 0$	$y > 0$	$y \neq 0$	$y > 0$
Continuous	yes	yes	yes	yes
Increasing		$(-\infty, 0)$		$(-\infty, 0)$
Decreasing	$(-\infty, 0), (0, \infty)$	$(0, \infty)$	$(-\infty, 0), (0, \infty)$	$(0, \infty)$
Symmetry	w.r.t. origin	w.r.t. y-axis	w.r.t. origin	w.r.t. y-axis
Bounded	not	below	not	below
Extrema	none	none	none	none
Asymptotes	x-axis, y-axis	x-axis, y-axis	x-axis, y-axis	x-axis, y-axis
End Behavior	$\lim_{x \to \pm\infty} f(x) = 0$	$\lim_{x \to \pm\infty} g(x) = 0$	$\lim_{x \to \pm\infty} h(x) = 0$	$\lim_{x \to \pm\infty} k(x) = 0$

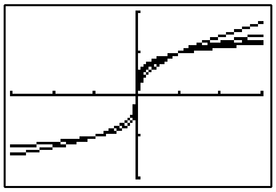
(b)



[0, 1] by [0, 1]



[0, 3] by [0, 2]

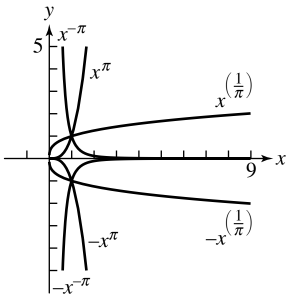


[−3, 3] by [−2, 2]

The graphs of $f(x) = x^{1/2}$ and $h(x) = x^{1/4}$ are similar and appear in the 1st quadrant only. The graphs of $g(x) = x^{1/3}$ and $k(x) = x^{1/5}$ are similar and appear in the 1st and 3rd quadrants only. The pairs (0, 0), (1, 1) are common to all four functions.

	f	g	h	k
Domain	$[0, \infty)$	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$
Range	$y \geq 0$	$(-\infty, \infty)$	$y \geq 0$	$(-\infty, \infty)$
Continuous	yes	yes	yes	yes
Increasing	$[0, \infty)$	$(-\infty, \infty)$	$[0, \infty)$	$(-\infty, \infty)$
Decreasing				
Symmetry	none	w.r.t. origin	none	w.r.t. origin
Bounded	below	not	below	not
Extrema	min at (0, 0)	none	min at (0, 0)	none
Asymptotes	none	none	none	none
End behavior	$\lim_{x \to \infty} f(x) = \infty$	$\lim_{x \to \infty} g(x) = \infty$ $\lim_{x \to -\infty} g(x) = -\infty$	$\lim_{x \to \infty} h(x) = \infty$	$\lim_{x \to \infty} k(x) = \infty$ $\lim_{x \to -\infty} k(x) = -\infty$

66.



The graphs look like those shown in Figure 2.14 on page 192.

$f(x) = x^{\pi}$ looks like the red graph in Figure 2.14(a) because $k = 1 > 0$ and $a = \pi > 1$.

$f(x) = x^{1/\pi}$ looks like the blue graph in Figure 2.14(a) because $k = 1 > 0$ and $0 < a = 1/\pi < 1$.

$f(x) = x^{-\pi}$ looks like the green graph in Figure 2.14(a) because $k = 1 < 0$ and $a = -\pi < 0$.

$f(x) = -x^{\pi}$ looks like the red graph in Figure 2.14(b) because $k = -1 < 0$ and $a = \pi > 1$.

$f(x) = -x^{1/\pi}$ looks like the blue graph in Figure 2.14(b) because $k = -1 < 0$ and $a = -\pi < 0$.

$f(x) = -x^{-\pi}$ looks like the green graph in Figure 2.14(b) because $k = -1 < 0$ and $a = -\pi < 0$.

67. Our new table looks like:

Table 2.10 (revised) Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Au)	Period of Orbit (yrs)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Using these new data, we find a power function model of: $y \approx 0.99995 \cdot x^{1.50115} \approx x^{1.5}$. Since y represents years, we set $y = T$ and since x represents distance, we set $x = a$ then, $y = x^{1.5} \Rightarrow T = a^{3/2} \Rightarrow (T)^2 = (a^{3/2})^2 \Rightarrow T^2 = a^3$.

68. Using the free-fall equations in Section 2.1, we know that $s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$ and that $v(t) = -gt + v_0$. If $t = 0$ is the time at which the object is dropped, then $v_0 = 0$. So $d = s_0 - s = s_0 - \left(-\frac{1}{2}gt^2 + s_0\right) = \frac{1}{2}gt^2$ and $p = |v| = |-gt|$. Solving $d = \frac{1}{2}gt^2$ for t, we have $t = \sqrt{\frac{2d}{g}}$. Then $p = \left|-g\sqrt{\frac{2d}{g}}\right| = \sqrt{\frac{2dg^2}{g}} = \sqrt{2dg}$.
The results of Example 6 approximate this formula.

69. If f is even,

$f(x) = f(-x)$, so $\frac{1}{f(x)} = \frac{1}{f(-x)}$, $(f(x) \neq 0)$.

Since $g(x) = \frac{1}{f(x)} = \frac{1}{f(-x)} = g(-x)$, g is also even.

If g is even,

$g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = g(x) = \frac{1}{f(x)}$.

Since $\frac{1}{f(-x)} = \frac{1}{f(x)}$, $f(-x) = f(x)$, and f is even.

If f is odd,

$f(x) = -f(x)$, so $\frac{1}{f(x)} = -\frac{1}{f(x)}$, $f(x) \neq 0$.

Since $g(x) = \frac{1}{f(x)} = -\frac{1}{f(x)} = -g(x)$, g is also odd.

If g is odd,

$g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = -g(x) = -\frac{1}{f(x)}$.

Since $\frac{1}{f(-x)} = -\frac{1}{f(x)}$, $f(-x) = -f(x)$, and f is odd.

70. Let $g(x) = x^{-a}$ and $f(x) = x^a$. Then $g(x) = 1/x^a = 1/f(x)$. Exercise 69 shows that $g(x) = 1/f(x)$ is even if and only if $f(x)$ is even, and $g(x) = 1/f(x)$ is odd if and only if $f(x)$ is odd. Therefore, $g(x) = x^{-a}$ is even if and only if $f(x) = x^a$ is even, and that $g(x) = x^{-a}$ is odd if and only if $f(x) = x^a$ is odd.

71. **(a)** The force F acting on an object varies jointly as the mass m of the object and the acceleration a of the object.

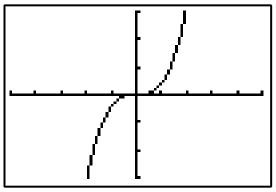
(b) The kinetic energy KE of an object varies jointly as the mass m of the object and the square of the velocity v of the object.

(c) The force of gravity F acting on two objects varies jointly as their masses m_1 and m_2 and inversely as the square of the distance r between their centers, with the constant of variation G, the universal gravitational constant.

■ Section 2.3 Polynomial Functions of Higher Degree with Modeling

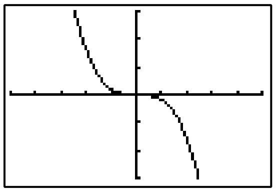
Exploration 1

1. (a) $\lim_{x\to\infty} 2x^3 = \infty$, $\lim_{x\to-\infty} 2x^3 = -\infty$



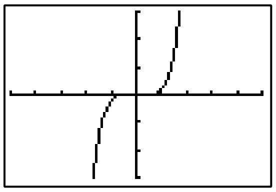
$[-5, 5]$ by $[-15, 15]$

(b) $\lim_{x\to\infty}(-x^3) = -\infty$, $\lim_{x\to-\infty}(-x^3) = \infty$



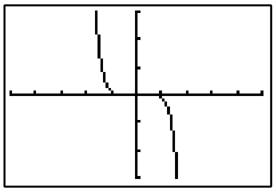
$[-5, 5]$ by $[-15, 15]$

(c) $\lim_{x\to\infty} x^5 = \infty$, $\lim_{x\to-\infty} x^5 = -\infty$



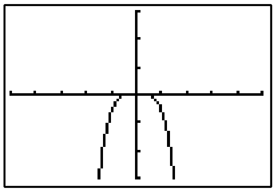
$[-5, 5]$ by $[-15, 15]$

(d) $\lim_{x\to\infty}(-0.5x^7) = -\infty$, $\lim_{x\to-\infty}(-0.5x^7) = \infty$



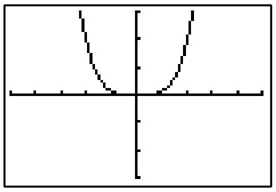
$[-5, 5]$ by $[-15, 15]$

2. (a) $\lim_{x\to\infty}(-3x^4) = -\infty$, $\lim_{x\to-\infty}(-3x^4) = -\infty$



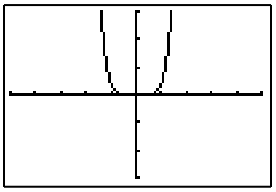
$[-5, 5]$ by $[-15, 15]$

(b) $\lim_{x\to\infty} 0.6x^4 = \infty$, $\lim_{x\to-\infty} 0.6x^4 = \infty$



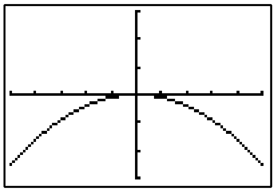
$[-5, 5]$ by $[-15, 15]$

(c) $\lim_{x\to\infty} 2x^6 = \infty$, $\lim_{x\to-\infty} 2x^6 = \infty$



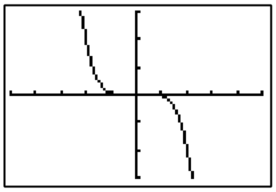
$[-5, 5]$ by $[-15, 15]$

(d) $\lim_{x\to\infty}(-0.5x^2) = -\infty$, $\lim_{x\to-\infty}(-0.5x^2) = -\infty$



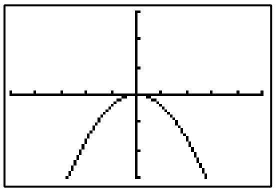
$[-5, 5]$ by $[-15, 15]$

3. (a) $\lim_{x\to\infty}(-0.3x^5) = -\infty$, $\lim_{x\to-\infty}(-0.3x^5) = \infty$



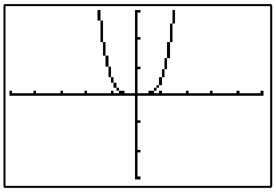
$[-5, 5]$ by $[-15, 15]$

(b) $\lim_{x\to\infty}(-2x^2) = -\infty$, $\lim_{x\to-\infty}(-2x^2) = -\infty$



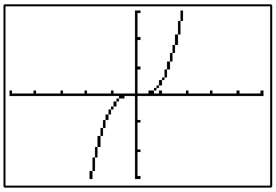
$[-5, 5]$ by $[-15, 15]$

(c) $\lim_{x\to\infty} 3x^4 = \infty$, $\lim_{x\to-\infty} 3x^4 = \infty$



$[-5, 5]$ by $[-15, 15]$

(d) $\lim_{x\to\infty} 2.5x^3 = \infty$, $\lim_{x\to-\infty} 2.5x^3 = -\infty$

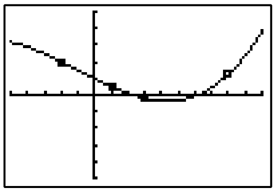


$[-5, 5]$ by $[-15, 15]$

If $a_n > 0$ and $n > 0$, $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. If $a_n < 0$ and $n > 0$, $\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to-\infty} f(x) = \infty$.

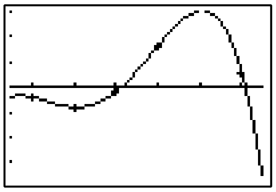
Exploration 2

1. $y = 0.0061x^3 + 0.0177x^2 - 0.5007x + 0.9769$ It is an exact fit, which we expect with only 4 data points!



[−5, 10] by [−5, 5]

2. $y = -0.375x^4 + 6.917x^3 - 44.125x^2 + 116.583x - 111$ It is an exact fit, exactly what we expect with only 5 data points!



[2.5, 8.5] by [−18, 15]

Quick Review 2.3

1. $(x - 4)(x + 3)$

2. $(x - 7)(x - 4)$

3. $(3x - 2)(x - 3)$

4. $(2x - 1)(3x - 1)$

5. $x(3x - 2)(x - 1)$

6. $2x(3x - 2)(x - 3)$

7. $x = 0, x = 1$

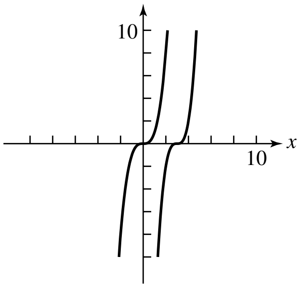
8. $x = 0, x = -2, x = 5$

9. $x = -6, x = -3, x = 1.5$

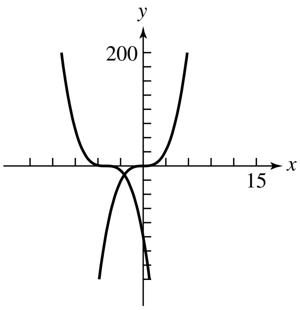
10. $x = -6, x = -4, x = 5$

Section 2.3 Exercises

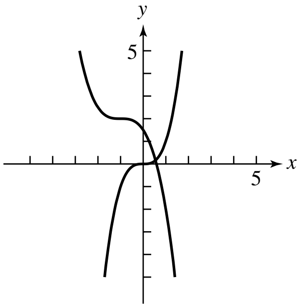
1. Start with $y = x^3$, shift to the right by 3 units, and then stretch vertically by 2. y-intercept: $(0, -54)$



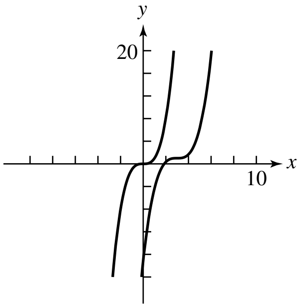
2. Start with $y = x^3$, shift to the left by 5 units, and then reflect over the x-axis. y-intercept: $(0, -125)$



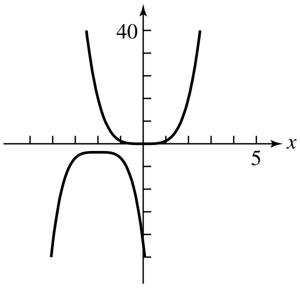
3. Start with $y = x^3$, shift to the left by 1 unit, vertically shrink by $\frac{1}{2}$, reflect over the x-axis, and then vertically shift up 2 units. y-intercept: $\left(0, \frac{3}{2}\right)$



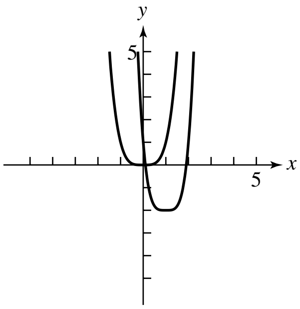
4. Start with $y = x^3$, shift to the right by 3 units, vertically shrink by $\frac{2}{3}$, and vertically shift up 1 unit. y-intercept: $(0, -17)$



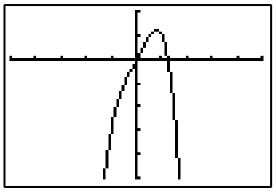
5. Start with $y = x^4$, shift to the left 2 units, vertically stretch by 2, reflect over the x-axis, and vertically shift down 3 units. y-intercept: $(0, -35)$



6. Start with $y = x^4$, shift to the right 1 unit, vertically stretch by 3, and vertically shift down 2 units. y-intercept: $(0, 1)$

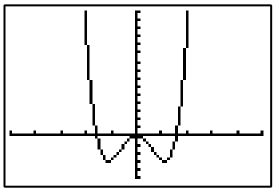


7. local maximum: $\approx (0.79, 1.19)$, zeros: $x = 0$ and $x \approx 1.26$. The general shape of f is like $y = -x^4$, but near the origin, f behaves a lot like its other term, $2x$. f is neither even nor odd.



$[-5, 5]$ by $[-5, 2]$

8. local maximum at $(0, 0)$ and local minima at $(1.12, -3.13)$ and $(-1.12, -3.13)$, zeros: $x = 0$, $x \approx 1.58$, $x \approx -1.58$. f behaves a lot like $y = 2x^4$ except in the interval $[-1.58, 1.58]$, where it behaves more like its second building block term, $-5x^2$.



$[-5, 5]$ by $[-5, 15]$

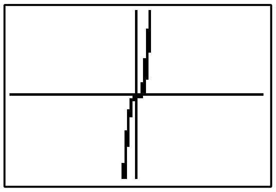
9. Cubic function, positive leading coefficient. The answer is (c).

10. Cubic function, negative leading coefficient. The answer is (b).

11. Higher than cubic, positive leading coefficient. The answer is (a).

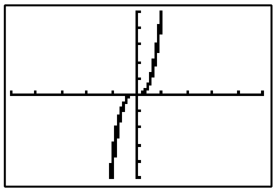
12. Higher than cubic, negative leading coefficient. The answer is (d).

13. One possibility:



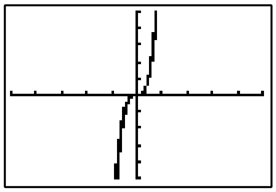
$[-100, 100]$ by $[-1000, 1000]$

14. One possibility:



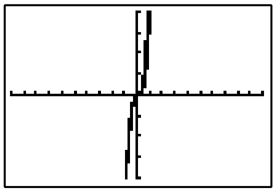
$[-50, 50]$ by $[-1000, 1000]$

15. One possibility:



$[-50, 50]$ by $[-1000, 1000]$

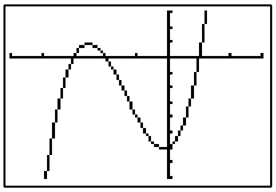
16. One possibility:



$[-100, 100]$ by $[-2000, 2000]$

For #17–24, when one end of a polynomial function's graph curves up into Quadrant I or II, this indicates a limit at ∞. And when an end curves down into Quadrant III or IV, this indicates a limit at $-\infty$.

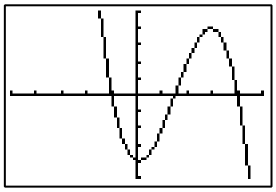
17.



$[-5, 3]$ by $[-8, 3]$

$$\lim_{x\to\infty} f(x) = \infty$$
$$\lim_{x\to-\infty} f(x) = -\infty$$

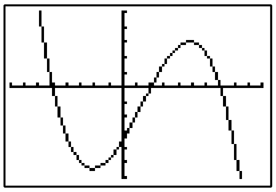
18.



$[-5, 5]$ by $[-15, 15]$

$$\lim_{x\to\infty} f(x) = -\infty$$
$$\lim_{x\to-\infty} f(x) = \infty$$

19.

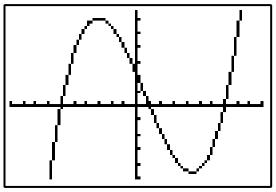


[−8, 10] by [−120, 100]

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

20.

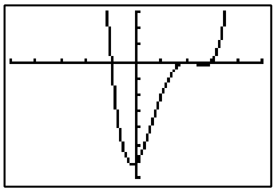


[−10, 10] by [−100, 130]

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

21.

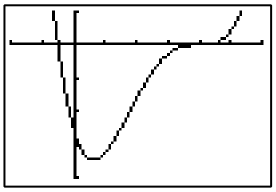


[−5, 5] by [−14, 6]

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

22.

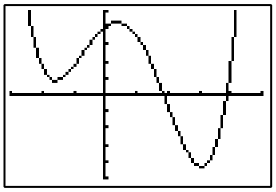


[−2, 6] by [−100, 25]

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

23.

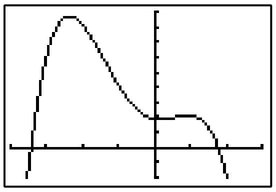


[−3, 5] by [−50, 50]

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

24.



[−4, 3] by [−20, 90]

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

For #25–28, the end behavior of a polynomial is governed by the highest-degree term.

25. $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = \infty$

26. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$

27. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$

28. $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

29. (a); There are 3 zeros: they are −2.5, 1, and 1.1.

30. (b); There are 3 zeros: they are 0.4, approximately 0.429 (actually 3/7), and 3.

31. (c); There are 3 zeros: approximately −0.273 (actually −3/11), −0.25, and 1.

32. (d); There are 3 zeros: −2, 0.5, and 3.

For #33–35, factor or apply the quadratic formula.

33. −4 and 2

34. −2 and 2/3

35. 2/3 and −1/3

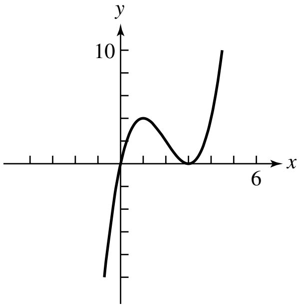
For #36–38, factor out x, then factor or apply the quadratic formula.

36. 0, −5, and 5

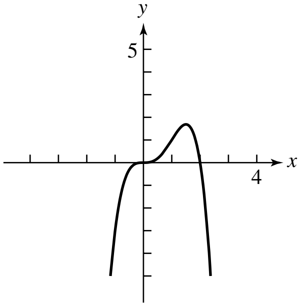
37. 0, −2/3, and 1

38. 0, −1, and 2

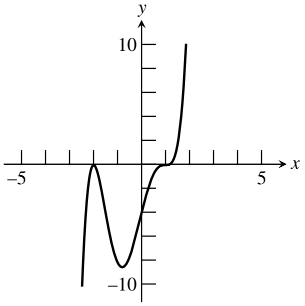
39. Degree 3; zeros: $x = 0$ (multiplicity 1, graph crosses x-axis), $x = 3$ (multiplicity 2, graph is tangent)



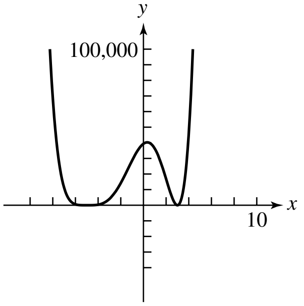
40. Degree 4; zeros: $x = 0$ (multiplicity 3, graph crosses x-axis), $x = 2$ (multiplicity 1, graph crosses x-axis)



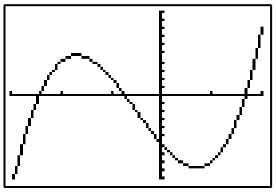
41. Degree 5; zeros: $x = 1$ (multiplicity 3, graph crosses x-axis), $x = -2$ (multiplicity 2, graph is tangent)



42. Degree 6; zeros: $x = 3$ (multiplicity 2, graph is tangent), $x = -5$ (multiplicity 4, graph is tangent)

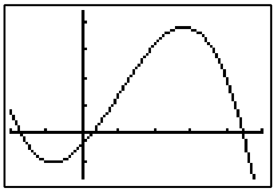


43. Zeros: −2.43, −0.74, 1.67



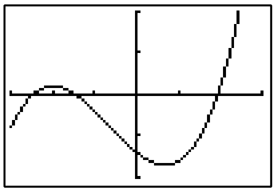
[−3, 2] by [−10, 10]

44. Zeros: −1.73, 0.26, 4.47



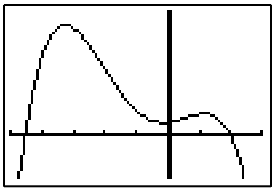
[−2, 5] by [−8, 22]

45. Zeros: −2.47, −1.46, 1.94



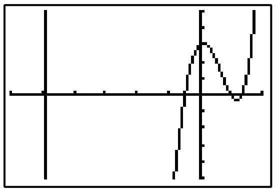
[−3, 3] by [−10, 10]

46. Zeros: −4.53, 2



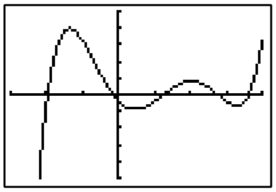
[−5, 3] by [−20, 90]

47. Zeros: −4.90, −0.45, 1, 1.35



[−6, 2] by [−5, 5]

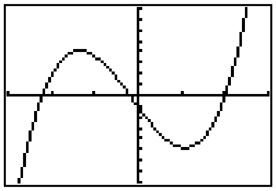
48. Zeros: −1.98, −0.16, 1.25, 2.77, 3.62



[−3, 4] by [−100, 100]

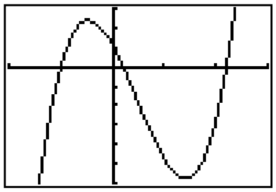
49. 0, −6, and 6. Algebraically — factor out x first.

50. −11, −1, and 10. Graphically. Cubic equations *can* be solved algebraically, but methods of doing so are more complicated than the quadratic formula.



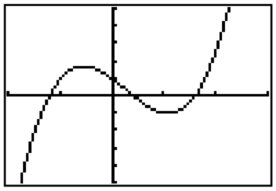
[–15, 15] by [–800, 800]

51. −5, 1, and 11. Graphically.



[–10, 15] by [–300, 150]

52. −6, 2, and 8. Graphically.



[–10, 15] by [–500, 500]

For #53–56, the "minimal" polynomials are given; any constant (or any other polynomial) can be multiplied by the answer given to give another answer.

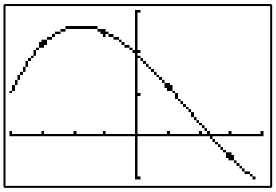
53. $f(x) = (x - 3)(x + 4)(x - 6)$
$= x^3 - 5x^2 - 18x + 72$

54. $f(x) = (x + 2)(x - 3)(x + 5) = x^3 + 4x^2 - 11x - 30$

55. $f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4)$
$= (x^2 - 3)(x - 4) = x^3 - 4x^2 - 3x + 12$

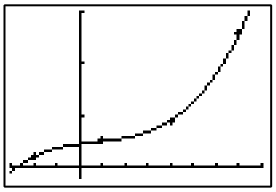
56. $f(x) = (x - 1)(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$
$= (x - 1)[(x - 1)^2 - 2] = x^3 - 3x^2 + x + 1$

57. $y = 0.25x^3 - 1.25x^2 - 6.75x + 19.75$



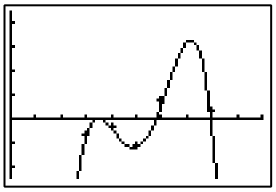
$[-4, 4]$ by $[-10, 30]$

58. $y = 0.074x^3 - 0.167x^2 + 0.611x + 4.48$



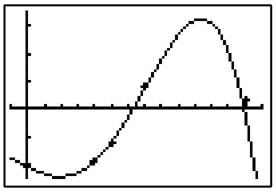
$[-3, 8]$ by $[-2, 30]$

59. $y = -2.21x^4 + 45.75x^3 - 339.79x^2 + 1075.25x - 1231$



$[0, 10]$ by $[-25, 45]$

60. $y = -0.017x^4 + 0.226x^3 + 0.289x^2 - 3.202x - 21$

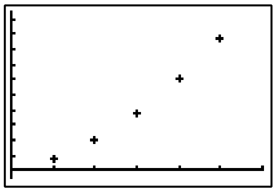


$[-1, 14]$ by $[-25, 35]$

61. $f(x) = x^7 + x + 100$ has an odd-degree leading term, which means that in its end behavior it will go toward $-\infty$ at one end and toward ∞ at the other. Thus the graph must cross the x-axis at least once. That is to say, $f(x)$ takes on both positive and negative values, and thus by the Intermediate Value Theorem, $f(x) = 0$ for some x.

62. $f(x) = x^9 - x + 50$ has an odd-degree leading term, which means that in its end behavior it will go toward $-\infty$ at one end and toward ∞ at the other. Thus the graph must cross the x-axis at least once. That is to say, $f(x)$ takes on both positive and negative values, and thus by the Intermediate Value Theorem, $f(x) = 0$ for some x.

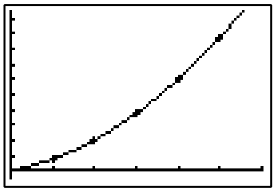
63. **(a)**



$[0, 60]$ by $[-10, 210]$

(b) $y = 0.051x^2 + 0.97x + 0.26$

(c)



$[0, 60]$ by $[-10, 210]$

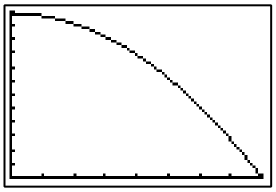
(d) $y(25) \approx 56.39$ ft

(e) Using the quadratic equation to solve $0 = 0.051x^2 + 0.97x + (0.26 - 300)$, we find two answers: $x = 67.74$ mph and $x = -86.76$ mph. Clearly the negative value is extraneous.

64. **(a)** $P(x) = R(x) - C(x)$ is positive if $29.73 < x < 541.74$ (approx.), so they need between 30 and 541 customers.

(b) $P(x) = 60000$ when $x = 200.49$ or $x = 429.73$. Either 201 or 429 customers gives a profit slightly over \$60,000; 200 or 430 customers both yield slightly less than \$60,000.

65. **(a)**

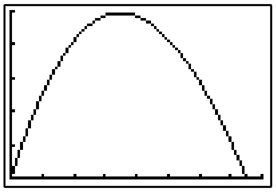


$[0, 0.8]$ by $[0, 1.20]$

(b) 0.3391 cm from the center of the artery

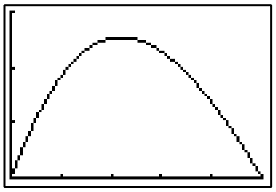
66. **(a)** The height of the box will be x, the width will be $15 - 2x$, and the length $60 - 2x$.

(b) Any value of x between approximately 0.550 and 6.786 inches.



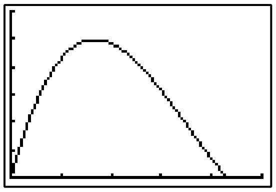
$[0, 8]$ by $[0, 1500]$

67. The volume is $V(x) = x(10 - 2x)(25 - 2x)$; use any x with $0 < x \leq 0.929$ or $3.644 \leq x < 5$.



$[0, 5]$ by $[0, 300]$

68. Determine where the function is positive: $0 < x < 21.5$. (The side lengths of the rectangle are 43 and 62 units.)



$[0, 25]$ by $[0, 12{,}000]$

69. True. Because f is continuous and $f(1) = (1)^3 - (1)^2 - 2 = -2 < 0$ while $f(2) = (2)^3 - (2)^2 - 2 = 2 > 0$, the Intermediate Value Theorem assures us that the graph of f crosses the x-axis ($f(x) = 0$) somewhere between $x = 1$ and $x = 2$.

70. False. If $a > 0$, the graph of $f(x) = (x + a)^2$ is obtained by translating the graph of $f(x) = x^2$ to the left by a units. Translation to the right corresponds to $a < 0$.

71. When $x = 0$, $f(x) = 2(x - 1)^3 + 5 = 2(-1)^3 + 5 = 3$. The answer is C.

72. In $f(x) = (x - 2)^2(x + 2)^3(x + 3)^7$, the factor $x - 2$ occurs twice. So $x = 2$ is a zero of multiplicity 2, and the answer is B.

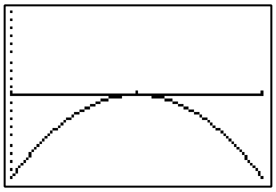
73. The graph indicates three zeros, each of multiplicity 1: $x = -2$, $x = 0$, and $x = 2$. The end behavior indicates a negative leading coefficient. So $f(x) = -x(x + 2)(x - 2)$, and the answer is B.

74. The graph indicates four zeros: $x = -2$ (multiplicity 2), $x = 0$ (multiplicity 1) and $x = 2$ (multiplicity 2). The end behavior indicates a positive leading coefficient. So $f(x) = x(x + 2)^2(x - 2)$, and the answer is A.

75. The first view shows the end behavior of the function, but obscures the fact that there are two local maxima and a local minimum (and 4 x-axis intersections) between -3 and 4. These are visible in the second view, but missing is the minimum near $x = 7$ and the x-axis intersection near $x = 9$. The second view suggests a degree 4 polynomial rather than degree 5.

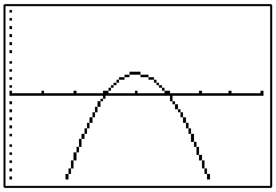
76. The end behavior is visible in the first window, but not the details of the behavior near $x = 1$. The second view shows those details, but at the loss of the end behavior.

77. The exact behavior near $x = 1$ is hard to see. A zoomed–in view around the point $(1, 0)$ suggests that the graph just touches the x-axis at 0 without actually crossing it — that is, $(1, 0)$ is a local maximum. One possible window is $[0.9999, 1.0001]$ by $[-1 \times 10^{-7}, 1 \times 10^{-7}]$.



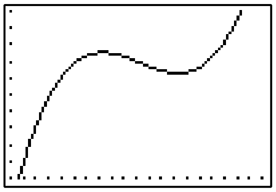
$[0.9999, 1.0001]$ by $[-1 \times 10^{-7}, 1 \times 10^{-7}]$

78. This also has a maximum near $x = 1$ — but this time a window such as $[0.6, 1.4]$ by $[-0.1, 0.1]$ reveals that the graph actually rises above the x-axis and has a maximum at $(0.999, 0.025)$.



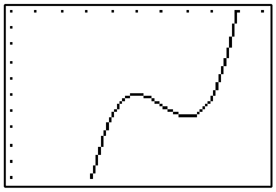
$[0.6, 1.4]$ by $[-0.1, 0.1]$

79. A maximum and minimum are not visible in the standard window, but can be seen on the window $[0.2, 0.4]$ by $[5.29, 5.3]$.



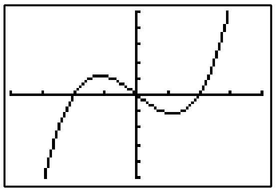
$[0.2, 0.4]$ by $[5.29, 5.30]$

80. A maximum and minimum are not visible in the standard window, but can be seen on the window $[0.95, 1.05]$ by $[-6.0005, -5.9995]$.



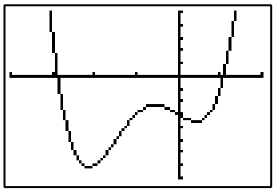
$[0.95, 1.05]$ by $[-6.0005, -5.9995]$

81. The graph of $y = 3(x^3 - x)$ (shown on the window $[-2, 2]$ by $[-5, 5]$) increases, then decreases, then increases; the graph of $y = x^3$ only increases. Therefore, this graph cannot be obtained from the graph of $y = x^3$ by the transformations studied in Chapter 1 (translations, reflections, and stretching/shrinking). Since the right side includes only these transformations, there can be no solution.



$[-2, 2]$ by $[-5, 5]$

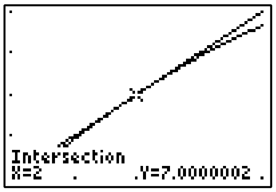
82. The graph of $y = x^4$ has a "flat bottom," while the graph of $y = x^4 + 3x^3 - 2x - 3$ (shown on $[-4, 2]$ by $[-8, 5]$) is "bumpy." Therefore, this graph cannot be obtained from the graph of $y = x^4$ through the transformations of Chapter 1. Since the right side includes only these transformations, there can be no solution.



$[-4, 2]$ by $[-8, 5]$

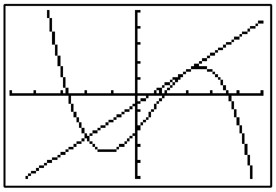
83. (a) Substituting $x = 2$, $y = 7$, we find that $7 = 5(2 - 2) + 7$, so Q is on line L, and also $f(2) = -8 + 8 + 18 - 11 = 7$, so Q is on the graph of $f(x)$.

(b) Window $[1.8, 2.2]$ by $[6, 8]$. Calculator output will not show the detail seen here.



$[1.8, 2.2]$ by $[6, 8]$

(c) The line L also crosses the graph of $f(x)$ at $(-2, -13)$.

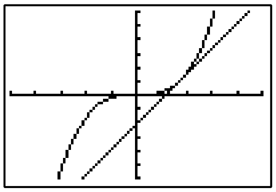


$[-5, 5]$ by $[-25, 25]$

84. (a) Note that $f(a) = a^n$ and $f(-a) = -a^n$; $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-a^n - a^n}{-a - a} = \frac{-2a^n}{-2a} = a^{n-1}$.

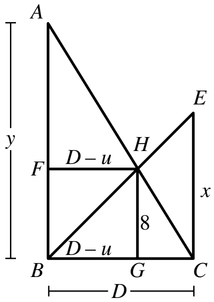
(b) First observe that $f(x_0) = (a^{1/(n-1)})^n = a^{n/(n-1)}$. Using point–slope form: $y - a^{n/(n-1)} = a^{n-1}(x - a^{1/(n-1)})$.

(c) With $n = 3$ and $a = 3$, this equation becomes $y - 3^{3/2} = 3^2(x - 3^{1/2})$, or $y = 9(x - \sqrt{3}) + 3\sqrt{3}$ $= 9x - 6\sqrt{3}$. So $y = x^3$.



$[-5, 5]$ by $[-30, 30]$

85. (a) Label the points of the diagram as shown, adding the horizontal segment $\overline{FH}$. Therefore, ΔECB is similar (in the geometric sense) to ΔHGB, and also ΔABC is similar to ΔAFH. Therefore: $\frac{HG}{EC} = \frac{BG}{BC}$, or $\frac{8}{x} = \frac{D - u}{D}$, and also $\frac{AF}{AB} = \frac{FH}{BC}$, or $\frac{y - 8}{y} = \frac{D - u}{D}$. Then $\frac{8}{x} = \frac{y - 8}{y}$.



(b) Equation (a) says $\frac{8}{x} = 1 - \frac{8}{y}$. Multiply both sides by xy: $8y = xy - 8x$. Subtract xy from both sides and factor: $y(8 - x) = -8$. Divide both sides by $8 - x$: $y = \frac{-8}{8 - x}$. Factor out -1 from numerator and denominator: $y = \frac{8}{x - 8}$.

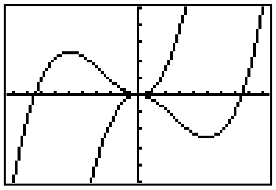
(c) Applying the Pythagorean Theorem to ΔEBC and ΔABC, we have $x^2 + D^2 = 20^2$ and $y^2 + D^2 = 30^2$, which combine to give $D^2 = 400 - x^2 = 900 - y^2$, or $y^2 - x^2 = 500$. Substituting $y = 8x/(x - 8)$, we get $\left(\frac{8x}{x - 8}\right)^2 - x^2 = 500$, so that $\frac{64x^2}{(x - 8)^2} - x^2 = 500$, or $64x^2 - x^2(x - 8)^2$ $= 500(x - 8)^2$. Expanding this gives $500x^2 - 8000x + 32{,}000 = 64x^2 - x^4 + 16x^3 - 64x^2$. This is equivalent to $x^4 - 16x^3 + 500x^2 - 8000x + 32{,}000 = 0$.

(d) The two solutions are $x \approx 5.9446$ and $x \approx 11.7118$. Based on the figure, x must be between 8 and 20 for this problem, so $x \approx 11.7118$. Then $D = \sqrt{20^2 - x^2}$ ≈ 16.2121 ft.

86. (a) Regardless of the value of b, $f(-b) = 1 - b$, $\lim_{x \to \infty} f(x) = +\infty$, $\lim_{x \to -\infty} f(x) = -\infty$, and the graph of f has a y-intercept of 1. If $|b| \leq \sqrt{3}$, the graph of f is strictly increasing. If $|b| > \sqrt{3}$, f has one local maximum and one local minimum. If $|b|$ is large, the graph of f *appears* to have a double root at 0 and a single root at $-b$, because $f(x) = x^3 + bx^2 = x^2(x + b)$ for large x.

(b) Answers will vary.

(c)



[–18.8, 18.8] by [–1000, 1000]

■ Section 2.4 Real Zeros of Polynomial Functions

Quick Review 2.4

1. $x^2 - 4x + 7$
2. $x^2 - \frac{5}{2}x - 3$
3. $7x^3 + x^2 - 3$
4. $2x^2 - \frac{2}{3}x + \frac{7}{3}$
5. $x(x^2 - 4) = x(x^2 - 2^2) = x(x + 2)(x - 2)$
6. $6(x^2 - 9) = 6(x^2 - 3^2) = 6(x + 3)(x - 3)$
7. $4(x^2 + 2x - 15) = 4(x + 5)(x - 3)$
8. $x(15x^2 - 22x + 8) = x(3x - 2)(5x - 4)$
9. $(x^3 + 2x^2) - (x + 2) = x^2(x + 2) - 1(x + 2)$
   $= (x + 2)(x^2 - 1) = (x + 2)(x + 1)(x - 1)$
10. $x(x^3 + x^2 - 9x - 9) = x[(x^3 + x^2) - (9x + 9)]$
    $= x([x^2(x + 1) - 9(x + 1)]$
    $= x(x + 1)(x^2 - 9) = x(x + 1)(x^2 - 3^2)$
    $= x(x + 1)(x + 3)(x - 3)$

Section 2.4 Exercises

1.

$$
\begin{array}{r@{\,}r}
 & x - 1 \\ \cline{2-2}
x - 1\big) & x^2 - 2x + 3 \\
 & \underline{x^2 - x\phantom{{}+3}} \\
 & -x + 3 \\
 & \underline{-x + 1} \\
 & 2
\end{array}
$$

$f(x) = (x - 1)^2 + 2;\ \dfrac{f(x)}{x - 1} = x - 1 + \dfrac{2}{x - 1}$

2.

$$
\begin{array}{r@{\,}r}
 & x^2 - x + 1 \\ \cline{2-2}
x + 1\big) & x^3 + 0x^2 + 0x - 1 \\
 & \underline{x^3 + x^2\phantom{{}+0x-1}} \\
 & -x^2 + 0x\phantom{{}-1} \\
 & \underline{-x^2 - x\phantom{{}-1}} \\
 & x - 1 \\
 & \underline{x + 1} \\
 & -2
\end{array}
$$

$f(x) = (x^2 - x + 1)(x + 1) - 2;$

$\dfrac{f(x)}{x + 1} = x^2 - x + 1 - \dfrac{2}{x + 1}$

3.

$$
\begin{array}{r@{\,}r}
 & x^2 + x + 4 \\ \cline{2-2}
x + 3\big) & x^3 + 4x^2 + 7x - 9 \\
 & \underline{x^3 + 3x^2\phantom{{}+7x-9}} \\
 & x^2 + 7x\phantom{{}-9} \\
 & \underline{x^2 + 3x\phantom{{}-9}} \\
 & 4x - 9 \\
 & \underline{4x + 12} \\
 & -21
\end{array}
$$

$f(x) = (x^2 + x + 4)(x + 3) - 21;$

$\dfrac{f(x)}{x + 3} = x^2 + x + 4 - \dfrac{21}{x + 3}$

4.

$$
\begin{array}{r@{\,}r}
 & 2x^2 - 5x + \frac{7}{2} \\ \cline{2-2}
2x + 1\big) & 4x^3 - 8x^2 + 2x - 1 \\
 & \underline{4x^3 + 2x^2\phantom{{}+2x-1}} \\
 & -10x^2 + 2x\phantom{{}-1} \\
 & \underline{-10x^2 - 5x\phantom{{}-1}} \\
 & 7x - 1 \\
 & \underline{7x + \frac{7}{2}} \\
 & -\frac{9}{2}
\end{array}
$$

$f(x) = \left(2x^2 - 5x + \dfrac{7}{2}\right)(2x + 1) - \dfrac{9}{2};$

$\dfrac{f(x)}{2x + 1} = 2x^2 - 5x + \dfrac{7}{2} - \dfrac{9/2}{2x + 1}$

5.

$$
\begin{array}{r@{\,}r}
 & x^2 - 4x + 12 \\ \cline{2-2}
x^2 + 2x - 1\big) & x^4 - 2x^3 + 3x^2 - 4x + 6 \\
 & \underline{x^4 + 2x^3 - x^2\phantom{{}-4x+6}} \\
 & -4x^3 + 4x^2 - 4x\phantom{{}+6} \\
 & \underline{-4x^3 - 8x^2 + 4x\phantom{{}+6}} \\
 & 12x^2 - 8x + 6 \\
 & \underline{12x^2 + 24x - 12} \\
 & -32x + 18
\end{array}
$$

$f(x) = (x^2 - 4x + 12)(x^2 + 2x - 1) - 32x + 18;$

$\dfrac{f(x)}{x^2 + 2x - 1} = x^2 - 4x + 12 + \dfrac{-32x + 18}{x^2 + 2x - 1}$

6.

$$
\begin{array}{r@{\,}r}
 & x^2 - 3x + 5 \\ \cline{2-2}
x^2 + 1\big) & x^4 - 3x^3 + 6x^2 - 3x + 5 \\
 & \underline{x^4 \phantom{{}-3x^3} + x^2\phantom{{}-3x+5}} \\
 & -3x^3 + 5x^2 - 3x\phantom{{}+5} \\
 & \underline{-3x^3 \phantom{{}+5x^2} - 3x\phantom{{}+5}} \\
 & 5x^2 \phantom{{}-3x} + 5 \\
 & \underline{5x^2 \phantom{{}-3x} + 5} \\
 & 0
\end{array}
$$

$f(x) = (x^2 - 3x + 5)(x^2 + 1);\ \dfrac{f(x)}{x^2 + 1} = x^2 - 3x + 5$

7. $\dfrac{x^3 - 5x^2 + 3x - 2}{x + 1} = x^2 - 6x + 9 + \dfrac{-11}{x + 1}$

$$
\begin{array}{r|rrrr}
\underline{-1} & 1 & -5 & 3 & -2 \\
 & & -1 & 6 & -9 \\ \cline{2-5}
 & 1 & -6 & 9 & -11
\end{array}
$$

8. $\dfrac{2x^4 - 5x^3 + 7x^2 - 3x + 1}{x - 3}$

$= 2x^3 + x^2 + 10x + 27 + \dfrac{82}{x - 3}$

$$
\begin{array}{r|rrrrr}
\underline{3} & 2 & -5 & 7 & -3 & 1 \\
 & & 6 & 3 & 30 & 81 \\ \cline{2-6}
 & 2 & 1 & 10 & 27 & 82
\end{array}
$$

9. $\dfrac{9x^3 + 7x^2 - 3x}{x - 10} = 9x^2 + 97x + 967 + \dfrac{9670}{x - 10}$

$$\begin{array}{r|rrrr} 10 & 9 & 7 & -3 & 0 \\ & & 90 & 970 & 9670 \\ \hline & 9 & 97 & 967 & 9670 \end{array}$$

10. $\dfrac{3x^4 + x^3 - 4x^2 + 9x - 3}{x + 5}$

$= 3x^3 - 14x^2 + 66x - 321 + \dfrac{1602}{x + 5}$

$$\begin{array}{r|rrrrr} -5 & 3 & 1 & -4 & 9 & -3 \\ & & -15 & 70 & -330 & 1605 \\ \hline & 3 & -14 & 66 & -321 & 1602 \end{array}$$

11. $\dfrac{5x^4 - 3x + 1}{4 - x}$

$= -5x^3 - 20x^2 - 80x - 317 + \dfrac{-1269}{4 - x}$

$$\begin{array}{r|rrrrr} 4 & -5 & 0 & 0 & 3 & -1 \\ & & -20 & -80 & -320 & -1268 \\ \hline & -5 & -20 & -80 & -317 & -1269 \end{array}$$

12. $\dfrac{x^8 - 1}{x + 2}$

$= x^7 - 2x^6 + 4x^5 - 8x^4 + 16x^3 - 32x^2 + 64x - 128$
$+ \dfrac{255}{x + 2}$

$$\begin{array}{r|rrrrrrrrr} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ & & -2 & 4 & -8 & 16 & -32 & 64 & -128 & 256 \\ \hline & 1 & -2 & 4 & -8 & 16 & -32 & 64 & -128 & 255 \end{array}$$

13. The remainder is $f(2) = 3$.

14. The remainder is $f(1) = -4$.

15. The remainder is $f(-3) = -43$.

16. The remainder is $f(-2) = 2$.

17. The remainder is $f(2) = 5$.

18. The remainder is $f(-1) = 23$.

19. Yes: 1 is a zero of the second polynomial.

20. Yes: 3 is a zero of the second polynomial.

21. No: when $x = 2$, the second polynomial evaluates to 10.

22. Yes: 2 is a zero of the second polynomial.

23. Yes: -2 is a zero of the second polynomial.

24. No: when $x = -1$, the second polynomial evaluates to 2.

25. From the graph it appears that $(x + 3)$ and $(x - 1)$ are factors.

$$\begin{array}{r|rrrr} -3 & 5 & -7 & -49 & 51 \\ & & -15 & 66 & -51 \\ \hline 1 & 5 & -22 & -17 & 0 \\ & & 5 & -17 & \\ \hline & 5 & -17 & 0 & \end{array}$$

$f(x) = (x + 3)(x - 1)(5x - 17)$

26. From the graph it appears that $(x + 2)$ and $(x - 3)$ are factors.

$$\begin{array}{r|rrrr} -2 & 5 & -12 & -23 & 42 \\ & & -10 & 44 & -42 \\ \hline 3 & 5 & -22 & 21 & 0 \\ & & 15 & -21 & \\ \hline & 5 & -7 & 0 & \end{array}$$

$f(x) = (x + 2)(x - 3)(5x - 7)$

27. $2(x + 2)(x - 1)(x - 4) = 2x^3 - 6x^2 - 12x + 16$

28. $2(x + 1)(x - 3)(x + 5) = 2x^3 + 6x^2 - 26x - 30$

29. $2(x - 2)\left(x - \dfrac{1}{2}\right)\left(x - \dfrac{3}{2}\right)$

$= \dfrac{1}{2}(x - 2)(2x - 1)(2x - 3)$

$= 2x^3 - 8x^2 + \dfrac{19}{2}x - 3$

30. $2(x + 3)(x + 1)(x)\left(x - \dfrac{5}{2}\right)$

$= x(x + 3)(x + 1)(2x - 5)$

$= 2x^4 + 3x^3 - 14x^2 - 15x$

31. Since $f(-4) = f(3) = f(5) = 0$, it must be that $(x + 4)$, $(x - 3)$, and $(x - 5)$ are factors of f. So $f(x) = k(x + 4)(x - 3)(x - 5)$ for some constant k.

Since $f(0) = 180$, we must have $k = 3$. So $f(x) = 3(x + 4)(x - 3)(x - 5)$

32. Since $f(-2) = f(1) = f(5) = 0$, it must be that $(x + 2)$, $(x - 1)$, and $(x - 5)$ are factors of f. So $f(x) = k(x + 2)(x - 1)(x - 5)$ for some constant k.

Since $f(-1) = 24$, we must have $k = 2$, so $f(x) = 2(x + 2)(x - 1)(x - 5)$

33. Possible rational zeros: $\dfrac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm\dfrac{1}{2}, \pm\dfrac{1}{3}, \pm\dfrac{1}{6}$; 1 is a zero.

34. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 3}$, or $\pm 1, \pm 2, \pm 7, \pm 14, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{7}{3}, \pm\dfrac{14}{3}$; $\dfrac{7}{3}$ is a zero.

35. Possible rational zeros: $\dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$, or $\pm 1, \pm 3, \pm 9, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{9}{2}$; $\dfrac{3}{2}$ is a zero.

36. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{4}{3}, \pm\dfrac{1}{6}$; $-\dfrac{4}{3}$ and $\dfrac{3}{2}$ are zeros.

37.

$$\begin{array}{r|rrrr} 3 & 2 & -4 & 1 & -2 \\ & & 6 & 6 & 21 \\ \hline & 2 & 2 & 7 & 19 \end{array}$$

Since all numbers in the last line are ≥ 0, 3 is an upper bound for the zeros of f.

38.
$$\begin{array}{r|rrrr} 5 & 2 & -5 & -5 & -1 \\ & & 10 & 25 & 100 \\ \hline & 2 & 5 & 20 & 99 \end{array}$$

Since all values in the last line are ≥ 0, 5 is an upper bound for the zeros of $f(x)$.

39.
$$\begin{array}{r|rrrrr} 2 & 1 & -1 & 1 & 1 & -12 \\ & & 2 & 2 & 6 & 14 \\ \hline & 1 & 1 & 3 & 7 & 2 \end{array}$$

Since all values in the last line are ≥ 0, 2 is an upper bound for the zeros of $f(x)$.

40.
$$\begin{array}{r|rrrrr} 3 & 4 & -6 & -7 & 9 & 2 \\ & & 12 & 18 & 33 & 126 \\ \hline & 4 & 6 & 11 & 42 & 128 \end{array}$$

Since all values in the last line are ≥ 0, 3 is an upper bound for the zeros of $f(x)$.

41.
$$\begin{array}{r|rrrr} -1 & 3 & -4 & 1 & 3 \\ & & -3 & 7 & -8 \\ \hline & 3 & -7 & 8 & -5 \end{array}$$

Since the values in the last line alternate signs, -1 is a lower bound for the zeros of $f(x)$.

42.
$$\begin{array}{r|rrrr} -3 & 1 & 2 & 2 & 5 \\ & & -3 & 3 & -15 \\ \hline & 1 & -1 & 5 & -10 \end{array}$$

Since the values in the last line alternate signs, -3 is a lower bound for the zeros of $f(x)$.

43.
$$\begin{array}{r|rrrr} 0 & 1 & -4 & 7 & -2 \\ & & 0 & 0 & 0 \\ \hline & 1 & -4 & 7 & -2 \end{array}$$

Since the values in the last line alternate signs, 0 is a lower bound for the zeros of $f(x)$.

44.
$$\begin{array}{r|rrrr} -4 & 3 & -1 & -5 & -3 \\ & & -12 & 52 & -188 \\ \hline & 3 & -13 & 47 & -191 \end{array}$$

Since the values in the last line alternate signs, -4 is a lower bound for the zeros of $f(x)$.

45. By the Upper and Lower Bound Tests, -5 is a lower bound and 5 is an upper bound. No zeros outside window.

$$\begin{array}{r|rrrrr} -5 & 6 & -11 & -7 & 8 & -34 \\ & & -30 & 205 & -990 & 4910 \\ \hline & 6 & -41 & 198 & -982 & 4876 \end{array}$$

$$\begin{array}{r|rrrrr} 5 & 6 & -11 & -7 & 8 & -34 \\ & & 30 & 95 & 440 & 2240 \\ \hline & 6 & 19 & 88 & 448 & 2206 \end{array}$$

46. By the Upper and Lower Bound Tests, -5 is a lower bound and 5 is an upper bound. No zeros outside window.

$$\begin{array}{r|rrrrrr} -5 & 1 & -1 & 0 & 21 & 19 & -3 \\ & & -5 & 30 & -150 & 645 & -3320 \\ \hline & 1 & -6 & 30 & -129 & 664 & -3323 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 1 & -1 & 0 & 21 & 19 & -3 \\ & & 5 & 20 & 100 & 605 & 3120 \\ \hline & 1 & 4 & 20 & 121 & 624 & -3117 \end{array}$$

47. Synthetic division shows that the Upper and Lower Bound Tests were not met. There *are* zeros not shown (approx. -11.002 and 12.003), because -5 and 5 are not bounds for zeros of $f(x)$.

$$\begin{array}{r|rrrrrr} -5 & 1 & -4 & -129 & 396 & -8 & 3 \\ & & -5 & 45 & 420 & -4080 & 20{,}440 \\ \hline & 1 & -9 & -84 & 816 & -4088 & -20{,}443 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 1 & -4 & -129 & 396 & -8 & 3 \\ & & 5 & 5 & -620 & -1120 & -5640 \\ \hline & 1 & 1 & -124 & -224 & -1128 & -5637 \end{array}$$

48. Synthetic division shows that the lower/upper bounds tests were not met. There *are* zeros not shown (approx. -8.036 and 9.038), because -5 and 5 are not bounds for zeros of $f(x)$.

$$\begin{array}{r|rrrrrr} -5 & 2 & -5 & -141 & 216 & -91 & 25 \\ & & -10 & 75 & 330 & -2730 & 14{,}105 \\ \hline & 2 & -15 & -66 & 546 & -2821 & 14{,}130 \end{array}$$

$$\begin{array}{r|rrrrrr} 5 & 2 & -5 & -141 & 216 & -91 & 25 \\ & & 10 & 25 & -580 & -1820 & -9555 \\ \hline & 2 & 5 & -116 & -364 & -1911 & -9530 \end{array}$$

For #49–56, determine the rational zeros using a grapher (and the Rational Zeros Test as necessary). Use synthetic division to reduce the function to a quadratic polynomial, which can be solved with the quadratic formula (or otherwise). The first two are done in detail; for the rest, we show only the synthetic division step(s).

49. Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$, or

$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$. The only rational zero is $\frac{3}{2}$. Synthetic division (below) leaves $2x^2 - 4$, so the irrational zeros are $\pm\sqrt{2}$.

$$\begin{array}{r|rrrr} 3/2 & 2 & -3 & -4 & 6 \\ & & 3 & 0 & -6 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

50. Possible rational zeros: $\pm 1, \pm 3, \pm 9$. The only rational zero is -3. Synthetic division (below) leaves $x^2 - 3$, so the irrational zeros are $\pm\sqrt{3}$.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -3 & -9 \\ & & -3 & 0 & 9 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

51. Rational: -3; irrational: $1 \pm \sqrt{3}$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -6 \\ & & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

52. Rational: 4; irrational: $1 \pm \sqrt{2}$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

53. Rational: -1 and 4; irrational: $\pm\sqrt{2}$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -6 & 6 & 8 \\ & & -1 & 4 & 2 & -8 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

54. Rational: -1 and 2; irrational: $\pm\sqrt{5}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -7 & 5 & 10 \\ & & -1 & 2 & 5 & -10 \\ \hline & 1 & -2 & -5 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

55. Rational: $-\frac{1}{2}$ and 4; irrational: none

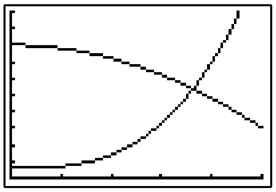
$$\begin{array}{r|rrrrr} 4 & 2 & -7 & -2 & -7 & -4 \\ & & 8 & 4 & 8 & 4 \\ \hline & 2 & 1 & 2 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1/2 & 2 & 1 & 2 & 1 \\ & & -1 & 0 & -1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

56. Rational: $\frac{2}{3}$; irrational: about -0.6823

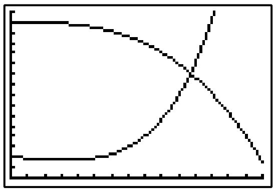
$$\begin{array}{r|rrrrr} 2/3 & 3 & -2 & 3 & 1 & -2 \\ & & 2 & 0 & 2 & 2 \\ \hline & 3 & 0 & 3 & 3 & 0 \end{array}$$

57. The supply and demand graphs are shown on the window $[0, 50]$ by $[0, 100]$. They intersect when $p = \$36.27$, at which point the supply and demand equal 53.7.



$[0, 50]$ by $[0, 100]$

58. The supply and demand graphs, shown on the window $[0, 150]$ by $[0, 1600]$, intersect when $p = \$106.99$. There $S(p) = D(p) = 1010.15$.



$[0, 150]$ by $[0, 1600]$

59. Using the Remainder Theorem, the remainder is $(-1)^{40} - 3 = -2$.

60. Using the Remainder Theorem, the remainder is $1^{63} - 17 = -16$.

61. **(a)** Lower bound:

$$\begin{array}{r|rrrrr} -5 & 1 & 2 & -11 & -13 & 38 \\ & & -5 & 15 & -20 & 165 \\ \hline & 1 & -3 & 4 & -33 & 203 \end{array}$$

Upper bound:

$$\begin{array}{r|rrrrr} 4 & 1 & 2 & -11 & -13 & 38 \\ & & 4 & 24 & 52 & 156 \\ \hline & 1 & 6 & 13 & 39 & 194 \end{array}$$

The Upper and Lower Bound Tests are met, so all real zeros of f lie on the interval $[-5, 4]$.

(b) Potential rational zeros:

$$\frac{\textit{Factors of } 38}{\textit{Factors of } 1} : \frac{\pm1, \pm2, \pm19, \pm38}{\pm1}$$

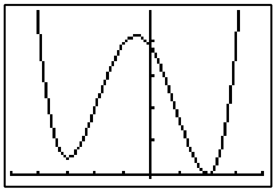
A graph shows that 2 is most promising, so we verify with synthetic division:

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -11 & -13 & 38 \\ & & 2 & 8 & -6 & -38 \\ \hline & 1 & 4 & -3 & -19 & 0 \end{array}$$

Use the Remainder Theorem:

$$\begin{array}{rr} f(-2) = 20 \neq 0 & f(-38) = 1{,}960{,}040 \\ f(-1) = 39 \neq 0 & f(38) = 2{,}178{,}540 \\ f(1) = 17 \neq 0 & f(-19) = 112{,}917 \\ & f(19) = 139{,}859 \end{array}$$

Since all possible rational roots besides 2 yield nonzero function values, there are no other rational roots.



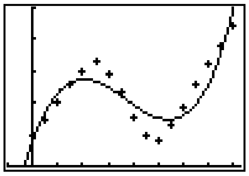
$[-5, 4]$ by $[-1, 49]$

(c) $f(x) = (x - 2)(x^3 + 4x^2 - 3x - 19)$

(d) From our graph, we find that one irrational zero of x is $x \approx 2.04$.

(e) $f(x) \approx (x - 2)(x - 2.04)(x^2 + 6.04x + 9.3216)$

62. **(a)** $D \approx 0.0669t^3 - 0.7420t^2 + 2.1759t + 0.8250$



$[1, 8.25]$ by $[0, 5]$

(b) When $t = 0$, $D \approx 0.8250$ m.

(c) The graph changes direction at $t \approx 2.02$ and at $t \approx 5.38$. Lewis is approximately 2.74 m from the motion detector at $t = 2.02$ and 1.47 m from the motion detector at $t = 5.38$.

63. False. $x - a$ is a factor if and only if $f(a) = 0$. So $(x + 2)$ is a factor if and only if $f(-2) = 0$.

64. True. By the Remainder Theorem, the remainder when $f(x)$ is divided by $x - 1$ is $f(1)$, which equals 3.

65. The statement $f(3) = 0$ means that $x = 3$ is a zero of $f(x)$ and that 3 is an x-intercept of the graph of $f(x)$. And it follows that $x - 3$ is a factor of $f(x)$ and thus that the remainder when $f(x)$ is divided by $x - 3$ is zero. So the answer is A.

66. By the Rational Zeros Theorem, every rational root of $f(x)$ must be among the numbers $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$.

The answer is E.

67. $f(x) = (x + 2)(x^2 + x - 1) - 3$ yields a remainder of -3 when divided by either $x + 2$ or $x^2 + x - 1$, from which it follows that $x + 2$ is not a factor of $f(x)$ and that $f(x)$ is not evenly divisible by $x + 2$. The answer is B.

68. Answers A through D can be verified to be true. And because $f(x)$ is a polynomial function of odd degree, its graph must cross the x-axis somewhere. The answer is E.

69. (a) The volume of a sphere is $V = \frac{4}{3}\pi r^3$. In this case, the radius of the buoy is 1, so the buoy's volume is $\frac{4}{3}\pi$.

(b) Total weight = volume $\cdot \frac{\text{weight}}{\text{unit volume}}$ = volume $\cdot$ density. In this case, the density of the buoy is $\frac{1}{4}d$, so, the weight W_b of the buoy is

$$W_b = \frac{4\pi}{3} \cdot \frac{1}{4}d = \frac{d\pi}{3}.$$

(c) The weight of the displaced water is W_{H_2O} = volume $\cdot$ density. We know from geometry that the volume of a spherical cap is

$$V = \frac{\pi}{6}(3r^2 + h^2)\,h, \text{ so,}$$

$$W_{H_2O} = \frac{\pi}{6}(3r^2 + x^2)\,x \cdot d = \frac{\pi d}{6}x(3r^2 + x^2)$$

(d) Setting the two weights equal, we have:

$$W_b = W_{H_2O}$$
$$\frac{\pi d}{3} = \frac{\pi d}{6}(3r^2 + x^2)x$$
$$2 = (3r^2 + x^2)x$$
$$0 = (6x - 3x^2 + x^2)x - 2$$
$$0 = -2x^3 + 6x^2 - 2$$
$$0 = x^3 - 3x^2 + 1$$

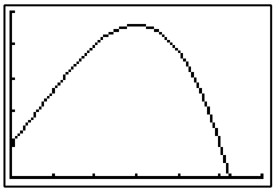
Solving graphically, we find that $x \approx 0.6527$ m, the depth that the buoy will sink.

70. The weight of the buoy, W_b, with density $\frac{1}{5}d$, is

$$W_b = \frac{4\pi}{3} \cdot \frac{1}{5}d = \frac{4\pi d}{15}. \text{ So,}$$
$$W_b = W_{H_2O}$$
$$\frac{4\pi d}{15} = \frac{\pi d}{6}(3r^2 + x^2)x$$
$$\frac{24}{15} = (3r^2 + x^2)x$$
$$0 = (6x - 3x^2 + x^2)x - \frac{8}{5}$$
$$0 = -2x^3 + 6x^2 - \frac{8}{5}$$
$$0 = x^3 - 3x^2 + \frac{4}{5}$$

Solving graphically, we find that $x \approx 0.57$ m, the depth the buoy would sink.

71. (a) Shown is one possible view, on the window [0, 600] by [0, 500].



[0, 600] by [0, 500]

(b) The maximum population, after 300 days, is 460 turkeys.

(c) $P = 0$ when $t \approx 523.22$ — about 523 days after release.

(d) Answers will vary. One possibility: After the population increases to a certain point, they begin to compete for food and eventually die of starvation.

72. (a) d is the independent variable.

(b) A good choice is [0, 172] by [0, 5].

(c) $s = 1.25$ when $d \approx 95.777$ ft. (found graphically)

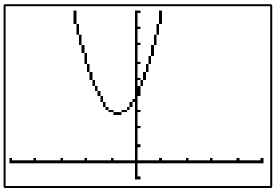
73. (a) 2 sign changes in $f(x)$, 1 sign change in $f(-x) = -x^3 + x^2 + x + 1$; 0 or 2 positive zeros, 1 negative zero.

(b) No positive zeros, 1 or 3 negative zeros.

(c) 1 positive zero, no negative zeros.

(d) 1 positive zero, 1 negative zero.

74.



[−5, 5] by [−1, 9]

The functions are not exactly the same, when $x \neq 3$, we have $f(x) = \frac{(x - 3)(2x^2 + 3x + 4)}{(x - 3)}$ $= 2x^2 + 3x + 4 = g(x)$

The domain of f is $(-\infty, 3) \cup (3, \infty)$ while the domain of g is $(-\infty, \infty)$. f is discontinuous at $x = 3$. g is continuous.

75. $\dfrac{4x^3 - 5x^2 + 3x + 1}{2x - 1}$

$= \dfrac{2x^3 - \frac{5}{2}x^2 + \frac{3}{2}x + \frac{1}{2}}{x - \frac{1}{2}}$ Divide numerator and denominator by 2.

$$\begin{array}{l|l|rrrr|l} \text{zero of divisor} & \frac{1}{2} & 2 & -\frac{5}{2} & \frac{3}{2} & \frac{1}{2} & \text{Write coefficients of dividend.} \\ \text{line for products} & & & 1 & -\frac{3}{4} & \frac{3}{8} & \\ \hline \text{line for sums} & & 2 & -\frac{3}{2} & \frac{3}{4} & \frac{7}{8} & \text{Quotient, remainder} \end{array}$$

Copy 2 into the first quotient position. Multiply $2 \cdot \frac{1}{2} = 1$ and add this to $-\frac{5}{2}$. Multiply $-\frac{3}{2} \cdot \frac{1}{2} = -\frac{3}{4}$ and add this to $\frac{3}{2}$. Multiply $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ and add this to $\frac{1}{2}$. The last line tells us $\left(x - \frac{1}{2}\right)\left(2x^2 - \frac{3}{2}x + \frac{3}{4}\right) + \frac{7}{8}$

$= 2x^3 - \frac{5}{4}x^2 + \frac{3}{2}x + \frac{1}{2}.$

76. Graph both functions in the same viewing window to see if they differ significantly. If the graphs lie on top of each other, then they are approximately equal in that viewng window.

77. (a) $g(x) = 3f(x)$, so the zeros of f and the zeros of g are identical. If the coefficients of a polynomial are rational, we may multiply that polynomial by the least common multiple (LCM) of the denominators of the coefficients to obtain a polynomial, with integer coefficients, that has the same zeros as the original.

(b) The zeros of $f(x)$ are the same as the zeros of

$6f(x) = 6x^3 - 7x^2 - 40x + 21$. Possible rational zeros: $\dfrac{\pm 1, \pm 3, \pm 7, \pm 21}{\pm 1, \pm 2, \pm 3, \pm 6}$, or

$\pm 1, \pm 3, \pm 7, \pm 21, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$. The actual zeros are $-7/3$, $1/2$, and 3.

(c) The zeros of $f(x)$ are the same as the zeros of $12f(x) = 12x^3 - 30x^2 - 37x + 30$.

Possible rational zeros:

$\dfrac{\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30}{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$, or

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{6}, \pm\frac{5}{6}, \pm\frac{1}{12}, \pm\frac{5}{12}$.

There are no rational zeros.

78. Let $f(x) = x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$. Notice that $\sqrt{2}$ is a zero of f. By the Rational Zeros Theorem, the only possible rational zeros of f are ± 1 and ± 2. Because $\sqrt{2}$ is none of these, it must be irrational.

79. (a) Approximate zeros: $-3.126, -1.075, 0.910, 2.291$

(b) $f(x) \approx g(x)$
$= (x + 3.126)(x + 1.075)(x - 0.910)(x - 2.291)$

(c) Graphically: Graph the original function and the approximate factorization on a variety of windows and observe their similarity. Numerically: Compute $f(c)$ and $g(c)$ for several values of c.

■ Section 2.5 Complex Zeros and the Fundamental Theorem of Algebra

Exploration 1

1. $f(2i) = (2i)^2 - i(2i) + 2 = -4 + 2 + 2 = 0$;
$f(-i) = (-i)^2 - i(-i) + 2 = -1 - 1 + 2 = 0$; no.

2. $g(i) = i^2 - i + (1 + i) = -1 - i + 1 + i = 0$;
$g(1 - i) = (1 - i)^2 - (1 - i) + (1 + i) =$
$-2i + 2i = 0$; no.

3. The Complex Conjugate Zeros Theorem does not necessarily hold true for a polynomial function with *complex* coefficients.

Quick Review 2.5

1. $(3 - 2i) + (-2 + 5i) = (3 - 2) + (-2 + 5)i$
$= 1 + 3i$

2. $(5 - 7i) - (3 - 2i) = (5 - 3) + (-7 - (-2))i$
$= 2 - 5i$

3. $(1 + 2i)(3 - 2i) = 1(3 - 2i) + 2i(3 - 2i)$
$= 3 - 2i + 6i - 4i^2$
$= 7 + 4i$

4. $\dfrac{2 + 3i}{1 - 5i} = \dfrac{2 + 3i}{1 - 5i} \cdot \dfrac{1 + 5i}{1 + 5i}$
$= \dfrac{2 + 10i + 3i + 15i^2}{1^2 + 5^2}$
$= \dfrac{-13 + 13i}{26}$
$= -\dfrac{1}{2} + \dfrac{1}{2}i$

5. $(2x - 3)(x + 1)$

6. $(3x + 1)(2x - 5)$

7. $x = \dfrac{5 \pm \sqrt{25 - 4(1)(11)}}{2} = \dfrac{5 \pm \sqrt{-19}}{2}$
$= \dfrac{5}{2} \pm \dfrac{\sqrt{19}}{2}i$

8. $x = \dfrac{-3 \pm \sqrt{9 - 4(2)(7)}}{4} = \dfrac{-3 \pm \sqrt{-47}}{4}$
$= -\dfrac{3}{4} \pm \dfrac{\sqrt{47}}{4}i$

9. $\dfrac{\pm 1, \ \pm 2}{\pm 1, \ \pm 3}$, or $\pm 1, \ \pm 2, \ \pm\frac{1}{3}, \ \pm\frac{2}{3}$

10. $\dfrac{\pm 1, \ \pm 3}{\pm 1, \ \pm 2, \ \pm 4}$, or $\pm 1, \ \pm 3, \ \pm\frac{1}{2}, \ \pm\frac{3}{2}, \ \pm\frac{1}{4}, \ \pm\frac{3}{4}$

Section 2.5 Exercises

1. $(x - 3i)(x + 3i) = x^2 - (3i)^2 = x^2 + 9$. The factored form shows the zeros to be $x = \pm 3i$. The absence of real zeros means that the graph has no x-intercepts.

2. $(x + 2)(x - \sqrt{3}\,i)(x + \sqrt{3}\,i) = (x + 2)(x^2 + 3)$ $= x^3 + 2x^2 + 3x + 6$. The factored form shows the zeros to be $x = -2$ and $x = \pm\sqrt{3}i$. The real zero $x = -2$ is the x-intercept of the graph.

3. $(x - 1)(x - 1)(x + 2i)(x - 2i)$
$= (x^2 - 2x + 1)(x^2 + 4)$
$= x^4 - 2x^3 + 5x^2 - 8x + 4$. The factored form shows the zeros to be $x = 1$ (multiplicity 2) and $x = \pm 2i$. The real zero $x = 1$ is the x-intercept of the graph.

4. $x(x - 1)(x - 1 - i)(x - 1 + i)$
$= (x^2 - x)[x - (1 + i)][x - (1 - i)]$
$= (x^2 - x)[x^2 - (1 - i + 1 + i)\,x + (1 + 1)]$
$= (x^2 - x)(x^2 - 2x + 2) = x^4 - 3x^3 + 4x^2 - 2x$.
The factored form shows the zeros to be $x = 0$, $x = 1$, and $x = 1 \pm i$. The real zeros $x = 0$ and $x = 1$ are the x-intercepts of the graph.

In #5–16, any constant multiple of the given polynomial is also an answer.

5. $(x - i)(x + i) = x^2 + 1$

6. $(x - 1 + 2i)(x - 1 - 2i) = x^2 - 2x + 5$

7. $(x - 1)(x - 3i)(x + 3i) = (x - 1)(x^2 + 9)$
$= x^3 - x^2 + 9x - 9$

8. $(x + 4)(x - 1 + i)(x - 1 - i)$
$= (x + 4)(x^2 - 2x + 2) = x^3 + 2x^2 - 6x + 8$

9. $(x - 2)(x - 3)(x - i)(x + i)$
$= (x - 2)(x - 3)(x^2 + 1)$
$= x^4 - 5x^3 + 7x^2 - 5x + 6$

10. $(x + 1)(x - 2)(x - 1 + i)(x - 1 - i)$
$= (x + 1)(x - 2)(x^2 - 2x + 2)$
$= x^4 - 3x^3 + 2x^2 + 2x - 4$

11. $(x - 5)(x - 3 - 2i)(x - 3 + 2i)$
$= (x - 5)(x^2 - 6x + 13) = x^3 - 11x^2 + 43x - 65$

12. $(x + 2)(x - 1 - 2i)(x - 1 + 2i)$
$= (x + 2)(x^2 - 2x + 5) = x^3 + x + 10$

13. $(x - 1)^2(x + 2)^3 = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

14. $(x + 1)^3(x - 3) = x^4 - 6x^2 - 8x - 3$

15. $(x - 2)^2(x - 3 - i)(x - 3 + i)$
$= (x - 2)^2(x^2 - 6x + 10)$
$= (x^2 - 4x + 4)(x^2 - 6x + 10)$
$= x^4 - 10x^3 + 38x^2 - 64x + 40$

16. $(x + 1)^2(x + 2 + i)(x + 2 - i)$
$= (x + 1)^2(x^2 + 4x + 5)$
$= (x^2 + 2x + 1)(x^2 + 4x + 5)$
$= x^4 + 6x^3 + 14x^2 + 14x + 5$

In #17–20, note that the graph crosses the x-axis at odd-multiplicity zeros, and "kisses" (touches but does not cross) the x-axis where the multiplicity is even.

17. (b)

18. (c)

19. (d)

20. (a)

In #21–26, the number of complex zeros is the same as the degree of the polynomial; the number of real zeros can be determined from a graph. The latter always differs from the former by an even number (when the coefficients of the polynomial are real).

21. 2 complex zeros; none real.

22. 3 complex zeros; all 3 real.

23. 3 complex zeros; 1 real.

24. 4 complex zeros; 2 real.

25. 4 complex zeros; 2 real.

26. 5 complex zeros; 1 real.

In #27–32, look for real zeros using a graph (and perhaps the Rational Zeros Test). Use synthetic division to factor the polynomial into one or more linear factors and a quadratic factor. Then use the quadratic formula to find complex zeros.

27. Inspection of the graph reveals that $x = 1$ is the only real zero. Dividing $f(x)$ by $x - 1$ leaves $x^2 + x + 5$ (below). The quadratic formula gives the remaining zeros of $f(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 4 & -5 \\ & & 1 & 1 & 5 \\ \hline & 1 & 1 & 5 & 0 \end{array}$$

Zeros: $x = 1$, $x = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i$

$f(x)$

$$= (x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{19}}{2}i\right)\right]\left[x - \left(-\frac{1}{2} + \frac{\sqrt{19}}{2}i\right)\right]$$

$$= \frac{1}{4}(x - 1)(2x + 1 + \sqrt{19}\,i)(2x + 1 - \sqrt{19}\,i)$$

28. Zeros: $x = 3$ (graphically) and $x = \frac{7}{2} \pm \frac{\sqrt{43}}{2}i$ (applying the quadratic formula to $x^2 - 7x + 23$).

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 4 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$f(x)$

$$= (x - 3)\left[x - \left(\frac{7}{2} - \frac{\sqrt{43}}{2}i\right)\right]\left[x - \left(\frac{7}{2} + \frac{\sqrt{43}}{2}i\right)\right]$$

$$= \frac{1}{4}(x - 3)(2x - 7 + \sqrt{43}\,i)(2x - 7 - \sqrt{43}\,i)$$

29. Zeros: $x = \pm 1$(graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$ (applying the quadratic formula to $x^2 + x + 6$).

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 5 & -1 & -6 \\ & & 1 & 2 & 7 & 6 \\ \hline & 1 & 2 & 7 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 7 & 6 \\ & & -1 & -1 & -6 \\ \hline & 1 & 1 & 6 & 0 \end{array}$$

$$f(x) = (x - 1)(x + 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{23}}{2}i\right)\right]$$
$$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{23}}{2}i\right)\right]$$

$$= \frac{1}{4}(x - 1)(x + 1)(2x + 1 + \sqrt{23}\,i)\,(2x + 1 - \sqrt{23}\,i)$$

30. Zeros: $x = -2$ and $x = \frac{1}{3}$ (graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ (applying the quadratic formula to $3x^2 + 3x + 3 = 3(x^2 + x + 1)$).

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & 6 & 3 & -2 \\ & & -6 & -4 & -4 & 2 \\ \hline & 3 & 2 & 2 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1/3 & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

$f(x) = (x + 2)(3x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\right]$
$= \frac{1}{4}(x + 2)(3x - 1)(2x + 1 + \sqrt{3}\, i).$
$(2x + 1 - \sqrt{3}\, i)$

31. Zeros: $x = -\frac{7}{3}$ and $x = \frac{3}{2}$ (graphically) and $x = 1 \pm 2i$ (applying the quadratic formula to $6x^2 - 12x + 30 = 6(x^2 - 2x + 5)$).

$$\begin{array}{r|rrrrr} -7/3 & 6 & -7 & -1 & 67 & -105 \\ & & -14 & 49 & -112 & 105 \\ \hline & 6 & -21 & 48 & -45 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3/2 & 6 & -21 & 48 & -45 \\ & & 9 & -18 & 45 \\ \hline & 6 & -12 & 30 & 0 \end{array}$$

$f(x) = (3x + 7)(2x - 3)[x - (1 - 2i)]$
$[x - (1 + 2i)]$
$= (3x + 7)(2x - 3)(x - 1 + 2i)(x - 1 - 2i)$

32. Zeros: $x = -\frac{3}{5}$ and $x = 5$ (graphically) and $x = \frac{3}{2} \pm i$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$
(applying the quadratic formula to $20x^2 - 60x + 65 = 5(4x^2 - 12x + 13)$).

$$\begin{array}{r|rrrrr} 5 & 20 & -148 & 269 & -106 & -195 \\ & & 100 & -240 & 145 & 195 \\ \hline & 20 & -48 & 29 & 39 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3/5 & 20 & -48 & 29 & 39 \\ & & -12 & 36 & -39 \\ \hline & 20 & -60 & 65 & 0 \end{array}$$

$f(x) = (5x + 3)(x - 5)[2x - (3 - 2i)]$
$[2x - (3 + 2i)]$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$

In #33–36, since the polynomials' coefficients are real, for the given zero $z = a + bi$, the complex conjugate $\bar{z} = a - bi$ must also be a zero. Divide $f(x)$ by $x - z$ and $x - \bar{z}$ to reduce to a quadratic.

33. First divide $f(x)$ by $x - (1 + i)$ (synthetically). Then divide the result, $x^3 + (-1 + i)x^2 - 3x + (3 - 3i)$, by $x - (1 - i)$. This leaves the polynominal $x^2 - 3$. Zeros: $x = \pm\sqrt{3}$, $x = 1 \pm i$

$$\begin{array}{r|rrrrr} 1+i & 1 & -2 & -1 & 6 & -6 \\ & & 1+i & -2 & -3-3i & 6 \\ \hline & 1 & -1+i & -3 & 3-3i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1-i & 1 & -1+i & -3 & 3-3i \\ & & 1-i & 0 & -3+3i \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x) = (x - \sqrt{3})(x + \sqrt{3})[x - (1 - i)][x - (1 + i)]$
$= (x - \sqrt{3})(x + \sqrt{3})(x - 1 + i)(x - 1 - i)$

34. First divide $f(x)$ by $x - 4i$. Then divide the result, $x^3 + 4ix^2 - 3x - 12i$, by $x + 4i$. This leaves the polynomial $x^2 - 3$. Zeros: $x = \pm\sqrt{3}$, $x = \pm 4i$

$$\begin{array}{r|rrrrr} 4i & 1 & 0 & 13 & 0 & -48 \\ & & 4i & -16 & -12i & 48 \\ \hline & 1 & 4i & -3 & -12i & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4i & 1 & 4i & -3 & -12i \\ & & -4i & 0 & 12i \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4i)(x + 4i)$

35. First divide $f(x)$ by $x - (3 - 2i)$. Then divide the result, $x^3 + (-3 - 2i)x^2 - 2x + 6 + 4i$, by $x - (3 + 2i)$. This leaves $x^2 - 2$. Zeros: $x = \pm\sqrt{2}$, $x = 3 \pm 2i$

$$\begin{array}{r|rrrrr} 3-2i & 1 & -6 & 11 & 12 & -26 \\ & & 3-2i & -13 & -6+4i & 26 \\ \hline & 1 & -3-2i & -2 & 6+4i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3+2i & 1 & -3-2i & -2 & 6+4i \\ & & 3+2i & 0 & -6-4i \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$f(x) = (x - \sqrt{2})(x + \sqrt{2})[x - (3 - 2i)]$
$[x - (3 + 2i)]$
$= (x - \sqrt{2})(x + \sqrt{2})(x - 3 + 2i)(x - 3 - 2i)$

36. First divide $f(x)$ by $x - (1 + 3i)$. Then divide the result, $x^3 + (-1 + 3i)x^2 - 5x + 5 - 15i$, by $x - (1 - 3i)$. This leaves $x^2 - 5$. Zeros: $x = \pm\sqrt{5}$, $x = 1 \pm 3i$

$$\begin{array}{r|rrrrr} 1+3i & 1 & -2 & 5 & 10 & -50 \\ & & 1+3i & -10 & -5-15i & 50 \\ \hline & 1 & -1+3i & -5 & 5-15i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1-3i & 1 & -1+3i & -5 & 5-15i \\ & & 1-3i & 0 & -5+15i \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$f(x) = (x - \sqrt{5})(x + \sqrt{5})[x - (1 - 3i)]$
$[x - (1 + 3i)]$
$= (x - \sqrt{5})(x + \sqrt{5})(x - 1 + 3i)(x - 1 - 3i)$

For #37–42, find real zeros graphically, then use synthetic division to find the quadratic factors. Only the synthetic divison step is shown.

37. $f(x) = (x - 2)(x^2 + x + 1)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

38. $f(x) = (x - 2)(x^2 + x + 3)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

39. $f(x) = (x - 1)(2x^2 + x + 3)$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 2 & -3 \\ & & 2 & 1 & 3 \\ \hline & 2 & 1 & 3 & 0 \end{array}$$

40. $f(x) = (x - 1)(3x^2 + x + 2)$

$$\begin{array}{r|rrrr} 1 & 3 & -2 & 1 & -2 \\ & & 3 & 1 & 2 \\ \hline & 3 & 1 & 2 & 0 \end{array}$$

41. $f(x) = (x - 1)(x + 4)(x^2 + 1)$

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -3 & 3 & -4 \\ & & 1 & 4 & 1 & 4 \\ \hline & 1 & 4 & 1 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ & & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

42. $f(x) = (x - 3)(x + 1)(x^2 + 4)$

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & 1 & -8 & -12 \\ & & 3 & 3 & 12 & 12 \\ \hline & 1 & 1 & 4 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 4 & 4 \\ & & -1 & 0 & -4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

43. Solve for h: $\frac{\pi}{3}(15h^2 - h^3)(62.5) = \frac{4}{3}\pi\,(125)(20)$, so that $15h^2 - h^3 = 160$. Of the three solutions (found graphically), only $h \approx 3.776$ ft makes sense in this setting.

44. Solve for h: $\frac{\pi}{3}(15h^2 - h^3)(62.5) = \frac{4}{3}\pi\,(125)(45)$, so that $15h^2 - h^3 = 360$. Of the three solutions (found graphically), only $h \approx 6.513$ ft makes sense in this setting.

45. Yes: $(x + 2)(x^2 + 1) = x^3 + 2x^2 + x + 2$ is one such polynomial. Other examples can be obtained by multiplying any other quadratic with no real zeros by $(x + 2)$.

46. No; by the Complex Conjugate Zeros Theorem, for such a polynomial, if $2i$ is a zero, so is $-2i$.

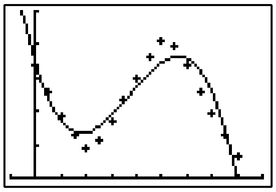
47. No: if all coefficients are real, $1 - 2i$ and $1 + i$ must also be zeros, giving 5 zeros for a degree 4 polynomial.

48. Yes: $f(x) = (x - 1 - 3i)(x - 1 + 3i)(x - 1 - i)(x - 1 + i) = x^4 - 4x^3 + 16x^2 - 24x + 20$ is one such polynomial; all other examples would be multiples of this polynomial.

49. $f(x)$ must have the form $a(x - 3)(x + 1)(x - 2 + i)\,(x - 2 - i)$; since $f(0) = a(-3)(1)(-2 + i)(-2 - i) = -15a = 30$, we know that $a = -2$. Multiplied out, this gives $f(x) = -2x^4 + 12x^3 - 20x^2 - 4x + 30$.

50. $f(x)$ must have the form $a(x - 1 - 2i)(x - 1 + 2i)\,(x - 1 - i)(x - 1 + i)$; since $f(0) = a(-1 - 2i)(-1 + 2i)(-1 - i)(-1 + i) = a(5)(2) = 10a = 20$, we know that $a = 2$. Multiplied out, this gives $f(x) = 2x^4 - 8x^3 + 22x^2 - 28x + 20$.

51. **(a)** The model is $D \approx -0.0820t^3 + 0.9162t^2 - 2.5126t + 3.3779$.

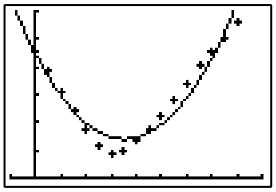


[−1, 9] by [0, 5]

(b) Sally walks toward the detector, turns and walks away (or walks backward), then walks toward the detector again.

(c) The model "changes direction" at $t \approx 1.81$ sec ($D \approx 1.35$ m) and $t \approx 5.64$ sec (when $D \approx 3.65$ m).

52. **(a)** $D \approx 0.2434t^2 - 1.7159t + 4.4241$



[−1, 9] by [0, 6]

(b) Jacob walks toward the detector, then turns and walks away (or walks backward).

(c) The model "changes direction" at $t \approx 3.52$ (when $D \approx 1.40$m).

53. False. Complex, nonreal solutions always come in conjugate pairs, so that if $1 - 2i$ is a zero, then $1 + 2i$ must also be a zero.

54. False. All three zeros could be real. For instance, the polynomial $f(x) = x(x - 1)(x - 2) = x^3 - 3x^2 + 2x$ has degree 3, real coefficients, and no non-real zeros. (The zeros are 0, 1, and 2.)

55. Both the sum and the product of two complex conjugates are real numbers, and the absolute value of a complex number is always real. The square of a complex number, on the other hand, need not be real. The answer is E.

56. Allowing for multiplicities other than 1, then, the polynomial can have anywhere from 1 to 5 distinct real zeros. But it cannot have no real zeros at all. The answer is A.

57. Because the complex, non-real zeros of a real-coefficient polynomial always come in conjugate pairs, a polynomial of degree 5 can have either 0, 2, or 4 non-real zeros. The answer is C.

58. A polynomial with real coefficients can never have an odd number of non-real complex zeros. The answer is E.

59. (a)

Power	Real Part	Imaginary Part
7	8	−8
8	16	0
9	16	16
10	0	32

(b) $(1 + i)^7 = 8 - 8i$

$(1 + i)^8 = 16$

$(1 + i)^9 = 16 + 16i$

$(1 + i)^{10} = 32i$

(c) Reconcile as needed.

60. (a) $(a + bi)(a + bi) = a^2 + 2abi + bi^2$
$= a^2 + 2abi - b^2$

(b) $a^2 - b^2 = 0$ (1)

$2abi = i$, so $2ab = 1$ (2)

(c) From (1), we have:

$$a^2 - b^2 = 0$$
$$(a + b)(a - b) = 0$$
$$a = -b, a = b$$

Substituting into (2), we find:

$$\begin{array}{ll} a = b\colon 2ab = 1 & a = -b\colon 2ab = 1 \\ 2b^2 = 1 & -2b^2 = 1 \\ b^2 = \frac{1}{2} & b^2 = -\frac{1}{2} \\ b = \pm\sqrt{\frac{1}{2}} & b^2 = \sqrt{\frac{1}{2}} \\ & b = \pm i\sqrt{\frac{1}{2}} \end{array}$$

Since a and b must be real, we have

$$(a, b) = \left\{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\right\}.$$

(d) Checking $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ first.

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2(1 + i)^2$$
$$= \frac{1}{2}(1 + 2i + i^2) = \frac{1}{2}(1 - 1 + 2i) = \frac{2i}{2} = i$$

Checking $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^2 = \left(-\frac{\sqrt{2}}{2}\right)^2(1 + i)^2$$
$$= \frac{1}{2}(1 + 2i + i^2) = \frac{1}{2}(1 - 1 + 2i) = \frac{2i}{2} = i$$

(e) The two square roots of i are:

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) \text{ and } \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

61. $f(i) = i^3 - i(i)^2 + 2i(i) + 2 = -i + i - 2 + 2 = 0.$
One can also take the last number of the bottom row from synthetic division.

62. $f(-2i) = (-2i)^3 - (2 - i)(-2i)^2 + (2 - 2i)(-2i) - 4$
$= 8i + (2 - i)(4) - (2 - 2i)(2i) - 4 =$
$8i + 8 - 4i - 4i - 4 - 4 = 0.$
One can also take the last number of the bottom row from synthetic division.

63. Synthetic division shows that $f(i) = 0$ (the remainder), and at the same time gives
$f(x) \div (x - i) = x^2 + 3x - i = h(x)$, so
$f(x) = (x - i)(x^2 + 3x - i).$

$$\begin{array}{r|rrrr} i & 1 & 3-i & -4i & -1 \\ & & i & 3i & 1 \\ \hline & 1 & 3 & -i & 0 \end{array}$$

64. Synthetic division shows that $f(1 + i) = 0$ (the remainder), and at the same time gives
$f(x) \div (x - 1 - i) = x^2 + 1 = h(x)$, so
$f(x) = (x - 1 - i)(x^2 + 1).$

$$\begin{array}{r|rrrr} 1+i & 1 & -1-i & 1 & -1-i \\ & & 1+i & 0 & 1+i \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

65. From graphing (or the Rational Zeros Test), we expect $x = 2$ to be a zero of $f(x) = x^3 - 8$. Indeed, $f(2) = 8 - 8 = 0$. So, $x = 2$ *is* a zero of $f(x)$. Using synthetic division we obtain:

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 + 2x + 4)$. We then apply the quadratic formula to find that the cube roots of $x^3 - 8$ are $2, -1 + \sqrt{3}i$, and $-1 - \sqrt{3}i$.

66. From graphing (or the Rational Zeros Test), we expect $x = -4$ to be a zero of $f(x) = x^3 + 64$. Indeed $f(-4) = -64 + 64 = 0$, so $x = -4$ is a zero of $f(x)$. Using synthetic division, we obtain:

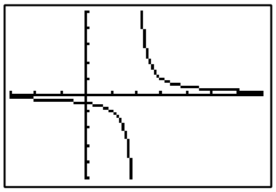
$$\begin{array}{r|rrrr} -4 & 1 & 0 & 0 & 64 \\ & & -4 & 16 & -64 \\ \hline & 1 & -4 & 16 & 0 \end{array}$$

$f(x) = (x + 4)(x^2 - 4x + 16)$. We then apply the quadratic formula to find that the cube roots of $x^3 + 64$ are $-4, 2 + 2\sqrt{3}i$, and $2 - 2\sqrt{3}i$.

■ Section 2.6 Graphs of Rational Functions

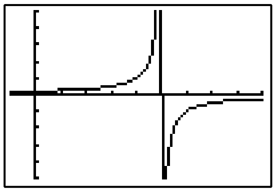
Exploration 1

1. $g(x) = \dfrac{1}{x - 2}$



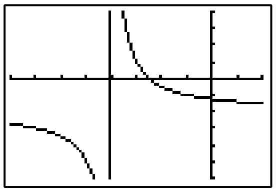
[−3, 7] by [−5, 5]

2. $h(x) = -\dfrac{1}{x-5}$



$[-1, 9]$ by $[-5, 5]$

3. $k(x) = \dfrac{3}{x+4} - 2$



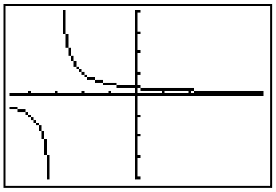
$[-8, 2]$ by $[-5, 5]$

Quick Review 2.6

1. $f(x) = (2x-1)(x+3) \Rightarrow x = -3$ or $x = \dfrac{1}{2}$
2. $f(x) = (3x+4)(x-2) \Rightarrow x = -\dfrac{4}{3}$ or $x = 2$
3. $g(x) = (x+2)(x-2) \Rightarrow x = \pm 2$
4. $g(x) = (x+1)(x-1) \Rightarrow x = \pm 1$
5. $h(x) = (x-1)(x^2+x+1) \Rightarrow x = 1$
6. $h(x) = (x-i)(x+i) \Rightarrow$ no real zeros
7. $$\begin{array}{r} 2 \\ x-3\overline{)\,2x+1} \\ \underline{2x-6} \\ 7 \end{array}$$
   Quotient: 2, Remainder: 7
8. $$\begin{array}{r} 2 \\ 2x-1\overline{)\,4x+3} \\ \underline{4x-2} \\ 5 \end{array}$$
   Quotient: 2, Remainder: 5
9. $$\begin{array}{r} 3 \\ x\overline{)\,3x-5} \\ \underline{3x} \\ -5 \end{array}$$
   Quotient: 3, Remainder: -5
10. $$\begin{array}{r} \frac{5}{2} \\ 2x\overline{)\,5x-1} \\ \underline{5x} \\ -1 \end{array}$$
    Quotient: $\dfrac{5}{2}$, Remainder: -1

Section 2.6 Exercises

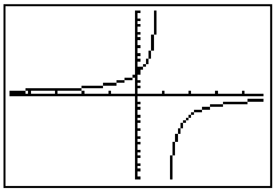
1. The domain of $f(x) = 1/(x+3)$ is all real numbers $x \neq -3$. The graph suggests that $f(x)$ has a vertical asymptote at $x = -3$.



[–4.7, 4.7] by [–4, 4]

As x approaches -3 from the left, the values of $f(x)$ decrease without bound. As x approaches -3 from the right, the values of $f(x)$ increase without bound. That is, $\lim_{x\to -3^-} f(x) = -\infty$ and $\lim_{x\to -3^+} f(x) = \infty$.

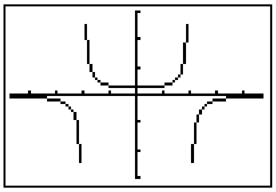
2. The domain of $f(x) = -3/(x-1)$ is all real numbers $x \neq 1$. The graph suggests that $f(x)$ has a vertical asymptote at $x = 1$.



[–4.7, 4.7] by [–12, 12]

As x approaches 1 from the left, the values of $f(x)$ increase without bound. As x approaches 1 from the right, the values of $f(x)$ decrease without bound. That is, $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1^+} f(x) = -\infty$.

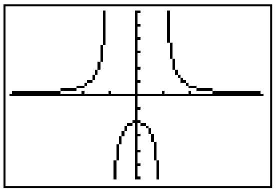
3. The domain of $f(x) = -1/(x^2-4)$ is all real numbers $x \neq -2, 2$. The graph suggests that $f(x)$ has vertical asymptotes at $x = -2$ and $x = 2$.



[–4.7, 4.7] by [–3, 3]

As x approaches -2 from the left, the values of $f(x)$ decrease without bound, and as x approaches -2 from the right, the values of $f(x)$ increase without bound. As x approaches 2 from the left, the values of $f(x)$ increase without bound, and as x approaches 2 from the right, the values of $f(x)$ decrease without bound. That is, $\lim_{x\to -2^-} f(x) = -\infty$, $\lim_{x\to -2^+} f(x) = \infty$, $\lim_{x\to 2^-} f(x) = \infty$, and $\lim_{x\to 2^+} f(x) = -\infty$.

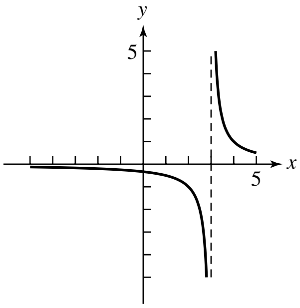
4. The domain of $f(x) = 2/(x^2 - 1)$ is all real numbers $x \neq -1, 1$. The graph suggests that $f(x)$ has vertical asymptotes at $x = -1$ and $x = 1$.



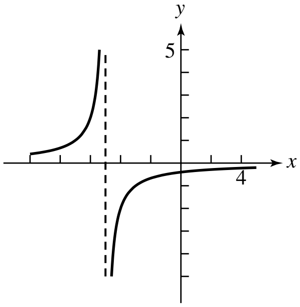
[–4.7, 4.7] by [–6, 6]

As x approaches -1 from the left, the values of $f(x)$ increase without bound, and as x approaches -1 from the right, the values of $f(x)$ decrease without bound. As x approaches 1 from the left, the values of $f(x)$ decrease without bound, and as x approaches 1 from the right, the values of $f(x)$ increase without bound. That is, $\lim_{x\to -1^-} f(x) = \infty$, $\lim_{x\to -1^+} f(x) = -\infty$, $\lim_{x\to 1^-} f(x) = -\infty$, and $\lim_{x\to 1^+} f(x) = \infty$.

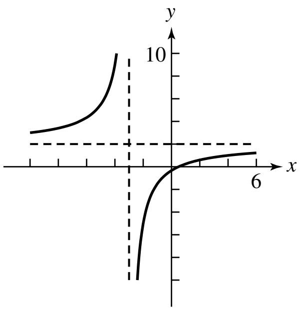
5. Translate right 3 units. Asymptotes: $x = 3$, $y = 0$.



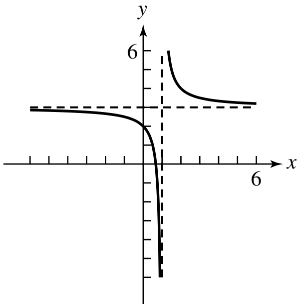
6. Translate left 5 units, reflect across x-axis, vertically stretch by 2. Asymptotes: $x = -5$, $y = 0$.



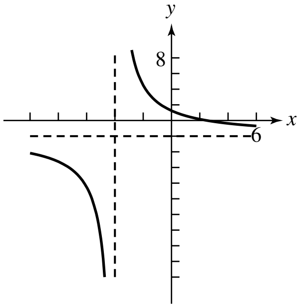
7. Translate left 3 units, reflect across x-axis, vertically stretch by 7, translate up 2 units.
Asymptotes: $x = -3$, $y = 2$.



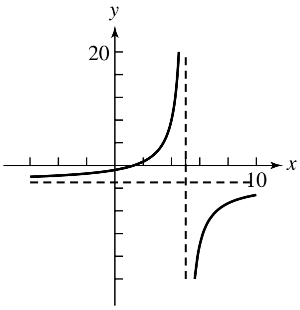
8. Translate right 1 unit, translate up 3 units.
Asymptotes: $x = 1$, $y = 3$.



9. Translate left 4 units, vertically stretch by 13, translate down 2 units. Asymptotes: $x = -4$, $y = -2$.



10. Translate right 5 units, vertically stretch by 11, reflect across x-axis, translate down 3 units. Asymptotes: $x = 5$, $y = -3$.



11. $\lim_{x\to 3^-} f(x) = \infty$

12. $\lim_{x\to 3^+} f(x) = -\infty$

13. $\lim_{x\to \infty} f(x) = 0$

14. $\lim_{x\to -\infty} f(x) = 0$

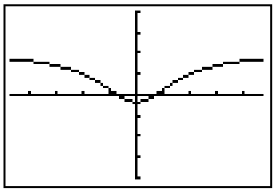
15. $\lim_{x\to -3^+} f(x) = \infty$

16. $\lim_{x\to -3^-} f(x) = -\infty$

17. $\lim_{x\to -\infty} f(x) = 5$

18. $\lim_{x\to \infty} f(x) = 5$

19. The graph of $f(x) = (2x^2 - 1)/(x^2 + 3)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 2$.



[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

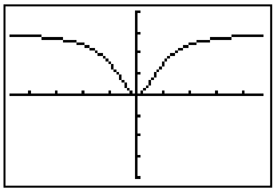
$$f(x) = \frac{2x^2 - 1}{x^2 + 3} = 2 - \frac{7}{x^2 + 3}$$

When the value of $|x|$ is large, the denominator $x^2 + 3$ is a large positive number, and $7/(x^2 + 3)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore,

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2,$$

so $y = 2$ is indeed a horizontal asymptote.

20. The graph of $f(x) = 3x^2/(x^2 + 1)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 3$.



[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

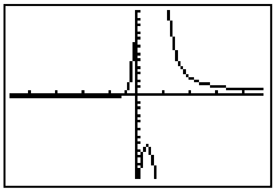
$$f(x) = \frac{3x^2}{x^2 + 1} = 3 - \frac{3}{x^2 + 1}$$

When the value of $|x|$ is large, the denominator $x^2 + 1$ is a large positive number, and $3/(x^2 + 1)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore,

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3,$$

so $y = 3$ is indeed a horizontal asymptote.

21. The graph of $f(x) = (2x + 1)/(x^2 - x)$ suggests that there are vertical asymptotes at $x = 0$ and $x = 1$, with $\lim_{x\to0^-} f(x) = \infty$, $\lim_{x\to0^+} f(x) = -\infty$, $\lim_{x\to1^-} f(x) = -\infty$, and $\lim_{x\to1^+} f(x) = \infty$, and that the horizontal asymptote is $y = 0$.



[–4.7, 4.7] by [–12, 12]

The domain of $f(x) = (2x + 1)/(x^2 - x) = (2x + 1)/[x(x - 1)]$ is all real numbers $x \neq 0, 1$, so there are indeed vertical asymptotes at $x = 0$ and $x = 1$. Rewriting one rational expression as two, we find that

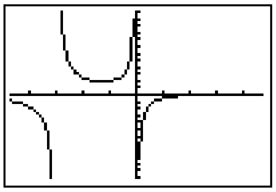
$$f(x) = \frac{2x + 1}{x^2 - x} = \frac{2x}{x^2 - x} + \frac{1}{x^2 - x}$$
$$= \frac{2}{x - 1} + \frac{1}{x^2 - x}$$

When the value of $|x|$ is large, both terms get close to zero. Therefore,

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0,$$

so $y = 0$ is indeed a horizontal asymptote.

22. The graph of $f(x) = (x - 3)/(x^2 + 3x)$ suggests that there are vertical asymptotes at $x = -3$ and $x = 0$, with $\lim_{x\to-3^-} f(x) = -\infty$, $\lim_{x\to-3^+} f(x) = \infty$, $\lim_{x\to0^-} f(x) = \infty$, and $\lim_{x\to0^+} f(x) = -\infty$, and that the horizontal asymptote is $y = 0$.



[–4.7, 4.7] by [–4, 4]

The domain of $f(x) = (x - 3)/(x^2 + 3x) = (x - 3)/[x(x + 3)]$ is all real numbers $x \neq -3, 0$, so there are indeed vertical asymptotes at $x = -3$ and $x = 0$. Rewriting one rational expression as two, we find that

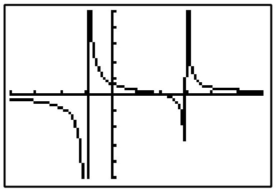
$$f(x) = \frac{x - 3}{x^2 + 3x} = \frac{x}{x^2 + 3x} - \frac{3}{x^2 + 3x}$$
$$= \frac{1}{x + 3} - \frac{3}{x^2 + 3x}$$

When the value of $|x|$ is large, both terms get close to zero. Therefore,

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0,$$

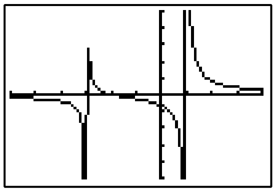
so $y = 0$ is indeed a horizontal asymptote.

23. Intercepts: $\left(0, \frac{2}{3}\right)$ and $(2, 0)$. Asymptotes: $x = -1$, $x = 3$, and $y = 0$.



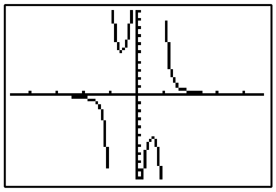
$[-4, 6]$ by $[-5, 5]$

24. Intercepts: $\left(0, -\frac{2}{3}\right)$ and $(-2, 0)$. Asymptotes: $x = -3$, $x = 1$, and $y = 0$.



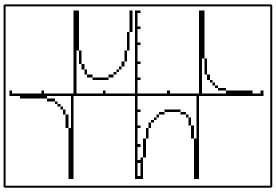
$[-6, 4]$ by $[-5, 5]$

25. No intercepts. Asymptotes: $x = -1$, $x = 0$, $x = 1$, and $y = 0$.



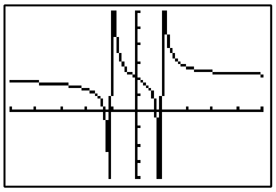
$[-4.7, 4.7]$ by $[-10, 10]$

26. No intercepts. Asymptotes: $x = -2$, $x = 0$, $x = 2$, and $y = 0$.



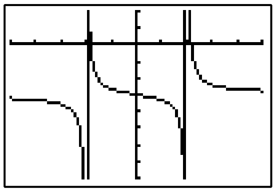
$[-4, 4]$ by $[-5, 5]$

27. Intercepts: $(0, 2)$, $(-1.28, 0)$, and $(0.78, 0)$. Asymptotes: $x = 1$, $x = -1$, and $y = 2$.



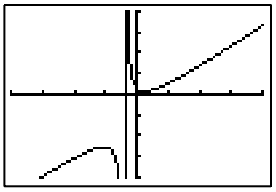
$[-5, 5]$ by $[-4, 6]$

28. Intercepts: $(0, -3)$, $(-1.84, 0)$, and $(2.17, 0)$. Asymptotes: $x = -2$, $x = 2$, and $y = -3$.



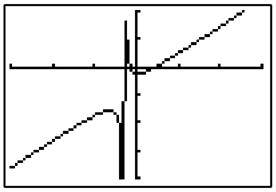
$[-5, 5]$ by $[-8, 2]$

29. Intercept: $\left(0, \frac{3}{2}\right)$. Asymptotes: $x = -2$, $y = x - 4$.



$[-20, 20]$ by $[-20, 20]$

30. Intercepts: $\left(0, -\frac{7}{3}\right)$, $(-1.54, 0)$, and $(4.54, 0)$. Asymptotes: $x = -3$, $y = x - 6$.



$[-30, 30]$ by $[-40, 20]$

31. (d); Xmin $= -2$, Xmax $= 8$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

32. (b); Xmin $= -6$, Xmax $= 2$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

33. (a); Xmin $= -3$, Xmax $= 5$, Xscl $= 1$, and Ymin $= -5$, Ymax $= 10$, Yscl $= 1$.

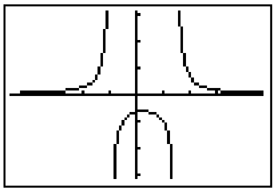
34. (f); Xmin $= -6$, Xmax $= 2$, Xscl $= 1$, and Ymin $= -5$, Ymax $= 5$, Yscl $= 1$.

35. (e); Xmin $= -2$, Xmax $= 8$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 3$, Yscl $= 1$.

36. (c); Xmin $= -3$, Xmax $= 5$, Xscl $= 1$, and Ymin $= -3$, Ymax $= 8$, Yscl $= 1$.

37. For $f(x) = 2/(2x^2 - x - 3)$, the numerator is never zero, and so $f(x)$ never equals zero and the graph has no x-intercepts. Because $f(0) = -2/3$, the y-intercept is $-2/3$. The denominator factors as $2x^2 - x - 3 = (2x - 3)(x + 1)$, so there are vertical asymptotes at $x = -1$ and $x = 3/2$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that $\lim_{x\to -1^-} f(x) = \infty$, $\lim_{x\to -1^+} f(x) = -\infty$, $\lim_{x\to (3/2)^-} f(x) = -\infty$, and $\lim_{x\to (3/2)^+} f(x) = \infty$.

The graph also shows a local maximum of $-16/25$ at $x = 1/4$.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Intercept: $\left(0, -\frac{2}{3}\right)$

Domain: $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

Range: $\left(-\infty, -\frac{16}{25}\right] \cup (0, \infty)$

Continuity: All $x \neq -1, \frac{3}{2}$

Increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{4}\right]$

Decreasing on $\left[\frac{1}{4}, \frac{3}{2}\right)$ and $\left(\frac{3}{2}, \infty\right)$

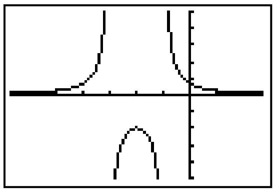
Not symmetric.
Unbounded.

Local maximum at $\left(\frac{1}{4}, -\frac{16}{25}\right)$

Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -1$ and $x = 3/2$
End behavior: $\lim_{x\to -\infty} f(x) = \lim_{x\to \infty} f(x) = 0$

38. For $g(x) = 2/(x^2 + 4x + 3)$, the numerator is never zero, and so $g(x)$ never equals zero and the graph has no x-intercepts. Because $g(0) = 2/3$, the y-intercept is $2/3$. The denominator factors as
$x^2 + 4x + 3 = (x + 1)(x + 3)$,
so there are vertical asymptotes at $x = -3$ and $x = -1$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that

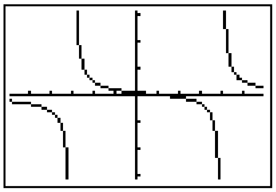
$\lim_{x\to-3^-} g(x) = \infty$, $\lim_{x\to-3^+} g(x) = -\infty$, $\lim_{x\to-1^-} g(x) = -\infty$, and $\lim_{x\to-1^+} g(x) = \infty$.
The graph also shows a local maximum of -2 at $x = -2$.



[–6.7, 2.7] by [–5, 5]

Intercept: $\left(0, \frac{2}{3}\right)$
Domain: $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$
Range: $(-\infty, -2] \cup (0, \infty)$
Continuity: All $x \neq -3, -1$
Increasing on $(-\infty, -3)$ and $(-3, -2]$
Decreasing on $[-2, -1)$ and $(-1, \infty)$
Not symmetric.
Unbounded.
Local maximum at $(-2, -2)$
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -3$ and $x = -1$
End behavior: $\lim_{x\to-\infty} g(x) = \lim_{x\to\infty} g(x) = 0$

39. For $h(x) = (x - 1)/(x^2 - x - 12)$, the numerator is zero when $x = 1$, so the x-intercept of the graph is 1. Because $h(0) = 1/12$, the y-intercept is $1/12$. The denominator factors as
$x^2 - x - 12 = (x + 3)(x - 4)$,
so there are vertical asymptotes at $x = -3$ and $x = 4$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x\to-3^-} h(x) = -\infty$, $\lim_{x\to-3^+} h(x) = \infty$, $\lim_{x\to4^-} h(x) = -\infty$, and $\lim_{x\to4^+} h(x) = \infty$.
The graph shows no local extrema.



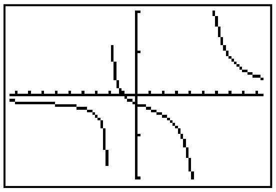
[–5.875, 5.875] by [–3.1, 3.1]

Intercepts: $\left(0, \frac{1}{12}\right)$, $(1, 0)$
Domain: $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -3, 4$
Decreasing on $(-\infty, -3)$, $(-3, 4)$, and $(4, \infty)$
Not symmetric.
Unbounded.
No local extrema.
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -3$ and $x = 4$
End behavior: $\lim_{x\to-\infty} h(x) = \lim_{x\to\infty} h(x) = 0$

40. For $k(x) = (x + 1)/(x^2 - 3x - 10)$, the numerator is zero when $x = -1$, so the x-intercept of the graph is -1. Because $k(0) = -1/10$, the y-intercept is $-1/10$. The denominator factors as
$x^2 - 3x - 10 = (x + 2)(x - 5)$,
so there are vertical asymptotes at $x = -2$ and $x = 5$.

And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that

$\lim_{x\to-2^-} k(x) = -\infty$, $\lim_{x\to2^+} k(x) = \infty$,
$\lim_{x\to5^-} k(x) = -\infty$, and $\lim_{x\to5^+} k(x) = \infty$.
The graph shows no local extrema.

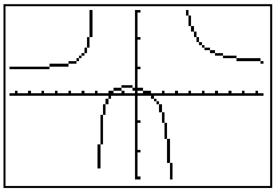


[–9.4, 9.4] by [–1, 1]

Intercepts: $(-1, 0)$, $(0, -0.1)$
Domain: $(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2, 5$
Decreasing on $(-\infty, -2)$, $(-2, 5)$, and $(5, \infty)$
Not symmetric.
Unbounded.
No local extrema.
Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -2$ and $x = 5$
End behavior: $\lim_{x\to-\infty} k(x) = \lim_{x\to\infty} k(x) = 0$

41. For $f(x) = (x^2 + x - 2)/(x^2 - 9)$, the numerator factors as
$x^2 + x - 2 = (x + 2)(x - 1)$,
so the x-intercepts of the graph are -2 and 1. Because $f(0) = 2/9$, the y-intercept is $2/9$. The denominator factors as
$x^2 - 9 = (x + 3)(x - 3)$,
so there are vertical asymptotes at $x = -3$ and $x = 3$. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 1, the horizontal asymptote is $y = 1$. The graph supports this information and allows us to conclude that
$\lim_{x\to-3^-} f(x) = \infty$, $\lim_{x\to-3^+} f(x) = -\infty$, $\lim_{x\to3^-} f(x) = -\infty$, and $\lim_{x\to3^+} f(x) = \infty$.

The graph also shows a local maximum of about 0.260 at about $x = -0.675$.

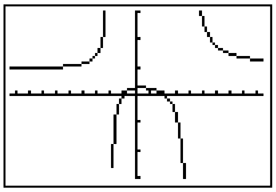


[–9.4, 9.4] by [–3, 3]

Intercepts: $(-2, 0)$, $(1, 0)$, $\left(0, \frac{2}{9}\right)$
Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
Range: $(-\infty, 0.260] \cup (1, \infty)$
Continuity: All $x \neq -3, 3$
Increasing on $(-\infty, -3)$ and $(-3, -0.675]$
Decreasing on $[-0.675, 3)$ and $(3, \infty)$
Not symmetric.
Unbounded.
Local maximum at about $(-0.675, 0.260)$
Horizontal asymptote: $y = 1$
Vertical asymptotes: $x = -3$ and $x = 3$
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 1.$

42. For $g(x) = (x^2 - x - 2)/(x^2 - 2x - 8)$, the numerator factors as
$x^2 - x - 2 = (x + 1)(x - 2)$,
so the x-intercepts of the graph are -1 and 2. Because $g(0) = 1/4$, the y-intercept is $1/4$. The denominator factors as
$x^2 - 2x - 8 = (x + 2)(x - 4)$,
so there are vertical asymptotes at $x = -2$ and $x = 4$. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 1, the horizontal asymptote is $y = 1$. The graph supports this information and allows us to conclude that $\lim_{x\to-2^-} g(x) = \infty$, $\lim_{x\to-2^+} g(x) = -\infty$, $\lim_{x\to4^-} g(x) = -\infty$, and $\lim_{x\to4^+} g(x) = \infty$.

The graph also shows a local maximum of about 0.260 at about $x = 0.324$.



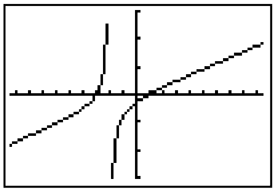
[–9.4, 9.4] by [–3, 3]

Intercepts: $(-1, 0)$, $(2, 0)$, $\left(0, \frac{1}{4}\right)$
Domain: $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$
Range: $(-\infty, 0.260] \cup (1, \infty)$
Continuity: All $x \neq -2, 4$
Increasing on $(-\infty, -2)$ and $(-2, 0.324]$
Decreasing on $[0.324, 4)$ and $(4, \infty)$
Not symmetric.
Unbounded.
Local maximum at about $(0.324, 0.260)$
Horizontal asymptote: $y = 1$
Vertical asymptotes: $x = -2$ and $x = 4$
End behavior: $\lim_{x\to-\infty} g(x) = \lim_{x\to\infty} g(x) = 1$

43. For $h(x) = (x^2 + 2x - 3)/(x + 2)$, the numerator factors as
$x^2 + 2x - 3 = (x + 3)(x - 1)$,
so the x-intercepts of the graph are -3 and 1. Because $h(0) = -3/2$, the y-intercept is $-3/2$. The denominator is zero when $x = -2$, so there is a vertical asymptote at $x = -2$. Using long division, we rewrite $h(x)$ as

$$h(x) = \frac{x^2 + 2x - 2}{x + 2} = x - \frac{2}{x + 2},$$

so the end-behavior asymptote of $h(x)$ is $y = x$. The graph supports this information and allows us to conclude that $\lim_{x\to-2^-} h(x) = \infty$ and $\lim_{x\to-2^+} h(x) = -\infty$.
The graph shows no local extrema.



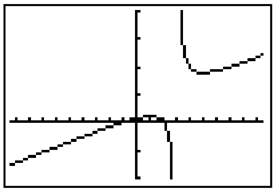
[–9.4, 9.4] by [–15, 15]

Intercepts: $(-3, 0)$, $(1, 0)$, $\left(0, -\frac{3}{2}\right)$
Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2$
Increasing on $(-\infty, -2)$ and $(-2, \infty)$
Not symmetric.
Unbounded.
No local extrema.
Horizontal asymptote: None
Vertical asymptote: $x = -2$
Slant asymptote: $y = x$
End behavior: $\lim_{x\to-\infty} h(x) = -\infty$ and $\lim_{x\to\infty} h(x) = \infty$.

44. For $k(x) = (x^2 - x - 2)/(x - 3)$, the numerator factors as
$x^2 - x - 2 = (x + 1)(x - 2)$,
so the x-intercepts of the graph are -1 and 2. Because $k(0) = 2/3$, the y-intercept is $2/3$. The denominator is zero when $x = 3$, so there is a vertical asymptote at $x = 3$. Using long division, we rewrite $k(x)$ as

$$h(x) = \frac{x^2 - x - 2}{x - 3} = x + 2 + \frac{4}{x - 3},$$

so the end-behavior asymptote of $k(x)$ is $y = x + 2$. The graph supports this information and allows us to conclude that $\lim_{x\to3^-} k(x) = -\infty$ and $\lim_{x\to3^+} k(x) = \infty$.
The graph shows a local maximum of 1 at $x = 1$ and a local minimum of 9 at $x = 5$.

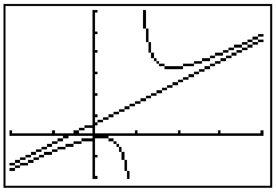


[–9.4, 9.4] by [–10, 20]

Intercepts: $(-1, 0)$, $(2, 0)$, $\left(0, \frac{2}{3}\right)$
Domain: $(-\infty, 3) \cup (3, \infty)$
Range: $(-\infty, 1] \cup [9, \infty)$
Continuity: All $x \neq 3$
Increasing on $(-\infty, 1]$ and $[5, \infty)$
Decreasing on $[1, 3)$ and $(3, 5]$
Not symmetric.
Unbounded.
Local maximum at $(1, 1)$; local minimum at $(5, 9)$
Horizontal asymptote: None
Vertical asymptote: $x = 3$
Slant asymptote: $y = x + 2$
End behavior: $\lim_{x\to-\infty} k(x) = -\infty$ and $\lim_{x\to\infty} k(x) = \infty$.

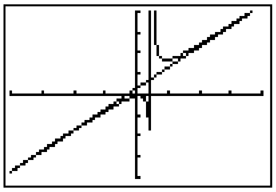
45. Divide $x^2 - 2x - 3$ by $x - 5$ to show that
$f(x) = \frac{x^2 - 2x + 3}{x - 5} = x + 3 + \frac{18}{x - 5}$.
The end-behavior asymptote of $f(x)$ is $y = x + 3$.

(a)



[–10, 20] by [–10, 30]

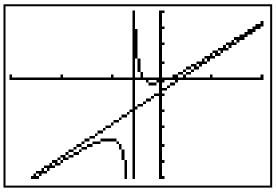
(b)



[–40, 40] by [–40, 40]

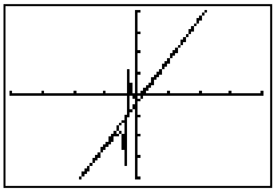
46. Divide $2x^2 + 2x - 3$ by $x + 3$ to show that
$f(x) = \frac{2x^2 + 2x - 3}{x + 3} = 2x - 4 + \frac{9}{x + 3}$.
The end-behavior asymptote of $f(x)$ is $y = 2x - 4$.

(a)



[–15, 10] by [–30, 20]

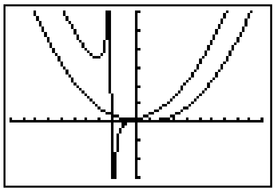
(b)



[–40, 40] by [–40, 40]

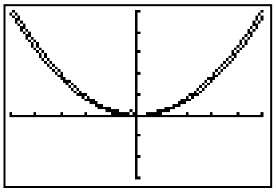
47. Divide $x^3 - x^2 + 1$ by $x + 2$ to show that
$f(x) = \frac{x^3 - x^2 + 1}{x + 2} = x^2 - 3x + 6 - \frac{11}{x + 2}$.
The end-behavior asymptote of $f(x)$ is $y = x^2 - 3x + 6$.

(a)



[–10, 10] by [–30, 60]

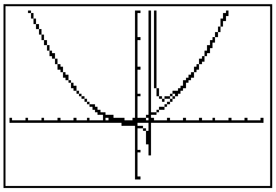
(b)



[–50, 50] by [–1500, 2500]

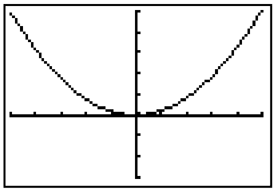
48. Divide $x^3 + 1$ by $x - 1$ to show that
$f(x) = \frac{x^3 + 1}{x - 1} = x^2 + x + 1 + \frac{2}{x - 1}$.
The end-behavior asymptote of $f(x)$ is $y = x^2 + x + 1$.

(a)



[–8, 8] by [–20, 40]

(b)



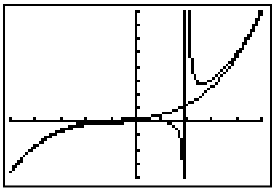
[–50, 50] by [–1500, 2500]

49. Divide $x^4 - 2x + 1$ by $x - 2$ to show that

$f(x) = \frac{x^4 - 2x + 1}{x - 2} = x^3 + 2x^2 + 4x + 6 + \frac{13}{x - 2}$.

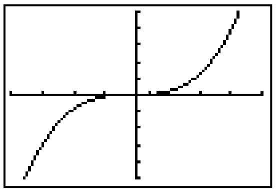
The end-behavior asymptote of $f(x)$ is
$y = x^3 + 2x^2 + 4x + 6$.

(a)



[–5, 5] by [–100, 200]

(b)



[–20, 20] by [–5000, 5000]

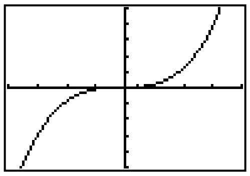
50. Divide $x^5 + 1$ by $x^2 + 1$ to show that

$$f(x) = \frac{x^5 + 1}{x^2 + 1} = x^3 - x + \frac{x + 1}{x^2 + 1}.$$

The end-behavior asymptote of $f(x)$ is $y = x^3 - x$.

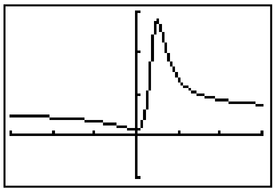
(a) There are no vertical asymptotes, since the denominator $x^2 + 1$ is never zero.

(b)



[–20, 20] by [–5000, 5000]

51. For $f(x) = (3x^2 - 2x + 4)/(x^2 - 4x + 5)$, the numerator is never zero, and so $f(x)$ never equals zero and the graph has no x-intercepts. Because $f(0) = 4/5$, the y-intercept is $4/5$. The denominator is never zero, and so there are no vertical asymptotes. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 3, the horizontal asymptote is $y = 3$. The graph supports this information. The graph also shows a local maximum of about 14.227 at about $x = 2.445$ and a local minimum of about 0.773 at about $x = -0.245$.



[–15, 15] by [–5, 15]

Intercept: $\left(0, \frac{4}{5}\right)$

Domain: $(-\infty, \infty)$

Range: $[0.773, 14.227]$

Continuity: $(-\infty, \infty)$

Increasing on $[-0.245, 2.445]$

Decreasing on $(-\infty, -0.245], [2.445, \infty)$

Not symmetric.

Bounded.

Local maximum at $(2.445, 14.227)$; local minimum at $(-0.245, 0.773)$

Horizontal asymptote: $y = 3$

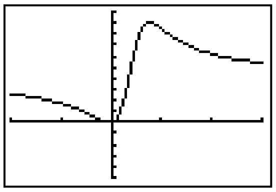
No vertical asymptotes.

End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 3$

52. For $g(x) = (4x^2 + 2x)/(x^2 - 4x + 8)$, the numerator factors as

$4x^2 + 2x = 2x(2x + 1)$,

so the x-intercepts of the graph are $-1/2$ and 0. Because $g(0) = 0$, the y-intercept is 0. The denominator is never zero, and so there are no vertical asymptotes. And because the degree of the numerator equals the degree of the denominator with a ratio of leading terms that equals 4, the horizontal asymptote is $y = 4$. The graph supports this information. The graph also shows a local maximum of about 9.028 at about $x = 3.790$ and a local minimum of about -0.028 at about $x = -0.235$.



[–10, 15] by [–5, 10]

Intercepts: $\left(-\frac{1}{2}, 0\right)$, $(0, 0)$

Domain: $(-\infty, \infty)$

Range: $[-0.028, 9.028]$

Continuity: $(-\infty, \infty)$

Increasing on $[-0.235, 3.790]$

Decreasing on $(-\infty, -0.235], [3.790, \infty)$.

Not symmetric.

Bounded.

Local maximum at $(3.790, 9.028)$; local minimum at $(-0.235, -0.028)$

Horizontal asymptote: $y = 4$

No vertical asymptotes.

End behavior: $\lim_{x\to-\infty} g(x) = \lim_{x\to\infty} g(x) = 4$

53. For $h(x) = (x^3 - 1)/(x - 2)$, the numerator factors as $x^3 - 1 = (x - 1)(x^2 + x + 1)$,

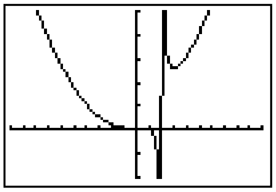
so the x-intercept of the graph is 1. The y-intercept is $h(0) = 1/2$. The denominator is zero when $x = 2$, so the vertical asymptote is $x = 2$. Because we can rewrite $h(x)$ as

$$h(x) = \frac{x^3 - 1}{x - 2} = x^2 + 2x + 4 + \frac{7}{x - 2},$$

we know that the end-behavior asymptote is $y = x^2 + 2x + 4$. The graph supports this information and allows us to conclude that

$\lim_{x\to 2^-} h(x) = -\infty$, $\lim_{x\to 2^+} h(x) = \infty$.

The graph also shows a local maximum of about 0.586 at about $x = 0.442$, a local minimum of about 0.443 at about $x = -0.384$, and another local minimum of about 25.970 at about $x = 2.942$.



[–10, 10] by [–20, 50]

Intercepts: $(1, 0)$, $\left(0, \frac{1}{2}\right)$

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, \infty)$

Continuity: All real $x \neq 2$

Increasing on $[-0.384, 0.442], [2.942, \infty)$

Decreasing on $(-\infty, -0.384], [0.442, 2), (2, 2.942]$

Not symmetric.

Unbounded.

Local maximum at $(0.442, 0.586)$; local minimum at $(-0.384, 0.443)$ and $(2.942, 25.970)$

No horizontal asymptote. End-behavior asymptote: $y = x^2 + 2x + 4$

Vertical asymptote: $x = 2$

End behavior: $\lim_{x\to-\infty} h(x) = \lim_{x\to\infty} h(x) = \infty$

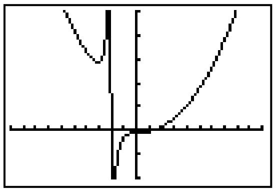
54. For $k(x) = (x^3 - 2)/(x + 2)$, the numerator is zero when $x = \sqrt[3]{2}$, so the x-intercept of the graph is $\sqrt[3]{2}$. The y-intercept is $k(0) = -1$. The denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$. Because we can rewrite $k(x)$ as

$$k(x) = \frac{x^3 - 2}{x + 2} = x^2 - 2x + 4 - \frac{10}{x - 2},$$

we know that the end-behavior asymptote is $y = x^2 - 2x + 4$. The graph supports this information and allows us to conclude that

$\lim_{x\to-2^-} k(x) = \infty$, $\lim_{x\to-2^+} k(x) = -\infty$.

The graph also shows a local minimum of about 28.901 at about $x = -3.104$.



[–10, 10] by [–20, 50]

Intercepts: $(\sqrt[3]{2}, 0)$, $(0, -1)$
Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, \infty)$
Continuity: All real $x \neq -2$
Increasing on $[-3.104, -2)$, $(-2, \infty)$
Decreasing on $(-\infty, -3.104]$
Not symmetric.
Unbounded.
Local minimum at $(-3.104, 28.901)$
No horizontal asymptote. End-behavior asymptote:
$y = x^2 - 2x + 4$
Vertical asymptote: $x = -2$
End behavior: $\lim_{x\to-\infty} k(x) = \lim_{x\to\infty} k(x) = \infty$

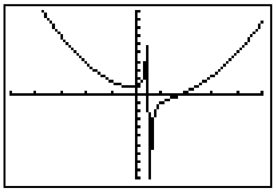
55. $f(x) = (x^3 - 2x^2 + x - 1)/(2x - 1)$ has only one x-intercept, and we can use the graph to show that it is about 1.755. The y-intercept is $f(0) = 1$. The denominator is zero when $x = 1/2$, so the vertical asymptote is $x = 1/2$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{x^3 - 2x^2 + x - 1}{2x - 1}$$
$$= \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8} - \frac{7}{16(2x - 1)},$$

we know that the end-behavior asymptote is $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$. The graph supports this information and allows us to conclude that

$\lim_{x\to1/2^-} f(x) = \infty$, $\lim_{x\to1/2^+} f(x) = -\infty$.

The graph also shows a local minimum of about 0.920 at about $x = -0.184$.



[–5, 5] by [–10, 10]

Intercepts: $(1.755, 0)$, $(0, 1)$
Domain: All $x \neq \frac{1}{2}$
Range: $(-\infty, \infty)$
Continuity: All $x \neq \frac{1}{2}$
Increasing on $[-0.184, 0.5)$, $(0.5, \infty)$
Decreasing on $(-\infty, -0.184]$
Not symmetric.
Unbounded.
Local minimum at $(-0.184, 0.920)$
No horizontal asymptote. End-behavior asymptote: $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{8}$
Vertical asymptote: $x = \frac{1}{2}$
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$

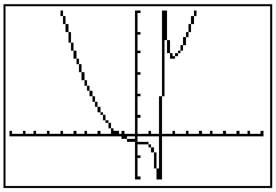
56. $g(x) = (2x^3 - 2x^2 - x + 5)/(x - 2)$ has only one x-intercept, and we can use the graph to show that it is about -1.189. The y-intercept is $g(0) = -5/2$. The denominator is zero when $x = 2$, so the vertical asymptote is $x = 2$. Because we can rewrite $g(x)$ as

$$g(x) = \frac{2x^3 - 2x^2 - x + 5}{x - 2} = 2x^2 + 2x + 3 + \frac{11}{x - 2},$$

we know that the end-behavior asymptote is $y = 2x^2 + 2x + 3$. The graph supports this information and allows us to conclude that

$\lim_{x\to2^-} g(x) = -\infty$, $\lim_{x\to2^+} g(x) = \infty$.

The graph also shows a local minimum of about 37.842 at about $x = 2.899$.



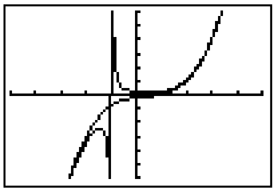
[–10, 10] by [–20, 60]

Intercepts: $(-1.189, 0)$, $(0, -2.5)$
Domain: All $x \neq 2$
Range: $(-\infty, \infty)$
Continuity: All $x \neq 2$
Increasing on $[2.899, \infty)$
Decreasing on $(-\infty, 2)$, $(2, 2.899]$
Not symmetric.
Unbounded.
Local minimum at $(2.899, 37.842)$
No horizontal asymptote. End-behavior asymptote:
$y = 2x^2 + 2x + 3$
Vertical asymptote: $x = 2$
End behavior: $\lim_{x\to-\infty} g(x) = \lim_{x\to\infty} g(x) = \infty$

57. For $h(x) = (x^4 + 1)/(x + 1)$, the numerator is never zero, and so $h(x)$ never equals zero and the graph has no x-intercepts. Because $h(0) = 1$, the y-intercept is 1. So the one intercept is the point $(0, 1)$. The denominator is zero when $x = -1$, so $x = -1$ is a vertical asymptote. Divide $x^4 + 1$ by $x + 1$ to show that

$$h(x) = \frac{x^4 + 1}{x + 1} = x^3 - x^2 + x - 1 + \frac{2}{x + 1}.$$

The end-behavior asymptote of $h(x)$ is $y = x^3 - x^2 + x - 1$.

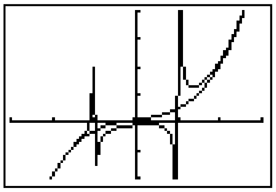


[–5, 5] by [–30, 30]

58. $k(x) = (2x^5 + x^2 - x + 1)/(x^2 - 1)$ has only one x-intercept, and we can use the graph to show that it is about -1.108. Because $k(0) = -1$, the y-intercept is -1. So the intercepts are $(-1.108, 0)$ and $(0, -1)$. The denominator is zero when $x = \pm 1$, so $x = -1$ and $x = 1$ are vertical asymptotes. Divide $2x^5 + x^2 - x + 1$ by $x^2 - 1$ to show that

$$k(x) = \frac{2x^5 + x^2 - x + 1}{x^2 - 1} = 2x^3 + 2x + 1 + \frac{x + 2}{x^2 - 1}.$$

The end-behavior asymptote of $k(x)$ is $y = 2x^3 + 2x + 1$.

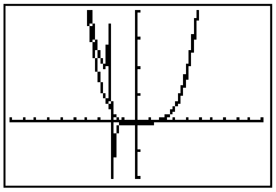


[–3, 3] by [–20, 40]

59. For $f(x) = (x^5 - 1)/(x + 2)$, the numerator factors as $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$, and since the second factor is never zero (as can be verified by Descartes' Rule of Signs or by graphing), the x-intercept of the graph is 1. Because $f(0) = -1/2$, the y-intercept is $-1/2$. So the intercepts are $(1, 0)$ and $(0, -1/2)$. The denominator is zero when $x = -2$, so $x = -2$ is a vertical asymptote. Divide $x^5 - 1$ by $x + 2$ to show that

$$f(x) = \frac{x^5 - 1}{x + 2} = x^4 - 2x^3 + 4x^2 - 8x + 16 - \frac{33}{x + 2}$$

The end-behavior asymptote of $f(x)$ is $y = x^4 - 2x^3 + 4x^2 - 8x + 16$.

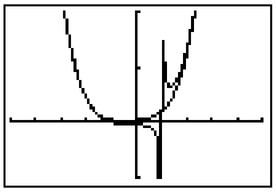


[–10, 10] by [–200, 400]

60. For $g(x) = (x^5 + 1)/(x - 1)$, the numerator factors as $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$, and since the second factor is never zero (as can be verified by graphing), the x-intercept of the graph is -1. Because $g(0) = -1$, the y-intercept is -1. So the intercepts are $(-1, 0)$ and $(0, -1)$. The denominator is zero when $x = 1$, so $x = 1$ is a vertical asymptote. Divide $x^5 + 1$ by $x - 1$ to show that

$$g(x) = \frac{x^5 + 1}{x - 1} = x^4 + x^3 + x^2 + x + 1 + \frac{2}{x - 1}.$$

The end-behavior asymptote of $g(x)$ is $y = x^4 + x^3 + x^2 + x + 1$.

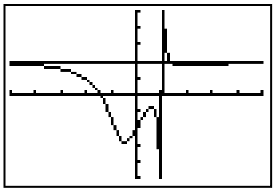


[–5, 5] by [–25, 50]

61. $h(x) = (2x^3 - 3x + 2)/(x^3 - 1)$ has only one x-intercept, and we can use the graph to show that it is about -1.476. Because $h(0) = -2$, the y-intercept is -2. So the intercepts are $(-1.476, 0)$ and $(0, -2)$. The denominator is zero when $x = 1$, so $x = 1$ is a vertical asymptote. Divide $2x^3 - 3x + 2$ by $x^3 - 1$ to show that

$$h(x) = \frac{2x^3 - 3x + 2}{x^3 - 1} = 2 - \frac{3x - 4}{x^3 - 1}.$$

The end-behavior asymptote of $h(x)$ is $y = 2$, a horizontal line.

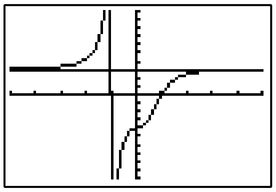


[–5, 5] by [–5, 5]

62. For $k(x) = (3x^3 + x - 4)/(x^3 + 1)$, the numerator factors as $3x^3 + x - 4 = (x - 1)(3x^2 + 3x + 4)$, and since the second factor is never zero (as can be verified by Descartes' Rule of Signs or by graphing), the x-intercept of the graph is 1. Because $k(0) = -4$, the y-intercept is -4. So the intercepts are $(1, 0)$ and $(0, -4)$. The denominator is zero when $x = -1$, so $x = -1$ is a vertical asymptote. Divide $3x^3 + x - 4$ by $x^3 + 1$ to show that

$$k(x) = \frac{3x^3 + x - 4}{x^3 + 1} = 3 + \frac{x - 7}{x^3 + 1}.$$

The end-behavior asymptote of $k(x)$ is $y = 3$, a horizontal line.



[–5, 5] by [–10, 10]

63. False. If the denominator is never zero, there will be no vertical asymptote. For example, $f(x) = 1/(x^2 + 1)$ is a rational function and has no vertical asymptotes.

64. False. A rational function is the quotient of two polynomials, and $\sqrt{x^2 + 4}$ is not a polynomial.

65. The excluded values are those for which $x^3 + 3x = 0$, namely 0 and -3. The answer is E.

66. $g(x)$ results from $f(x)$ by replacing x with $x + 3$, which represents a shift of 3 units to the left. The answer is A.

67. Since $x + 5 = 0$ when $x = -5$, there is a vertical asymptote. And because $x^2/(x + 5) = x - 5 + 25/(x + 5)$, the end behavior is characterized by the slant asymptote $y = x - 5$. The answer is D.

68. The quotient of the leading terms is x^4, so the answer is E.

69. **(a)** No: the domain of f is $(-\infty, 3) \cup (3, \infty)$; the domain of g is all real numbers.

(b) No: while it is not defined at 3, it does not tend toward $\pm\infty$ on either side.

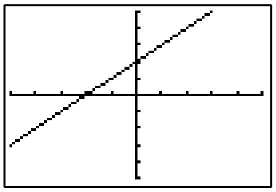
(c) Most grapher viewing windows do not reveal that f is undefined at 3.

(d) Almost—but not quite; they are equal for all $x \neq 3$.

70. **(a)** $f(x) = \dfrac{x^2 + x - 2}{x - 1} = \dfrac{(x + 2)(x - 1)}{x - 1} = x + 2$
$= g(x)$ when $x \neq 1$

	f	g
Asymptotes	$x = 1$	none
Intercepts	$(0, 2)$ $(-2, 0)$	$(0, 2)$ $(-2, 0)$
Domain	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, \infty)$

The functions are identical at all points except $x = 1$, where f has a discontinuity.

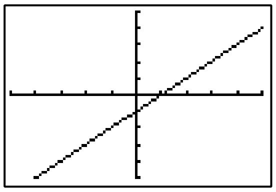


$[-5, 5]$ by $[-5, 5]$

(b) $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x + 1)(x - 1)}{x + 1} = x - 1$
$= g(x)$ when $x \neq -1$

	f	g
Asymptotes	$x = -1$	none
Intercepts	$(0, -1)$ $(1, 0)$	$(0, -1)$ $(1, 0)$
Domain	$(-\infty, -1) \cup (-1, \infty)$	$(-\infty, \infty)$

The functions are identical except at $x = -1$, where f has a discontinuity.

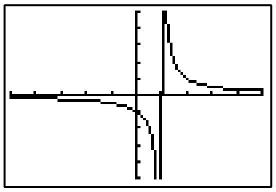


$[-5, 5]$ by $[-5, 5]$

(c) $f(x) = \dfrac{x^2 - 1}{x^3 - x^2 - x + 1} = \dfrac{x^2 - 1}{(x^2 - 1)(x - 1)}$
$= \dfrac{1}{x - 1} = g(x)$ when $x \neq -1$

	f	g
Asymptotes	$x = 1, x = -1$	$x = 1$
Intercepts	$(0, -1)$	$(0, -1)$
Domain	$(-\infty, -1) \cup (-1, 1)$ $\cup (1, \infty)$	$(-\infty, 1) \cup (1, \infty)$

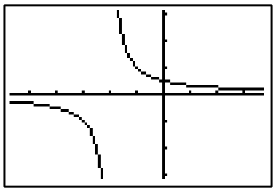
The functions are identical except at $x = -1$, where $f(x)$ has a discontinuity.



$[-5, 5]$ by $[-5, 5]$

	f	g
Asymptotes	$x = 1, x = -2$	$x = -2$
Intercepts	$\left(0, \frac{1}{2}\right)$	$\left(0, \frac{1}{2}\right)$
Domain	$(-\infty, -2) \cup$ $(-2, 1) \cup (1, \infty)$	$(-\infty, -2) \cup (-2, \infty)$

(d) $f(x) = \dfrac{x - 1}{x^2 + x - 2} = \dfrac{x - 1}{(x + 2)(x - 1)} = \dfrac{1}{x + 2}$
$= g(x)$ when, $x \neq -2$
Except at $x = 1$, where f has a discontinuity, the functions are identical.



$[-5.7, 3.7]$ by $[-3.1, 3.1]$

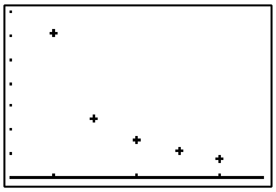
71. **(a)** The volume is $f(x) = k/x$, where x is pressure and k is a constant. $f(x)$ is a quotient of polynomials and hence is rational, but $f(x) = k \cdot x^{-1}$, so is a power function with constant of variation k and pressure -1.

(b) If $f(x) = kx^a$, where a is a negative integer, then the power function f is also a rational function.

(c) $V = \dfrac{k}{P}$, so $k = (2.59)(0.866) = 2.24294$.

If $P = 0.532$, then $V = \dfrac{2.24294}{0.532} \approx 4.22$ L.

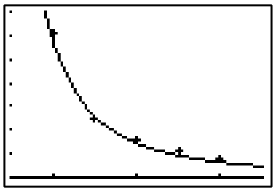
72. **(a)**



$[0.5, 3.5]$ by $[0, 7]$

(b) One method for determining k is to find the power regression for the data points using a calculator, discussed in previous sections. By this method, we find that a good approximation of the data points is given by the curve $y \approx 5.81 \cdot x^{-1.88}$. Since -1.88 is very close to -2, we graph the curve to see if $k = 5.81$ is reasonable.

(c)

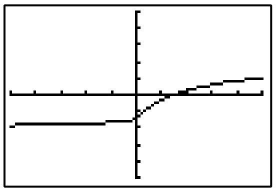


[0.5, 3.5] by [0, 7]

(d) At 2.2 m, the light intensity is approximately 1.20 W/m^2.
At 4.4 m, the light intensity is approximately 0.30 W/m^2.

73. Horizontal asymptotes: $y = -2$ and $y = 2$.
Intercepts: $\left(0, -\frac{3}{2}\right), \left(\frac{3}{2}, 0\right)$

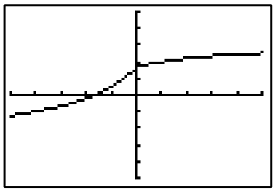
$$h(x) = \begin{cases} \dfrac{2x-3}{x+2} & x \ge 0 \\ \dfrac{2x-3}{-x+2} & x < 0 \end{cases}$$



[−5, 5] by [−5, 5]

74. Horizontal asymptotes: $y = \pm 3$.
Intercepts: $\left(0, \frac{5}{3}\right), \left(-\frac{5}{3}, 0\right)$

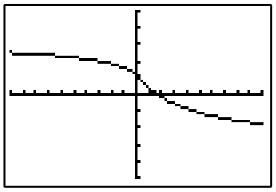
$$h(x) = \begin{cases} \dfrac{3x+5}{x+3} & x \ge 0 \\ \dfrac{3x+5}{-x+3} & x < 0 \end{cases}$$



[−5, 5] by [−5, 5]

75. Horizontal asymptotes: $y = \pm 3$.
Intercepts: $\left(0, \frac{5}{4}\right), \left(\frac{5}{3}, 0\right)$

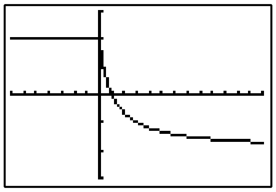
$$f(x) = \begin{cases} \dfrac{5-3x}{x+4} & x \ge 0 \\ \dfrac{5-3x}{-x+4} & x < 0 \end{cases}$$



[−10, 10] by [−5, 5]

76. Horizontal asymptotes: $y = \pm 2$.
Intercepts: $(0, 2), (1, 0)$

$$f(x) = \begin{cases} \dfrac{2-2x}{x+1} & x \ge 0 \\ 2 & x < 0 \end{cases}$$



[−7, 13] by [−3, 3]

77. The graph of f is the graph of $y = \frac{1}{x}$ shifted horizontally $-d/c$ units, stretched vertically by a factor of $|bc - ad|/c^2$, reflected across the x-axis if and only if $bc - ad < 0$, and then shifted vertically by a/c.

78. Yes, domain $= (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$; range $= (-\infty, 1) \cup (1, \infty)$; continuous and decreasing on each interval within their common domain; x-intercepts $= (-1 \pm \sqrt{5})/2$; no y-intercepts; hole at $(0, 1)$; horizontal asymptote of $y = 1$; vertical asymptotes of $x = \pm 1$; neither even nor odd; unbounded; no local extrema; end behavior: $\lim_{|x|\to\infty} f(x) = \lim_{|x|\to\infty} g(x) = 1$.

■ Section 2.7 Solving Equations in One Variable

Quick Review 2.7

1. The denominator is $x^2 + x - 12 = (x - 3)(x + 4)$, so the new numerator is $2x(x + 4) = 2x^2 + 8x$.
2. The numerator is $x^2 - 1 = (x - 1)(x + 1)$, so the new denominator is $(x + 1)(x + 1) = x^2 + 2x + 1$.
3. The LCD is the LCM of 12, 18, and 6, namely 36.
$$\frac{5}{12} + \frac{7}{18} - \frac{5}{6} = \frac{15}{36} + \frac{14}{36} - \frac{30}{36}$$
$$= -\frac{1}{36}$$
4. The LCD is $x(x - 1)$.
$$\frac{3}{x-1} - \frac{1}{x} = \frac{3x}{x(x-1)} - \frac{x-1}{x(x-1)}$$
$$= \frac{3x - x + 1}{x(x-1)}$$
$$= \frac{2x+1}{x^2 - x}$$
5. The LCD is $(2x + 1)(x - 3)$.
$$\frac{x}{2x+1} - \frac{2}{x-3} = \frac{x(x-3)}{(2x+1)(x-3)} - \frac{2(2x+1)}{(2x+1)(x-3)}$$
$$= \frac{x^2 - 3x - 4x - 2}{(2x+1)(x-3)}$$
$$= \frac{x^2 - 7x - 2}{(2x+1)(x-3)}$$

6. $x^2 - 5x + 6 = (x - 2)(x - 3)$ and $x^2 - x - 6 = (x + 2)(x - 3)$, so the LCD is $(x - 2)(x - 3)(x + 2)$.

$$\begin{aligned}
&\frac{x+1}{x^2-5x+6} - \frac{3x+11}{x^2-x-6} \\
&= \frac{(x+1)(x+2)}{(x-2)(x-3)(x+2)} - \frac{(3x+11)(x-2)}{(x-2)(x-3)(x+2)} \\
&= \frac{x^2+3x+2-3x^2-5x+22}{(x-2)(x-3)(x+2)} \\
&= \frac{-2x^2-2x+24}{(x-2)(x-3)(x+2)} \\
&= \frac{-2(x-3)(x+4)}{(x-2)(x-3)(x+2)} \\
&= \frac{-2x-8}{(x-2)(x+2)}, x \neq 3
\end{aligned}$$

7. For $2x^2 - 3x - 1 = 0$: $a = 2, b = -3$, and $c = -1$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-(-3) \pm \sqrt{(-3)^2-4(2)(-1)}}{2(2)} \\
&= \frac{3 \pm \sqrt{9-(-8)}}{4} = \frac{3 \pm \sqrt{17}}{4}
\end{aligned}$$

8. For $2x^2 - 5x - 1 = 0$: $a = 2, b = -5$, and $c = -1$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-(-5) \pm \sqrt{(-5)^2-4(2)(-1)}}{2(2)} \\
&= \frac{5 \pm \sqrt{25-(-8)}}{4} = \frac{5 \pm \sqrt{33}}{4}
\end{aligned}$$

9. For $3x^2 + 2x - 2 = 0$: $a = 3, b = 2$, and $c = -2$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-2 \pm \sqrt{(2)^2-4(3)(-2)}}{2(3)} \\
&= \frac{-2 \pm \sqrt{4-(-24)}}{6} = \frac{-2 \pm \sqrt{28}}{6} \\
&= \frac{-2 \pm 2\sqrt{7}}{6} = \frac{-1 \pm \sqrt{7}}{3}
\end{aligned}$$

10. For $x^2 - 3x - 9 = 0$: $a = 1, b = -3$, and $c = -9$.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(-9)}}{2(1)} \\
&= \frac{3 \pm \sqrt{9-(-36)}}{2} = \frac{3 \pm \sqrt{45}}{2} \\
&= \frac{3 \pm 3\sqrt{5}}{2}
\end{aligned}$$

Section 2.7 Exercises

1. Algebraically: $\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}$

$$\begin{aligned}
(x-2) + (x+5) &= 1 \\
2x + 3 &= 1 \\
2x &= -2 \\
x &= -1
\end{aligned}$$

Numerically: For $x = -1$,

$$\begin{aligned}
\frac{x-2}{3} + \frac{x+5}{3} &= \frac{-1-2}{3} + \frac{-1+5}{3} \\
&= \frac{-3}{3} + \frac{4}{3} \\
&= \frac{1}{3}
\end{aligned}$$

2. Algebraically: $x + 2 = \frac{15}{x}$

$$\begin{aligned}
x^2 + 2x &= 15 \ (x \neq 0) \\
x^2 + 2x - 15 &= 0 \\
(x-3)(x+5) &= 0
\end{aligned}$$

$x - 3 = 0$ or $x + 5 = 0$

$x = 3$ or $x = -5$

Numerically: For $x = 3$,
$x + 2 = 3 + 2 = 5$ and
$\frac{15}{x} = \frac{15}{3} = 5.$
For $x = -5$,
$x + 2 = -5 + 2 = -3$ and
$\frac{15}{x} = \frac{15}{-5} = -3.$

3. Algebraically: $x + 5 = \frac{14}{x}$

$$\begin{aligned}
x^2 + 5x &= 14 \quad (x \neq 0) \\
x^2 + 5x - 14 &= 0 \\
(x-2)(x+7) &= 0
\end{aligned}$$

$x - 2 = 0$ or $x + 7 = 0$

$x = 2$ or $x = -7$

Numerically: For $x = 2$,
$x + 5 = 2 + 5 = 7$ and
$\frac{14}{x} = \frac{14}{2} = 7.$
For $x = -7$,
$x + 5 = -7 + 5 = -2$ and
$\frac{14}{x} = \frac{14}{-7} = -2.$

4. Algebraically: $\frac{1}{x} - \frac{2}{x-3} = 4$

$$\begin{aligned}
(x-3) - 2x &= 4x(x-3) \ (x \neq 0, 3) \\
-x - 3 &= 4x^2 - 12x \\
-4x^2 + 11x - 3 &= 0 \\
x &= \frac{-11 \pm \sqrt{11^2-4(-4)(-3)}}{2(-4)} \\
&= \frac{-11 \pm \sqrt{73}}{-8}
\end{aligned}$$

$$x = \frac{11+\sqrt{73}}{8} \approx 2.443 \text{ or } x = \frac{11-\sqrt{73}}{8} \approx 0.307$$

Numerically: Use a graphing calculator to support your answers numerically.

5. Algebraically: $x + \dfrac{4x}{x-3} = \dfrac{12}{x-3}$

$x(x-3) + 4x = 12 \ (x \neq 3)$
$x^2 - 3x + 4x = 12$
$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$x + 4 = 0$ or $x - 3 = 0$
$x = -4$ or $x = 3$ — but $x = 3$ is extraneous.

Numerically: For $x = -4$,
$x + \dfrac{4x}{x-3} = -4 + \dfrac{4(-4)}{-4-3} = -4 + \dfrac{16}{7} = -\dfrac{12}{7}$ and
$\dfrac{12}{x-3} = \dfrac{12}{-4-3} = -\dfrac{12}{7}$.

6. Algebraically: $\dfrac{3}{x-1} + \dfrac{2}{x} = 8$

$3x + 2(x-1) = 8x(x-1) \quad (x \neq 0, 1)$
$5x - 2 = 8x^2 - 8x$
$-8x^2 + 13x - 2 = 0$

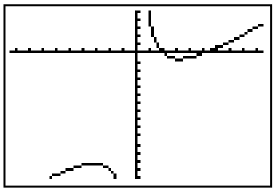
$$x = \frac{-13 \pm \sqrt{13^2 - 4(-8)(-2)}}{2(-8)}$$
$$= \frac{-13 \pm \sqrt{105}}{-16}$$
$$x = \frac{13 + \sqrt{105}}{16} \approx 1.453 \text{ or } x = \frac{13 - \sqrt{105}}{16} \approx 0.172$$

Numerically: Use a graphing calculator to support your answers numerically.

7. Algebraically: $x + \dfrac{10}{x} = 7$

$x^2 + 10 = 7x \quad (x \neq 0)$
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x - 2 = 0$ or $x - 5 = 0$
$x = 2$ or $x = 5$

Graphically: The graph of $f(x) = x + \dfrac{10}{x} - 7$ suggests that the x-intercepts are 2 and 5.



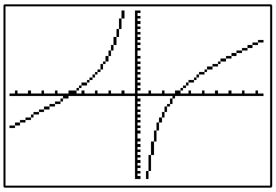
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 2$ and $x = 5$.

8. Algebraically: $x + 2 = \dfrac{15}{x}$

$x^2 + 2x = 15 \quad (x \neq 0)$
$x^2 + 2x - 15 = 0$
$(x+5)(x-3) = 0$
$x + 5 = 0$ or $x - 3 = 0$
$x = -5$ or $x = 3$

Graphically: The graph of $f(x) = x + 2 - \dfrac{15}{x}$ suggests that the x-intercepts are -5 and 3.



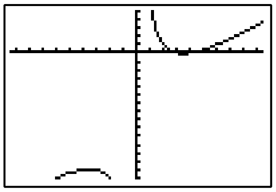
[–9.4, 9.4] by [–15, 15]

Then the solutions are $x = -5$ and $x = 3$.

9. Algebraically: $x + \dfrac{12}{x} = 7$

$x^2 + 12 = 7x \quad (x \neq 0)$
$x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
$x - 3 = 0$ or $x - 4 = 0$
$x = 3$ or $x = 4$

Graphically: The graph of $f(x) = x + \dfrac{12}{x} - 7$ suggests that the x-intercepts are 3 and 4.



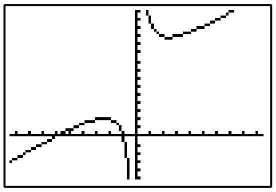
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 3$ and $x = 4$.

10. Algebraically: $x + \dfrac{6}{x} = -7$

$x^2 + 6 = -7x \quad (x \neq 0)$
$x^2 + 7x + 6 = 0$
$(x+6)(x+1) = 0$
$x + 6 = 0$ or $x + 1 = 0$
$x = -6$ or $x = -1$

Graphically: The graph of $f(x) = x + \dfrac{6}{x} + 7$ suggests that the x-intercepts are -6 and -1.



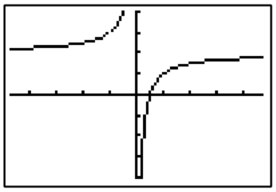
[–9.4, 9.4] by [–5, 15]

Then the solutions are $x = -6$ and $x = -1$.

11. Algebraically: $2 - \dfrac{1}{x+1} = \dfrac{1}{x^2+x}$

$[\text{and } x^2 + x = x(x+1)]$
$2(x^2 + x) - x = 1 \quad (x \neq 0, -1)$
$2x^2 + x - 1 = 0$
$(2x-1)(x+1) = 0$
$2x - 1 = 0$ or $x + 1 = 0$
$x = \dfrac{1}{2}$ or $x = -1$
— but $x = -1$ is extraneous.

Graphically: The graph of $f(x) = 2 - \dfrac{1}{x+1} - \dfrac{1}{x^2+x}$ suggests that the x-intercept is $\dfrac{1}{2}$. There is a hole at $x = -1$.



[–4.7, 4.7] by [–4, 4]

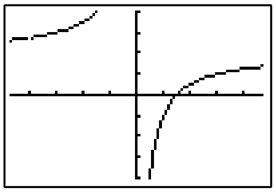
Then the solution is $x = \frac{1}{2}$.

12. Algebraically: $2 - \frac{3}{x+4} = \frac{12}{x^2 + 4x}$
$[\text{and } x^2 + 4x = x(x+4)]$

$$\begin{aligned} &2(x^2 + 4x) - 3x = 12 \quad (x \neq 0, -4) \\ &2x^2 + 5x - 12 = 0 \\ &(2x - 3)(x + 4) = 0 \\ &2x - 3 = 0 \quad \text{or} \quad x + 4 = 0 \\ &x = \frac{3}{2} \quad \text{or} \quad x = -4 \end{aligned}$$

— but $x = -4$ is extraneous.

Graphically: The graph of $f(x) = 2 - \frac{3}{x+4} - \frac{12}{x^2 + 4x}$ suggests that the x-intercept is $\frac{3}{2}$. There is a hole at $x = -4$.



[–4.7, 4.7] by [–4, 4]

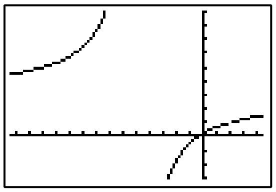
Then the solution is $x = \frac{3}{2}$.

13. Algebraically: $\frac{3x}{x+5} + \frac{1}{x-2} = \frac{7}{x^2 + 3x - 10}$
$[\text{and } x^2 + 3x - 10 = (x+5)(x-2)]$

$$\begin{aligned} &3x(x-2) + (x+5) = 7 \quad (x \neq -5, 2) \\ &3x^2 - 5x - 2 = 0 \\ &(3x + 1)(x - 2) = 0 \\ &3x + 1 = 0 \quad \text{or} \quad x - 2 = 0 \\ &x = -\frac{1}{3} \quad \text{or} \quad x = 2 \end{aligned}$$

— but $x = 2$ is extraneous.

Graphically: The graph of $f(x) = \frac{3x}{x+5} + \frac{1}{x-2} - \frac{7}{x^2 + 3x - 10}$ suggests that the x-intercept is $-\frac{1}{3}$. There is a hole at $x = 2$.



[–14.4, 4.4] by [–3, 9]

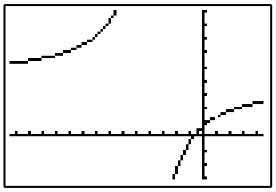
Then the solution is $x = -\frac{1}{3}$.

14. Algebraically: $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2 + 3x - 4}$
$[\text{and } x^2 + 3x - 4 = (x+4)(x-1)]$

$$\begin{aligned} &4x(x-1) + 3(x+4) = 15 \ (x \neq -4, 1) \\ &4x^2 - x - 3 = 0 \\ &(4x + 3)(x - 1) = 0 \\ &4x + 3 = 0 \quad \text{or} \quad x - 1 = 0 \\ &x = -\frac{3}{4} \quad \text{or} \quad x = 1 \end{aligned}$$

— but $x = 1$ is extraneous.

Graphically: The graph of $f(x) = \frac{4x}{x+4} + \frac{3}{x-1} - \frac{15}{x^2 + 3x - 4}$ suggests that the x-intercept is $-\frac{3}{4}$. There is a hole at $x = 1$.



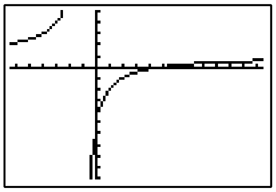
[–12.4, 6.4] by [–5, 10]

Then the solution is $x = -\frac{3}{4}$.

15. Algebraically: $\frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2 + x} = 0$
$[\text{and } x^2 + x = x(x+1)]$

$$\begin{aligned} &(x-3)(x+1) - 3x + 3 = 0 \quad (x \neq 0, -1) \\ &x^2 - 5x = 0 \\ &x(x - 5) = 0 \\ &x = 0 \quad \text{or} \quad x - 5 = 0 \\ &x = 0 \quad \text{or} \quad x = 5 \text{ — but } x = 0 \text{ is extraneous.} \end{aligned}$$

Graphically: The graph of $f(x) = \frac{x-3}{x} - \frac{3}{x+1} + \frac{3}{x^2 + x}$ suggests that the x-intercept is 5. The x-axis hides a hole at $x = 0$.



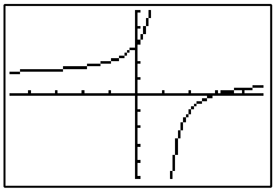
[–6.4, 12.4] by [–10, 5]

Then the solution is $x = 5$.

16. Algebraically: $\frac{x+2}{x} - \frac{4}{x-1} + \frac{2}{x^2 - x} = 0$
$[\text{and } x^2 - x = x(x-1)]$

$$\begin{aligned} &(x+2)(x-1) - 4x + 2 = 0 \quad (x \neq 0, 1) \\ &x^2 - 3x = 0 \\ &x(x - 3) = 0 \\ &x = 0 \quad \text{or} \quad x - 3 = 0 \\ &x = 0 \quad \text{or} \quad x = 3 \text{ — but } x = 0 \text{ is extraneous.} \end{aligned}$$

Graphically: The graph of
$f(x) = \frac{x+2}{x} - \frac{4}{x-1} + \frac{2}{x^2-x}$
suggests that the x-intercept is 3. The x-axis hides a hole at $x = 0$.



[–4.7, 4.7] by [–5, 5]

Then the solution is $x = 3$.

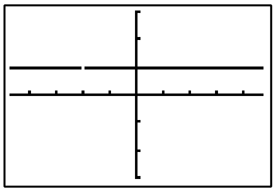
17. Algebraically: $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$

[and $x^2 + 2x = x(x+2)$]

$$\begin{aligned} 3x + 6 &= (3-x)(x+2) \quad (x \neq -2, 0) \\ 3x + 6 &= -x^2 + x + 6 \\ x^2 + 2x &= 0 \\ x(x+2) &= 0 \\ x = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = 0 \quad &\text{or} \quad x = -2 \end{aligned}$$

— but both solutions are extraneous.
No real solutions.

Graphically: The graph of
$f(x) = \frac{3}{x+2} + \frac{6}{x^2+2x} - \frac{3-x}{x}$ suggests that there are no x-intercepts. There is a hole at $x = -2$, and the x-axis hides a "hole" at $x = 0$.



[–4.7, 4.7] by [–3, 3]

Then there are no real solutions.

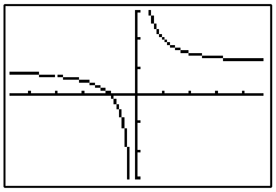
18. Algebraically: $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

[and $x^2 + 3x = x(x+3)$]

$$\begin{aligned} (x+3)^2 - 2x &= 6 \quad (x \neq -3, 0) \\ x^2 + 4x + 3 &= 0 \\ (x+1)(x+3) &= 0 \\ x + 1 = 0 \quad &\text{or} \quad x + 3 = 0 \\ x = -1 \quad &\text{or} \quad x = -3 \end{aligned}$$

— but $x = -3$ is extraneous.

Graphically: The graph of
$f(x) = \frac{x+3}{x} - \frac{2}{x+3} - \frac{6}{x^2+3x}$
suggests that the x-intercept is -1. There is a hole at $x = -3$.



[–4.7, 4.7] by [–3, 3]

Then the solution is $x = -3$.

19. There is no x-intercept at $x = -2$. That is the extraneous solution.

20. There is no x-intercept at $x = 3$. That is the extraneous solution.

21. Neither possible solution corresponds to an x-intercept of the graph. Both are extraneous.

22. There is no x-intercept at $x = 3$. That is the extraneous solution.

23.
$$\begin{aligned} \frac{2}{x-1} + x &= 5 \\ 2 + x(x-1) &= 5(x-1) \quad (x \neq 1) \\ x^2 - x + 2 &= 5x - 5 \\ x^2 - 6x + 7 &= 0 \end{aligned}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$$

$x = 3 + \sqrt{2} \approx 4.414$ or

$x = 3 - \sqrt{2} \approx 1.586$

24.
$$\begin{aligned} \frac{x^2 - 6x + 5}{x^2 - 2} &= 3 \\ x^2 - 6x + 5 &= 3(x^2 - 2) \quad (x \neq \pm\sqrt{2}) \\ -2x^2 - 6x + 11 &= 0 \end{aligned}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-2)(11)}}{2(-2)}$$

$$x = \frac{6 \pm \sqrt{124}}{-4} = \frac{-3 \pm \sqrt{31}}{2}$$

$$x = \frac{-3 + \sqrt{31}}{2} \approx 1.284 \text{ or}$$

$$x = \frac{-3 - \sqrt{31}}{2} \approx -4.284$$

25.
$$\begin{aligned} \frac{x^2 - 2x + 1}{x + 5} &= 0 \\ x^2 - 2x + 1 &= 0 \quad (x \neq 5) \\ (x-1)^2 &= 0 \\ x - 1 &= 0 \\ x &= 1 \end{aligned}$$

26. $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2+x-2}$

[and $x^2 + x - 2 = (x+2)(x-1)$]

$$\begin{aligned} 3x(x-1) + 2(x+2) &= 5 \quad (x \neq -2, 1) \\ 3x^2 - x - 1 &= 0 \end{aligned}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{13}}{6}$$

$$x = \frac{1 + \sqrt{13}}{6} \approx 0.768 \text{ or}$$

$$x = \frac{1 - \sqrt{13}}{6} \approx -0.434$$

27. $\dfrac{4x}{x+4} + \dfrac{5}{x-1} = \dfrac{15}{x^2+3x-4}$

[and $x^2 + 3x - 4 = (x + 4)(x - 1)$]

$4x(x - 1) + 5(x + 4) = 15 \quad (x \neq -4, 1)$

$4x^2 + x + 5 = 0$

The discriminant is $b^2 - 4ac = 1^2 - 4(4)(5) = -79 < 0$. There are no real solutions.

28. $\dfrac{3x}{x+1} + \dfrac{5}{x-2} = \dfrac{15}{x^2-x-2}$

[and $x^2 - x - 2 = (x + 1)(x - 2)$]

$3x(x - 2) + 5(x + 1) = 15 \quad (x \neq -1, 2)$

$3x^2 - x - 10 = 0$

$(3x + 5)(x - 2) = 0$

$3x + 5 = 0 \quad \text{or} \quad x - 2 = 0$

$x = -\dfrac{5}{3} \quad \text{or} \quad x = 2$

— but $x = 2$ is extraneous.

The solution is $x = -\dfrac{5}{3}$.

29. $x^2 + \dfrac{5}{x} = 8$

$x^3 + 5 = 8x \quad (x \neq 0)$

Using a graphing calculator to find the x-intercepts of $f(x) = x^3 - 8x + 5$ yields the solutions $x \approx -3.100$, $x \approx 0.661$, and $x \approx 2.439$.

30. $x^2 - \dfrac{3}{x} = 7$

$x^3 - 3 = 7x \quad (x \neq 0)$

Using a graphing calculator to find the x-intercepts of $f(x) = x^3 - 7x - 3$ yields the solutions $x \approx -2.398$, $x \approx -0.441$, and $x \approx 2.838$.

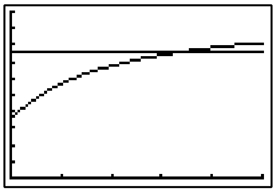
31. **(a)** The total amount of solution is $(125 + x)$ mL; of this, the amount of acid is x plus 60% of the original amount, or $x + 0.6(125)$.

(b) $y = 0.83$

(c) $C(x) = \dfrac{x + 75}{x + 125} = 0.83$. Multiply both sides by $x + 125$, then rearrange to get $0.17x = 28.75$, so that $x \approx 169.12$ mL.

32. **(a)** $C(x) = \dfrac{x + 0.35(100)}{x + 100} = \dfrac{x + 35}{x + 100}$

(b) Graph $C(x)$ along with $y = 0.75$; observe where the first graph intersects the second.



[0, 250] by [0, 1]

For $x = 160$, $C(x) = 0.75$. Use 160 mL.

(c) Starting from $\dfrac{x + 35}{x + 100} = 0.75$, multiply by $x + 100$ and rearrange to get $0.25x = 40$, so that $x = 160$ mL That is how much pure acid must be added.

33. **(a)** $C(x) = \dfrac{3000 + 2.12x}{x}$

(b) A profit is realized if $C(x) < 2.75$, or $3000 + 2.12x < 2.75x$. Then $3000 < 0.63x$, so that $x > 4761.9$—4762 hats per week.

(c) They must have $2.75x - (3000 + 2.12x) > 1000$ or $0.63x > 4000$: 6350 hats per week.

34. **(a)** $P(10) = 200$, $P(40) = 350$, $P(100) = 425$

(b) As $t \to \infty$, $P(t) \to 500$. So, yes. The horizontal asymptote is $y = 500$.

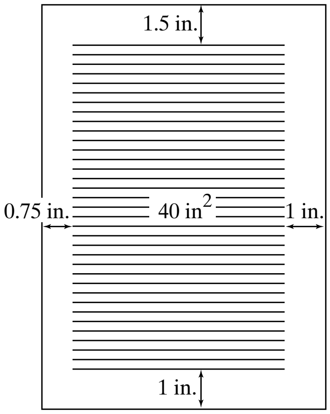
(c) $\lim\limits_{t\to\infty} P(t) = \lim\limits_{t\to\infty}\left(500 - \dfrac{9000}{t + 20}\right) = 500$, so the bear population will never exceed 500.

35. **(a)** If x is the length, then $182/x$ is the width.

$P(x) = 2x + 2\left(\dfrac{182}{x}\right) = 2x + \dfrac{364}{x}$

(b) The graph of $P(x) = 2x + 364/x$ has a minimum when $x \approx 13.49$, so that the rectangle is square. Then $P(13.49) = 2(13.49) + 364/13.49 \approx 53.96$ ft.

36. **(a)**



The height of the print material is $40/x$. The total area is

$$A(x) = (x + 0.75 + 1)\left(\frac{40}{x} + 1.5 + 1\right)$$
$$= (x + 1.75)\left(\frac{40}{x} + 2.5\right)$$

(b) The graph of $A(x) = (x + 1.75)\left(\dfrac{40}{x} + 2.5\right)$ has a minimum when $x \approx 5.29$, so the dimensions are about $5.29 + 1.75 = 7.04$ in. wide by $40/5.29 + 2.5 \approx 10.06$ in. high. And $A(5.29) \approx 70.8325$ in.^{2}

37. **(a)** Since $V = \pi r^2 h$, the height here is $V/(\pi r^2)$. And since in general, $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2V/r$, here $S(x) = 2\pi x^2 + 1000/x$ $(0.5\ L = 500\ \text{cm}^3)$.

(b) Solving $2\pi x^2 + 1000/x = 900$ graphically by finding the zeros of $f(x) = 2\pi x^2 + 1000/x - 900$ yields two solutions: either $x \approx 1.12$ cm, in which case $h \approx 126.88$ cm, or $x \approx 11.37$ cm, in which case $h \approx 1.23$ cm.

38. (a) If x is the length, then $1000/x$ is the width. The total area is $A(x) = (x + 4)(1000/x + 4)$.

(b) The least area comes when the pool is square, so that $x = \sqrt{1000} \approx 31.62$ ft. With dimensions 35.62 ft × 35.62 ft for the plot of land, $A(31.62) \approx 1268.98\ \text{ft}^2$.

39. (a)

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \frac{1}{R} &= \frac{1}{2.3} + \frac{1}{x} \\ 2.3x &= xR + 2.3R \\ R(x) &= \frac{2.3x}{x + 2.3} \end{aligned}$$

(b)

$$\begin{aligned} 2.3x &= xR + 2.3R \\ x &= \frac{2.3R}{2.3 - R} \end{aligned}$$

For $R = 1.7$, $x \approx 6.52$ ohms.

40. (a) If x is the length, then $200/x$ is the width.

$$P(x) = 2x + 2\left(\frac{200}{x}\right) = 2x + \frac{400}{x}$$

(b)

$$\begin{aligned} 70 &= 2x + \frac{400}{x} \\ 70x &= 2x^2 + 400 \\ 2x^2 - 70x + 400 &= 0 \end{aligned}$$

The quadratic formula gives $x \approx 7.1922$ or $x \approx 27.8078$. When one of those values is considered as the length, the other is the width. The dimensions are 7.1922 m × 27.8078 m.

41. (a) Drain A can drain $1/4.75$ of the pool per hour, while drain B can drain $1/t$ of the pool per hour. Together, they can drain a fraction

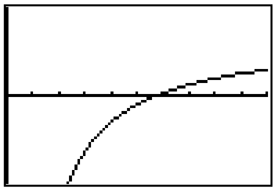
$$D(t) = \frac{1}{4.75} + \frac{1}{t} = \frac{t + 4.75}{4.75t}$$

of the pool in 1 hour.

(b) The information implies that $D(t) = 1/2.6$, so we solve

$$\frac{1}{2.6} = \frac{1}{4.75} + \frac{1}{t}.$$

Graphically: The function $f(t) = \frac{1}{2.6} - \frac{1}{4.75} - \frac{1}{t}$ has a zero at $t \approx 5.74$ h, so that is the solution.



[0, 10] by [–0.25, 0.25]

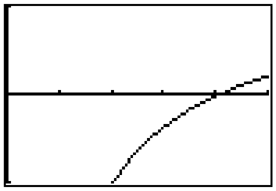
Algebraically:

$$\begin{aligned} \frac{1}{2.6} &= \frac{1}{4.75} + \frac{1}{t} \\ 4.75t &= 2.6t + 2.6(4.75) \\ t &= \frac{2.6\,(4.75)}{4.75 - 2.6} \\ &\approx 5.74 \end{aligned}$$

42. (a) With x as the bike speed, $x + 43$ is the car speed. Biking time $= 17/x$ and driving time $= 53/(x + 43)$, so

$$T = \frac{17}{x} + \frac{53}{x + 43}.$$

(b) Graphically: The function $f(x) = \frac{5}{3} - \frac{17}{x} - \frac{53}{x + 43}$ has a zero at $x \approx 20.45$.



[0, 25] by [–1, 1]

Algebraically: 1 h 40 min $= 1\frac{2}{3}\text{ h} = \frac{5}{3}\text{ h}$

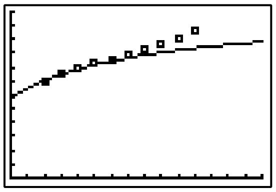
$$\begin{aligned} \frac{5}{3} &= \frac{17}{x} + \frac{53}{x + 43} \\ 5x(x + 43) &= 51(x + 43) + 159x \\ 5x^2 + 215x &= 51x + 2193 + 159x \\ 5x^2 + 5x - 2193 &= 0 \end{aligned}$$

Using the quadratic formula and selecting the positive solution yields

$$x = \frac{-5 + \sqrt{43{,}885}}{10} \approx 20.45.$$

The rate of the bike was about 20.45 mph.

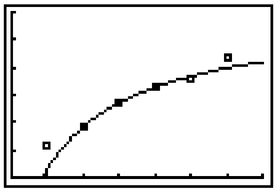
43. (a)



[0, 15] by [0, 120]

(b) When $x = 15$, $y = 120 - 500/(15 + 8) \approx 98.3$. In 2005, sales are estimated at about \$98.3 billion.

44. (a)



[0, 35] by [0, 3000]

(b) When $x = 35$, $y = 3000 - 39{,}500/(35 + 9) \approx 2102$. In 2005, the number of wineries is estimated to be 2102.

45. False. An extraneous solution is a value that, though generated by the solution-finding process, does not work in the original equation. In an equation containing rational expressions, an extraneous solution is typically a solution to the version of the equation that has been cleared of fractions but not to the original version.

46. True. For a fraction to equal zero, the numerator has to be zero, and 1 is not zero.

47. $x - \dfrac{3x}{x+2} = \dfrac{6}{x+2}$

$x(x+2) - 3x = 6 \quad (x \neq -2)$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$ — but $x = -2$ is extraneous.

The answer is D.

48. $1 - \dfrac{3}{x} = \dfrac{6}{x^2 + 2x}$ [and $x^2 + 2x = x(x+2)$]

$x^2 + 2x - 3(x+2) = 6 \quad (x \neq -2, 0)$

$x^2 - x - 12 = 0$

$(x+3)(x-4) = 0$

$x = -3$ or $x = 4$

The answer is C.

49. $\dfrac{x}{x+2} + \dfrac{2}{x-5} = \dfrac{14}{x^2 - 3x - 10}$

[and $x^2 - 3x - 10 = (x+2)(x-5)$]

$x(x-5) + 2(x+2) = 14 \quad (x \neq -2, 5)$

$x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

$x = -2$ or $x = 5$ — but both solutions are extraneous.

The answer is E.

50. 0.2×10 or $2 =$ liters of pure acid in 20% solution

0.30×30 or $9 =$ liters of pure acid in 30% solution

$\dfrac{\text{L of pure acid}}{\text{L of mixture}} =$ concentration of acid

$\dfrac{2+9}{10+30} = \dfrac{11}{40} = 0.275 = 27.5\%$

The answer is D.

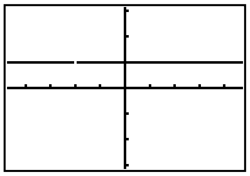
51. **(a)** The LCD is $x^2 + 2x = x(x+2)$.

$$\begin{aligned} f(x) &= \frac{x-3}{x} + \frac{3}{x+2} + \frac{6}{x^2+2x} \\ &= \frac{(x-3)(x+2)}{x^2+2x} + \frac{3x}{x^2+2x} + \frac{6}{x^2+2x} \\ &= \frac{x^2 - x - 6 + 3x + 6}{x^2+2x} \\ &= \frac{x^2+2x}{x^2+2x} \end{aligned}$$

(b) All $x \neq 0, -2$

(c) $f(x) = \begin{cases} 1 & x \neq -2, 0 \\ \text{undefined} & x = -2 \text{ or } x = 0 \end{cases}$

(d) The graph appears to be the horizontal line $y = 1$ with holes at $x = -2$ and $x = 0$.



[–4.7, 4.7] by [–3.1, 3.1]

This matches the definition in part (c).

52. $y = 1 + \dfrac{1}{1+x}$

$y(1+x) = (1+x) + 1$

$y + xy = x + 2$

$xy - x = 2 - y$

$x = \dfrac{2-y}{y-1}$

53. $y = 1 - \dfrac{1}{1-x}$

$y(1-x) = (1-x) - 1$

$y - xy = -x$

$y = xy - x$

$x = \dfrac{y}{y-1}$

54. $y = 1 + \dfrac{1}{1 + \dfrac{1}{x}}$

$y = 1 + \dfrac{x}{1+x}$

$y(1+x) = (1+x) + x$

$y + xy = 2x + 1$

$xy - 2x = 1 - y$

$x = \dfrac{1-y}{y-2}$

55. $y = 1 + \dfrac{1}{1 + \dfrac{1}{1-x}}$

$y = 1 + \dfrac{1-x}{1-x+1}$

$y = 1 + \dfrac{1-x}{2-x}$

$y(2-x) = (2-x) + (1-x)$

$2y - xy = 3 - 2x$

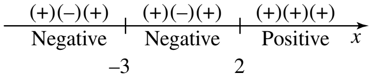
$2y - 3 = xy - 2x$

$x = \dfrac{2y-3}{y-2}$

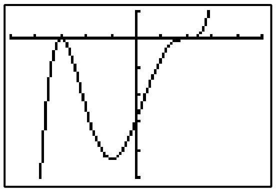
■ Section 2.8 Solving Inequalities in One Variable

Exploration 1

1. (a)

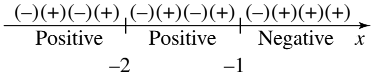


(b)

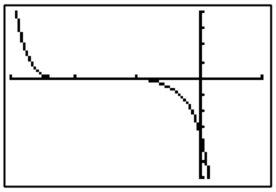


[–5, 5] by [–250, 50]

2. (a)

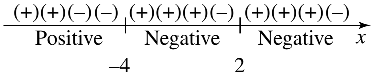


(b)

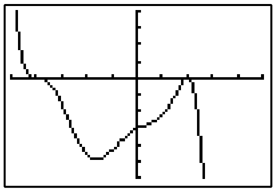


$[-3, 1]$ by $[-30, 20]$

3. (a)



(b)



$[-5, 5]$ by $[-3000, 2000]$

Quick Review 2.8

1. $\lim\limits_{x\to\infty} f(x) = \infty, \lim\limits_{x\to-\infty} f(x) = -\infty$

2. $\lim\limits_{x\to\infty} f(x) = -\infty, \lim\limits_{x\to-\infty} f(x) = -\infty$

3. $\lim\limits_{x\to\infty} g(x) = \infty, \lim\limits_{x\to-\infty} g(x) = \infty$

4. $\lim\limits_{x\to\infty} g(x) = \infty, \lim\limits_{x\to-\infty} g(x) = -\infty$

5. $\dfrac{x^3 + 5}{x}$

6. $\dfrac{x^3 - 3}{x}$

7. $$\frac{x(x-3) - 2(2x+1)}{(2x+1)(x-3)} = \frac{x^2 - 3x - 4x - 2}{(2x+1)(x-3)} = \frac{x^2 - 7x - 2}{(2x+1)(x-3)} = \frac{x^2 - 7x - 2}{2x^2 - 5x - 3}$$

8. $$\frac{x(3x-4) + (x+1)(x-1)}{(x-1)(3x-4)} = \frac{3x^2 - 4x + x^2 - 1}{(x-1)(3x-4)} = \frac{4x^2 - 4x - 1}{(x-1)(3x-4)} = \frac{4x^2 - 4x - 1}{3x^2 - 7x + 4}$$

9. (a) $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2}$ or $\pm 1, \pm\dfrac{1}{2}, \pm 3, \pm\dfrac{3}{2}$

(b) A graph suggests that -1 and $\dfrac{3}{2}$ are good candidates for zeros.

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -4 & -3 \\ & & -2 & 1 & 3 \\ \hline 3/2 & 2 & -1 & -3 & 0 \\ & & 3 & 3 & \\ \hline & 2 & 2 & 0 & \end{array}$$

$$2x^3 + x^2 - 4x - 3 = (x+1)\left(x - \frac{3}{2}\right)(2x+2) = (x+1)(2x-3)(x+1)$$

10. (a) $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$

or $\pm 1, \pm\dfrac{1}{3}, \pm 2, \pm\dfrac{2}{3}, \pm 4, \pm\dfrac{4}{3}, \pm 8, \pm\dfrac{8}{3}$

(b) A graph suggests that -2 and 1 are good candidates for zeros.

$$\begin{array}{r|rrrr} -2 & 3 & -1 & -10 & 8 \\ & & -6 & 14 & -8 \\ \hline 1 & 3 & -7 & 4 & 0 \\ & & 3 & -4 & \\ \hline & 3 & -4 & 0 & \end{array}$$

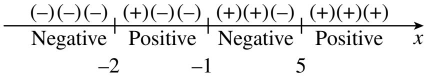
$3x^2 - x^2 - 10x + 8 = (x+2)(x-1)(3x-4)$

Section 2.8 Exercises

1. (a) $f(x) = 0$ when $x = -2, -1, 5$

(b) $f(x) > 0$ when $-2 < x < -1$ or $x > 5$

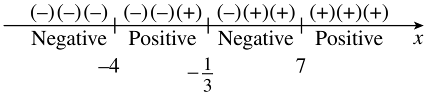
(c) $f(x) < 0$ when $x < -2$ or $-1 < x < 5$



2. (a) $f(x) = 0$ when $x = 7, -\dfrac{1}{3}, -4$

(b) $f(x) > 0$ when $-4 < x < -\dfrac{1}{3}$ or $x > 7$

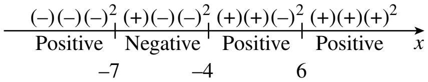
(c) $f(x) < 0$ when $x < -4$ or $-\dfrac{1}{3} < x < 7$



3. (a) $f(x) = 0$ when $x = -7, -4, 6$

(b) $f(x) > 0$ when $x < -7$ or $-4 < x < 6$ or $x > 6$

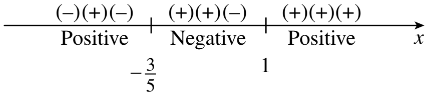
(c) $f(x) < 0$ when $-7 < x < -4$



4. (a) $f(x) = 0$ when $x = -\dfrac{3}{5}, 1$

(b) $f(x) > 0$ when $x < -\dfrac{3}{5}$ or $x > 1$

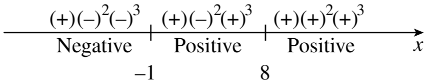
(c) $f(x) < 0$ when $-\dfrac{3}{5} < x < 1$



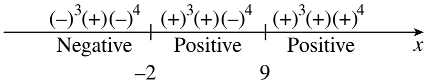
5. (a) $f(x) = 0$ when $x = 8, -1$

(b) $f(x) > 0$ when $-1 < x < 8$ or $x > 8$

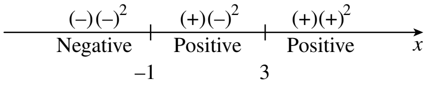
(c) $f(x) < 0$ when $x < -1$



6. **(a)** $f(x) = 0$ when $x = -2, 9$
(b) $f(x) > 0$ when $-2 < x < 9$ or $x > 9$
(c) $f(x) < 0$ when $x < -2$

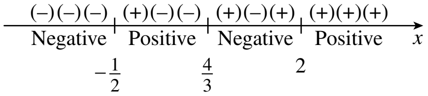


7. $(x + 1)(x - 3)^2 = 0$ when $x = -1, 3$



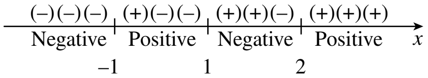
By the sign chart, the solution of $(x + 1)(x - 3)^2 > 0$ is $(-1, 3) \cup (3, \infty)$.

8. $(2x + 1)(x - 2)(3x - 4) = 0$ when $x = -\frac{1}{2}, 2, \frac{4}{3}$



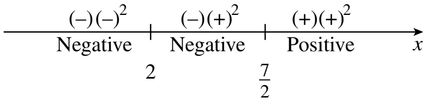
By the sign chart, the solution of $(2x + 1)(x - 2)(3x - 4) \le 0$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{4}{3}, 2\right]$.

9. $(x + 1)(x^2 - 3x + 2) = (x + 1)(x - 1)(x - 2) = 0$ when $x = -1, 1, 2$



By the sign chart, the solution of $(x + 1)(x - 1)(x - 2) < 0$ is $(-\infty, -1) \cup (1, 2)$.

10. $(2x - 7)(x^2 - 4x + 4) = (2x - 7)(x - 2)^2 = 0$ when $x = \frac{7}{2}, 2$

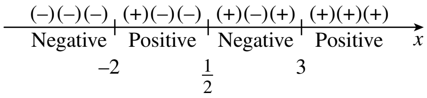


By the sign chart, the solution of $(2x - 7)(x - 2)^2 > 0$ is $\left(\frac{7}{2}, \infty\right)$.

11. By the Rational Zeros Theorem, the possible rational zeros are $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6$. A graph suggests that $-2, \frac{1}{2}$, and 3 are good candidates to be zeros.

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline 3 & 2 & -7 & 3 & 0 \\ & & 6 & -3 & \\ \hline & 2 & -1 & 0 & \end{array}$$

$2x^3 - 3x^2 - 11x + 6 = (x + 2)(x - 3)(2x - 1) = 0$ when $x = -2, 3, \frac{1}{2}$

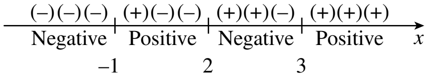


By the sign chart, the solution of $(x + 2)(x - 3)(2x - 1) \ge 0$ is $\left[-2, \frac{1}{2}\right] \cup [3, \infty)$.

12. By the Rational Zeros Theorem, the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6$. A graph suggests that -1, 2, and 3 are good candidates to be zeros.

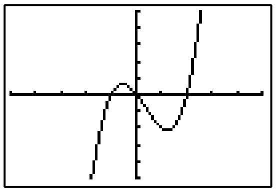
$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline 2 & 1 & -5 & 6 & 0 \\ & & 2 & -6 & \\ \hline & 1 & -3 & 0 & \end{array}$$

$x^3 - 4x^2 + x + 6 = (x + 1)(x - 2)(x - 3) = 0$ when $x = -1, 2, 3$.



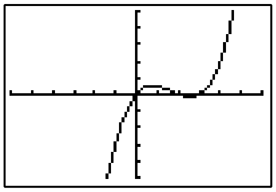
By the sign chart, the solution of $(x + 1)(x - 2)(x - 3) \le 0$ is $(-\infty, -1] \cup [2, 3]$.

13. The zeros of $f(x) = x^3 - x^2 - 2x$ appear to be -1, 0, and 2. Substituting these values into f confirms this. The graph shows that the solution of $x^3 - x^2 - 2x \ge 0$ is $[-1, 0] \cup [2, \infty)$.



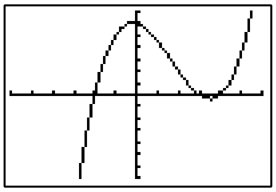
$[-5, 5]$ by $[-5, 5]$

14. The zeros of $f(x) = 2x^3 - 5x^2 + 3x$ appear to be 0, 1, and $\frac{3}{2}$. Substituting these values into f confirms this. The graph shows that the solution of $2x^3 - 5x^2 + 3x < 0$ is $(-\infty, 0) \cup \left(1, \frac{3}{2}\right)$.



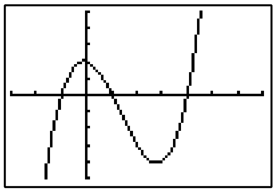
$[-3, 3]$ by $[-5, 5]$

15. The zeros of $f(x) = 2x^3 - 5x^2 - x + 6$ appear to be $-1, \frac{3}{2}$ and 2. Substituting these values into f confirms this. The graph shows that the solution of $2x^3 - 5x^2 - x + 6 > 0$ is $\left(-1, \frac{3}{2}\right) \cup (2, \infty)$.



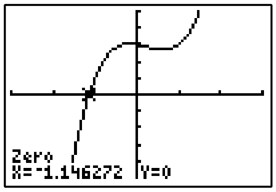
$[-3, 3]$ by $[-7, 7]$

16. The zeros of $f(x) = x^3 - 4x^2 - x + 4$ appear to be -1, 1, and 4. Substituting these values into f confirms this. The graph shows that the solution of $x^3 - 4x^2 - x + 4 \le 0$ is $(-\infty, -1] \cup [1, 4]$.



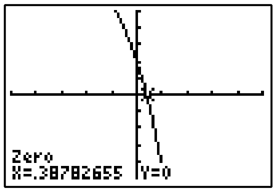
$[-3, 7]$ by $[-10, 10]$

17. The only zero of $f(x) = 3x^3 - 2x^2 - x + 6$ is found graphically to be $x \approx -1.15$. The graph shows that the solution of $3x^3 - 2x^2 - x + 6 \ge 0$ is approximately $[-1.15, \infty)$.



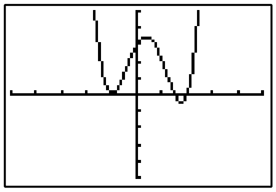
$[-3, 3]$ by $[-10, 10]$

18. The only zero of $f(x) = -x^3 - 3x^2 - 9x + 4$ is found graphically to be $x \approx 0.39$. The graph shows that the solution of $-x^3 - 3x^2 - 9x + 4 < 0$ is approximately $(0.39, \infty)$.



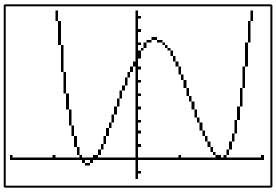
$[-5, 5]$ by $[-10, 10]$

19. The zeros of $f(x) = 2x^4 - 3x^3 - 6x^2 + 5x + 6$ appear to be $-1, \frac{3}{2}$, and 2. Substituting these into f confirms this. The graph shows that the solution of $2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0$ is $\left(\frac{3}{2}, 2\right)$.



$[-5, 5]$ by $[-10, 10]$

20. The zero of $f(x) = 3x^4 - 5x^3 - 12x^2 + 12x + 16$ appear to be $-\frac{4}{3}$,-1, and 2. Substituting these into f confirms this. The graph shows that the solution of $3x^4 - 5x^3 - 12x^2 + 12x + 16 \ge 0$ is $\left(-\infty, -\frac{4}{3}\right] \cup [-1, \infty)$.



$[-3, 3]$ by $[-3, 23]$

21. $f(x) = (x^2 + 4)(2x^2 + 3)$

(a) The solution is $(-\infty, \infty)$, because both factors of $f(x)$ are always positive.

(b) $(-\infty, \infty)$, for the same reason as in part (a).

(c) There are no solutions, because both factors of $f(x)$ are always positive.

(d) There are no solutions, for the same reason as in part (c).

22. $f(x) = (x^2 + 1)(-2 - 3x^2)$

(a) There are no solutions, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

(b) There are no solutions, for the same reason as in part (a).

(c) $(-\infty, \infty)$, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

(d) $(-\infty, \infty)$, for the same reason as in part (c).

23. $f(x) = (2x^2 - 2x + 5)(3x - 4)^2$
The first factor is always positive because the leading term has a positive coefficient and the discriminant $(-2)^2 - 4(2)(5) = -36$ is negative. The only zero is $x = 4/3$, with multiplicity two, since that is the solution for $3x - 4 = 0$.

(a) True for all $x \ne \frac{4}{3}$

(b) $(-\infty, \infty)$

(c) There are no solutions.

(d) $x = \frac{4}{3}$

24. $f(x) = (x^2 + 4)(3 - 2x)^2$
The first factor is always positive. The only zero is $x = 3/2$, with multiplicity two, since that is the solution for $3 - 2x = 0$.

(a) True for all $x \ne \frac{3}{2}$

(b) $(-\infty, \infty)$

(c) There are no solutions.

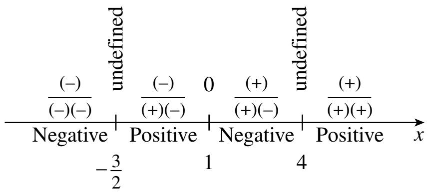
(d) $x = \frac{3}{2}$

25. **(a)** $f(x) = 0$ when $x = 1$

(b) $f(x)$ is undefined when $x = -\frac{3}{2}, 4$

(c) $f(x) > 0$ when $-\frac{3}{2} < x < 1$ or $x > 4$

(d) $f(x) < 0$ when $x < -\frac{3}{2}$ or $1 < x < 4$

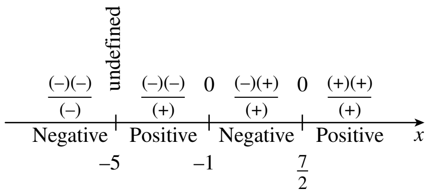


26. **(a)** $f(x) = 0$ when $x = \frac{7}{2}, -1$

(b) $f(x)$ is undefined when $x = -5$

(c) $f(x) > 0$ when $-5 < x < -1$ or $x > \frac{7}{2}$

(d) $f(x) < 0$ when $x < -5$ or $-1 < x < \frac{7}{2}$

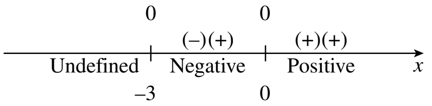


27. **(a)** $f(x) = 0$ when $x = 0, -3$

(b) $f(x)$ is undefined when $x < -3$

(c) $f(x) > 0$ when $x > 0$

(d) $f(x) < 0$ when $-3 < x < 0$

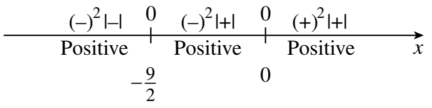


28. **(a)** $f(x) = 0$ when $x = 0, -\frac{9}{2}$

(b) None. $f(x)$ is never undefined

(c) $f(x) > 0$ when $x \neq -\frac{9}{2}, 0$

(d) None. $f(x)$ is never negative

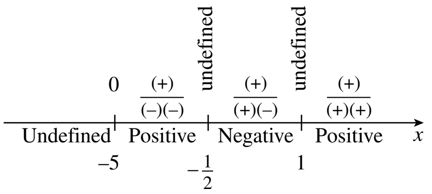


29. **(a)** $f(x) = 0$ when $x = -5$

(b) $f(x)$ is undefined when $x = -\frac{1}{2}, x = 1, x < -5$

(c) $f(x) > 0$ when $-5 < x < -\frac{1}{2}$ or $x > 1$

(d) $f(x) < 0$ when $-\frac{1}{2} < x < 1$

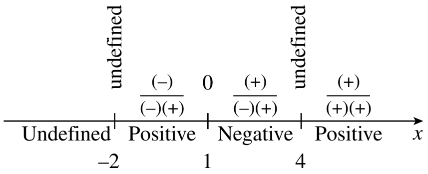


30. **(a)** $f(x) = 0$ when $x = 1$

(b) $f(x)$ is undefined when $x = 4, x \leq -2$

(c) $f(x) > 0$ when $-2 < x < 1$ or $x > 4$

(d) $f(x) < 0$ when $1 < x < 4$

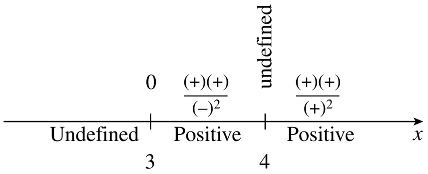


31. **(a)** $f(x) = 0$ when $x = 3$

(b) $f(x)$ is undefined when $x = 4, x < 3$

(c) $f(x) > 0$ when $3 < x < 4$ or $x > 4$

(d) None. $f(x)$ is never negative

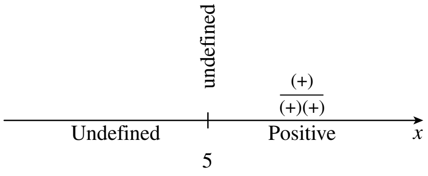


32. **(a)** None. $f(x)$is never 0

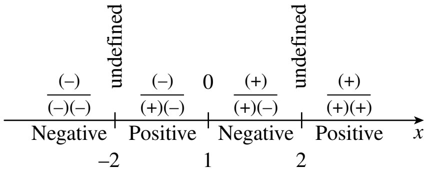
(b) $f(x)$ is undefined when $x \leq 5$

(c) $f(x) > 0$ when $5 < x < \infty$

(d) None. $f(x)$ is never negative

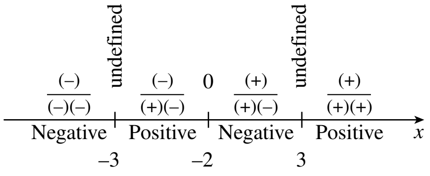


33. $f(x) = \frac{x-1}{x^2-4} = \frac{x-1}{(x+2)(x-2)}$ has points of potential sign change at $x = -2, 1, 2$.



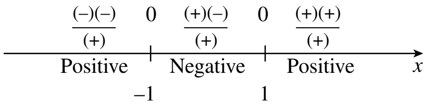
By the sign chart, the solution of $\frac{x-1}{x^2-4} < 0$ is $(-\infty, -2) \cup (1, 2)$.

34. $f(x) = \frac{x+2}{x^2-9} = \frac{x+2}{(x+3)(x-3)}$ has points of potential sign change at $x = -3, -2, 3$.



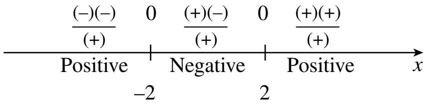
By the sign chart, the solution of $\frac{x+2}{x^2-9} < 0$ is $(-\infty, -3) \cup (-2, 3)$.

35. $f(x) = \dfrac{x^2 - 1}{x^2 + 1} = \dfrac{(x + 1)(x - 1)}{(x^2 + 1)}$ has points of potential sign change at $x = -1, 1$.



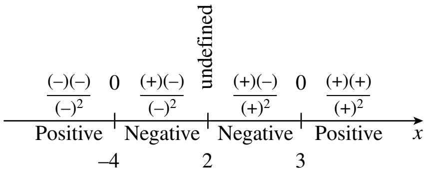
By the sign chart, the solution of $\dfrac{x^2 - 1}{x^2 + 1} \le 0$ is $[-1, 1]$.

36. $f(x) = \dfrac{x^2 - 4}{x^2 + 4} = \dfrac{(x + 2)(x - 2)}{x^2 + 4}$ has points of potential sign change at $x = -2, 2$.



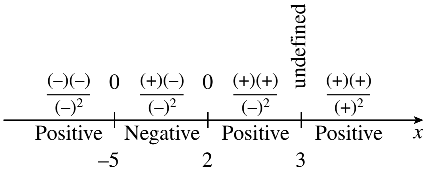
By the sign chart, the solution of $\dfrac{x^2 - 4}{x^2 + 4} > 0$ is $(-\infty, -2) \cup (2, \infty)$.

37. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4x + 4} = \dfrac{(x + 4)(x - 3)}{(x - 2)^2}$ has points of potential sign change at $x = -4, 2, 3$.



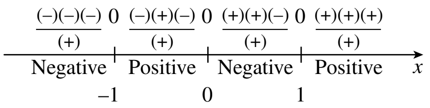
By the sign chart, the solution of $\dfrac{x^2 + x - 12}{x^2 - 4x + 4} > 0$ is $(-\infty, -4) \cup (3, \infty)$.

38. $f(x) = \dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} = \dfrac{(x + 5)(x - 2)}{(x - 3)^2}$ has points of potential sign change at $x = -5, 2, 3$.



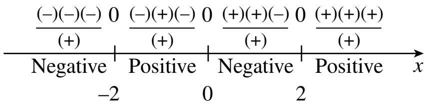
By the sign chart, the solution of $\dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$ is $(-5, 2)$.

39. $f(x) = \dfrac{x^3 - x}{x^2 + 1} = \dfrac{x\,(x + 1)(x - 1)}{x^2 + 1}$ has points of potential sign change at $x = -1, 0, 1$.



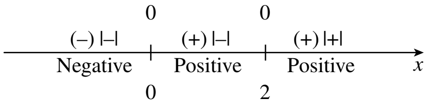
By the sign chart, the solution of $\dfrac{x^3 - x}{x^2 + 1} \ge 0$ is $[-1, 0] \cup [1, \infty)$.

40. $f(x) = \dfrac{x^3 - 4x}{x^2 + 2} = \dfrac{x(x + 2)(x - 2)}{x^2 + 2}$ has points of potential sign change at $x = -2, 0, 2$.



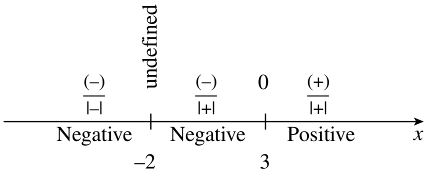
By the sign chart, the solution of $\dfrac{x^3 - 4x}{x^2 + 2} \le 0$ is $(-\infty, -2] \cup [0, 2]$.

41. $f(x) = x|x - 2|$ has points of potential sign change at $x = 0, 2$.



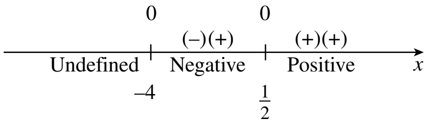
By the sign chart, the solution of $x|x - 2| > 0$ is $(0, 2) \cup (2, \infty)$.

42. $f(x) = \dfrac{x - 3}{|x + 2|}$ has points of potential sign change at $x = -2, 3$.



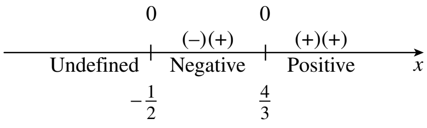
By the sign chart, the solution of $\dfrac{x - 3}{|x + 2|} < 0$ is $(-\infty, -2) \cup (-2, 3)$.

43. $f(x) = (2x - 1)\sqrt{x + 4}$ has a point of potential sign change at $x = \dfrac{1}{2}$. Note that the domain of f is $[-4, \infty)$.



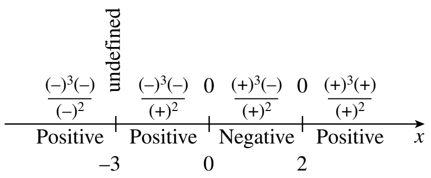
By the sign chart, the solution of $(2x - 1)\sqrt{x + 4} < 0$ is $\left(-4, \dfrac{1}{2}\right)$.

44. $f(x) = (3x - 4)\sqrt{2x + 1}$ has a point of potential sign change at $x = \dfrac{4}{3}$. Note that the domain of f is $\left[-\dfrac{1}{2}, \infty\right)$.



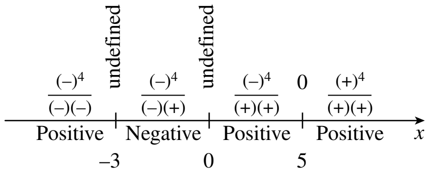
By the sign chart, the solution of $(3x - 4)\sqrt{2x + 1} \ge 0$ is $\left[\dfrac{4}{3}, \infty\right)$.

45. $f(x) = \dfrac{x^3(x-2)}{(x+3)^2}$ has points of potential sign change at $x = -3, 0, 2$.



By the sign chart, the solution of $\dfrac{x^3(x-2)}{(x+3)^2} < 0$ is $(0, 2)$.

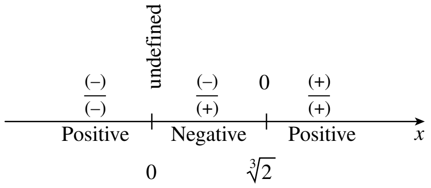
46. $f(x) = \dfrac{(x-5)^4}{x(x+3)} \geq 0$ has points of potential sign change at $x = -3, 0, 5$.



By the sign chart, the solution of $\dfrac{(x-5)^4}{x\,(x+3)} \geq 0$ is

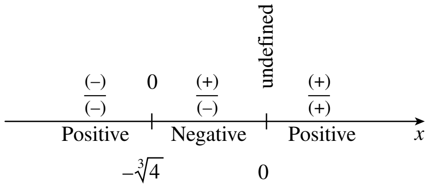
$(-\infty, -3) \cup (0, \infty)$.

47. $f(x) = x^2 - \dfrac{2}{x} = \dfrac{x^3 - 2}{x}$ has points of potential sign change at $x = 0, \sqrt[3]{2}$.



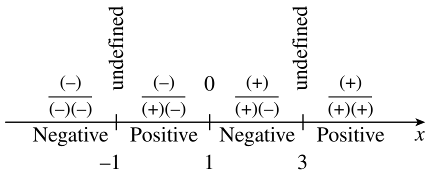
By the sign chart, the solution of $x^2 - \dfrac{2}{x} > 0$ is $(-\infty, 0) \cup (\sqrt[3]{2}, \infty)$.

48. $f(x) = x^2 + \dfrac{4}{x} = \dfrac{x^3 + 4}{x}$ has points of potential sign change at $x = 0, -\sqrt[3]{4}$.



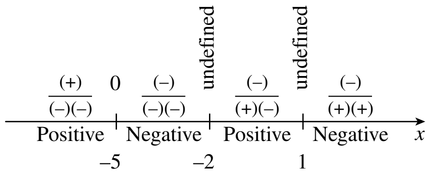
By the sign chart, the solution of $x^2 + \dfrac{4}{x} \geq 0$ is $(-\infty, -\sqrt[3]{4}] \cup (0, \infty)$.

49. $f(x) = \dfrac{1}{x+1} + \dfrac{1}{x-3} = \dfrac{2(x-1)}{(x+1)(x-3)}$ has points of potential sign change at $x = -1, 1, 3$.



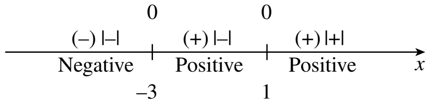
By the sign chart, the solution of $\dfrac{1}{x+1} + \dfrac{1}{x-3} \leq 0$ is $(-\infty, -1) \cup [1, 3)$.

50. $f(x) = \dfrac{1}{x+2} - \dfrac{2}{x-1} = \dfrac{-x-5}{(x+2)(x-1)}$ has points of potential sign change at $x = -5, -2, 1$.



By the sign chart, the solution of $\dfrac{1}{x+2} - \dfrac{2}{x-1} > 0$ is $(-\infty, -5) \cup (-2, 1)$.

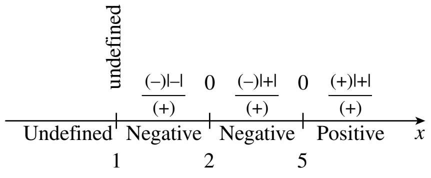
51. $f(x) = (x+3)|x-1|$ has points of potential sign change at $x = -3, 1$.



By the sign chart, the solution of $(x+3)|x-1| \geq 0$ is $[-3, \infty)$.

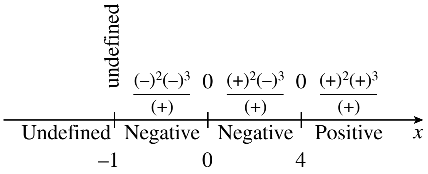
52. $f(x) = (3x+5)^2|x-2|$ is always 0 or positive since $(3x+5)^2 \geq 0$ for all real x and $|x-2| \geq 0$ for all real x. Thus the inequality $(3x+5)^2\,|x-2| < 0$ has no solution.

53. $f(x) = \dfrac{(x-5)|x-2|}{\sqrt{2x-2}}$ has points of potential sign change at $x = 2, 5$. Note that the domain of f is $(1, \infty)$.



By the sign chart, the solution of $\dfrac{(x-5)|x-2|}{\sqrt{2x-2}} \geq 0$ is $[5, \infty)$.

54. $f(x) = \dfrac{x^2(x-4)^3}{\sqrt{x+1}}$ has points of potential sign change at $x = 0, 4$. Note that the domain of f is $(-1, \infty)$.



By the sign chart, the solution of $\dfrac{x^2(x-4)^3}{\sqrt{x+1}} < 0$ is $(-1, 0) \cup (0, 4)$.

55. One way to solve the inequality is to graph $y = 3(x - 1) + 2$ and $y = 5x + 6$ together, then find the interval along the x-axis where the first graph is below or intersects the second graph. Another way is to solve for x algebraically.

56. Let x be the number of hours worked. The repair charge is $25 + 18x$; this must be less than \$100. Starting with $25 + 18x < 100$, we have $18x < 75$, so $x < 4.166\ldots$ Therefore, the electrician worked no more than 4 hours 7.5 minutes (which rounds to 4 hours).

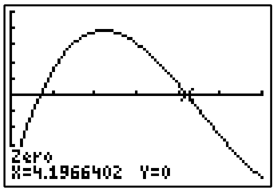
57. Let $x > 0$ be the width of a rectangle; then the length is $2x - 2$ and the perimeter is $P = 2[x + (2x - 2)]$. Solving $P < 200$ and $2x - 2 > 0$ (below) gives 1 in. $< x <$ 34 in.

$$\begin{aligned} 2[x + (2x-2)] &< 200 \quad \text{and} \quad & 2x - 2 &> 0 \\ 2(3x-2) &< 200 & 2x &> 2 \\ 6x - 4 &< 200 & x &> 1 \\ 6x &< 204 & & \\ x &< 34 & & \end{aligned}$$

58. Let x be the number of candy bars made. Then the costs are $C = 0.13x + 2000$, and the income is $I = 0.35x$. Solving $C < I$ (below) gives $x > 9090.91$. The company will need to make and sell 9091 candy bars to make a profit.

$$\begin{aligned} 0.13x + 2000 &< 0.35x \\ 2000 &< 0.22x \\ x &> 9090.91 \end{aligned}$$

59. The lengths of the sides of the box are x, $12 - 2x$, and $15 - 2x$, so the volume is $x(12 - 2x)(15 - 2x)$. To solve $x(12 - 2x)(15 - 2x) \le 100$, graph $f(x) = x(12 - 2x)(15 - 2x) - 100$ and find the zeros: $x \approx 0.69$ and $x \approx 4.20$.

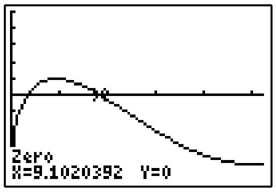


[0, 6] by [−100, 100]

From the graph, the solution of $f(x) \le 0$ is approximately $[0, 0.69] \cup [4.20, 6]$. The squares should be such that either 0 in. $\le x \le$ 0.69 in. or 4.20 in. $\le x \le$ 6 in.

60. The circumference of the base of the cone is $8\pi - x$, $r = 4 - \dfrac{x}{2\pi}$, and $h = \sqrt{16 - \left(4 - \dfrac{x}{2\pi}\right)^2}$. The volume is $v = \dfrac{1}{3}\pi\left(4 - \dfrac{x}{2\pi}\right)^2\sqrt{16 - \left(4 - \dfrac{x}{2\pi}\right)^2}$.

To solve $v \ge 21$, graph $v - 21$ and find the zeros: $x \approx 1.68$ in. or $x \approx 9.10$.



[0, 26] by [−25, 25]

From the graph, the solution of $v - 21 \ge 0$ is approximately $[1.68, 9.10]$. The arc length should be in the range of 1.68 in. $\le x \le$ 9.10 in.

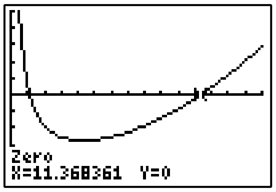
61. **(a)** $\dfrac{1}{2}L = 500 \text{ cm}^3$

$$V = \pi x^2 h = 500 \Rightarrow h = \frac{500}{\pi x^2}$$

$$S = 2\pi x h + 2\pi x^2 = 2\pi x\left(\frac{500}{\pi x^2}\right) + 2\pi x^2$$

$$= \frac{1000}{x} + 2\pi x^2 = \frac{1000 + 2\pi x^3}{x}$$

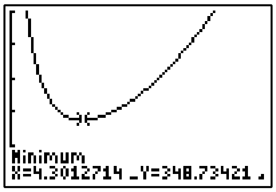
(b) Solve $S < 900$ by graphing $\dfrac{1000 + 2\pi x^3}{x} - 900$ and finding its zeros: $x \approx 1.12$ and $x \approx 11.37$



[0, 15] by [−1000, 1000]

From the graph, the solution of $S - 900 < 0$ is approximately $(1.12, 11.37)$. So the radius is between 1.12 cm and 11.37 cm. The corresponding height must be between 1.23 cm and 126.88 cm.

(c) Graph S and find the minimum graphically.



[0, 15] by [0, 1000]

The minimum surface area is about 348.73 cm^2.

62. **(a)** $\dfrac{1}{R} = \dfrac{1}{2.3} + \dfrac{1}{x}$

$$2.3x = Rx + 2.3R = R(x + 2.3)$$

$$R = \frac{2.3x}{x + 2.3}$$

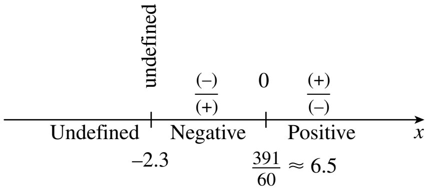
(b) $R \geq 1.7 \Rightarrow \dfrac{2.3x}{x+2.3} \geq 1.7$

$$\frac{2.3x}{x+2.3} - 1.7 \geq 0$$

$$\frac{2.3x - 1.7(x+2.3)}{x+2.3} \geq 0$$

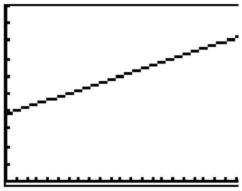
$$\frac{0.6x - 3.91}{x+2.3} \geq 0$$

The function $f(x) = \dfrac{0.6x - 3.91}{x+2.3}$ has a point of potential sign change at $x = \dfrac{391}{60} \approx 6.5$. Note that the domain of f is $(-2.3, \infty)$.



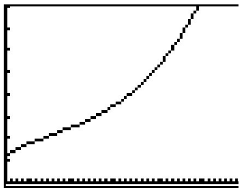
By the sign chart, the solution of $f(x) \geq 0$ is about $[6.5, \infty)$. The resistance in the second resistor is at least 6.5 ohms.

63. (a) $y \approx 993.870x + 19{,}025.768$



(b) From the graph of $y = 993.870x + 19{,}025.768$, we find that $y = 40{,}000$ when $x \approx 21.2$. The per capita income will exceed $40,000 in the year 2011.

64. (a) $y \approx 7.883x^3 - 214.972x^2 + 6479.797x + 62{,}862.278$



(b) From the graph of $y \approx 7.883x^3 - 214.972x^2 + 6479.797x + 62{,}862.278$, we find that $y = 250{,}000$ when $x \approx 28$. The per capita income will exceed $250,000 in the year 2008.

65. False. Because the factor x^4 has an even power, it does not change sign at $x = 0$.

66. True. Because the denominator factor $(x + 2)$ has an odd power (namely 1), it changes sign at $x = -2$.

67. x must be positive but less than 1. The answer is C.

68. The statement is true so long as the numerator does not equal zero. The answer is B.

69. The statement is true so long as the denominator is negative and the numerator is nonzero. Thus x must be less than three but nonzero. The answer is D.

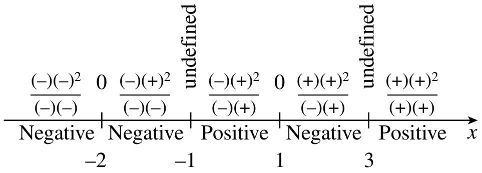
70. The expression $(x^2 - 1)^2$ cannot be negative for any real x, and it can equal zero only for $x = \pm 1$. The answer is A.

71. $f(x) = \dfrac{(x-1)(x+2)^2}{(x-3)(x+1)}$

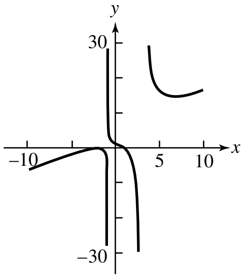
Vertical asymptotes: $x = -1$, $x = 3$

x-intercepts: $(-2, 0)$, $(1, 0)$

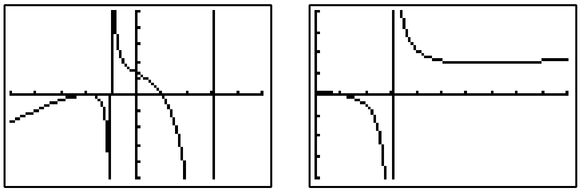
y-intercept: $\left(0, \dfrac{4}{3}\right)$



By hand:



Grapher:



[–5, 5] by [–5, 5]

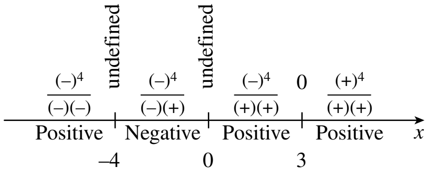
[0, 10] by [–40, 40]

72. $g(x) = \dfrac{(x-3)^4}{x^2 + 4x} = \dfrac{(x-3)^4}{x(x+4)}$

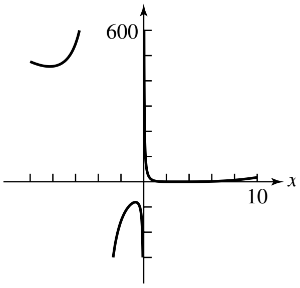
Vertical asymptotes: $x = -4$, $x = 0$

x-intercept: $(3, 0)$

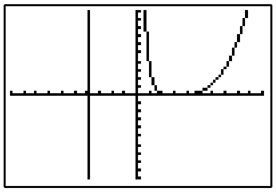
y-intercept: None



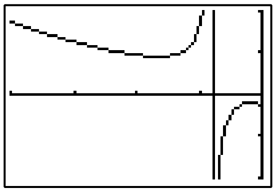
Sketch:



Grapher:



[−10, 10] by [−10, 10]



[−20, 0] by [−1000, 1000]

73. **(a)** $|x - 3| < 1/3 \Rightarrow |3x - 9| < 1 \Rightarrow$ $|3x - 5 - 4| < 1 \Rightarrow |f(x) - 4| < 1$.

For example:

$$|f(x) - 4| = |(3x - 5) - 4| = |3x - 9|$$
$$= 3|x - 3| < 3\left(\frac{1}{3}\right) = 1$$

(b) If x stays within the dashed vertical lines, $f(x)$ will stay within the dashed horizontal lines. For the example in part (a), the graph shows that for $\frac{8}{3} < x < \frac{10}{3}$ $\left(\text{that is, } |x - 3| < \frac{1}{3}\right)$, we have $3 < f(x) < 5$ (that is, $|f(x) - 4| < 1$).

(c) $|x - 3| < 0.01 \Rightarrow |3x - 9| < 0.03 \Rightarrow$ $|3x - 5 - 4| < 0.03 \Rightarrow |f(x) - 4| < 0.03$. The dashed lines would be closer to $x = 3$ and $y = 4$.

74. When $x^2 - 4 \geq 0$, $y = 1$, and when $x^2 - 4 \ngeq 0$, $y = 0$.

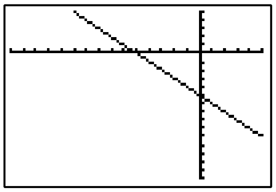
75. One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by a gives $a^2 < ab$; multiplying by b gives $ab < b^2$. Then, by the transitive property of inequality, we have $a^2 < b^2$.

76. One possible answer: Given $0 < a < b$, multiplying both sides of $a < b$ by $\frac{1}{ab}$ gives $\frac{1}{b} < \frac{1}{a}$, which is equivalent to $\frac{1}{a} > \frac{1}{b}$.

■ Chapter 2 Review

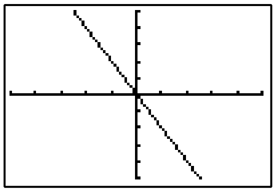
For #1 and 2, first find the slope of the line. Then use algebra to put into $y = mx + b$ format.

1. $m = \frac{-9 - (-2)}{4 - (-3)} = \frac{-7}{7} = -1, (y + 9) = -1(x - 4),$ $y = -x - 5$



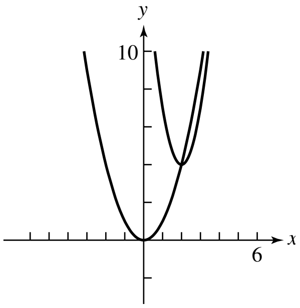
[−15, 5] by [−15, 5]

2. $m = \frac{-2 - 6}{1 - (-3)} = \frac{-8}{4} = -2, (y + 2) = -2(x - 1),$ $y = -2x$

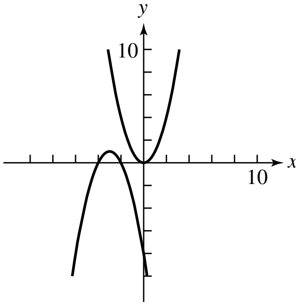


[−5, 5] by [−5, 5]

3. Starting from $y = x^2$, translate right 2 units and vertically stretch by 3 (either order), then translate up 4 units.



4. Starting from $y = x^2$, translate left 3 units and reflect across x-axis (either order), then translate up 1 unit.



5. Vertex: $(-3, 5)$; axis: $x = -3$

6. Vertex: $(5, -7)$; axis: $x = 5$

7. $f(x) = -2(x^2 + 8x) - 31$
$= -2(x^2 + 8x + 16) + 32 - 31 = -2(x + 4)^2 + 1$;
Vertex: $(-4, 1)$; axis: $x = -4$

8. $g(x) = 3(x^2 - 2x) + 2 = 3(x^2 - 2x + 1) - 3 + 2 =$ $3(x - 1)^2 - 1$; Vertex: $(1, -1)$; axis: $x = 1$

For #9–12, use the form $y = a(x - h)^2 + k$, where (h, k), the vertex, is given.

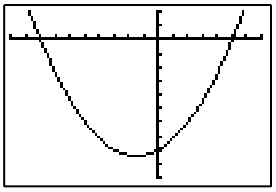
9. $h = -2$ and $k = -3$ are given, so $y = a(x + 2)^2 - 3$. Using the point $(1, 2)$, we have $2 = 9a - 3$, so $a = \frac{5}{9}$: $y = \frac{5}{9}(x + 2)^2 - 3$.

10. $h = -1$ and $k = 1$ are given, so $y = a(x + 1)^2 + 1$. Using the point $(3, -2)$, we have $-2 = 16a + 1$, so $a = -\frac{3}{16}$: $y = -\frac{3}{16}(x + 1)^2 + 1$.

11. $h = 3$ and $k = -2$ are given, so $y = a(x - 3)^2 - 2$. Using the point $(5, 0)$, we have $0 = 4a - 2$, so $a = \frac{1}{2}$: $y = \frac{1}{2}(x - 3)^2 - 2$.

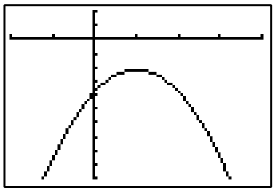
12. $h = -4$ and $k = 5$ are given, so $y = a(x + 4)^2 + 5$. Using the point $(0, -3)$, we have $-3 = 16a + 5$, so $a = -\frac{1}{2}$; $y = -\frac{1}{2}(x + 4)^{-2} + 5$.

13.



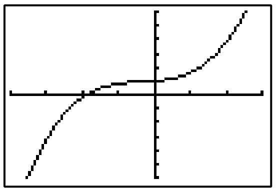
$[-10, 7]$ by $[-50, 10]$

14.



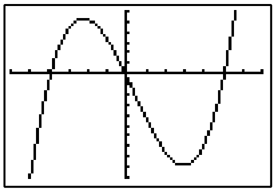
$[-2, 4]$ by $[-50, 10]$

15.



$[-4, 3]$ by $[-30, 30]$

16.



$[-6, 7]$ by $[-50, 30]$

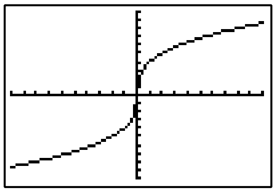
17. $S = kr^2 \quad (k = 4\pi)$

18. $F = \dfrac{k}{d^2} \quad (k = \text{gravitational constant})$

19. The force F needed varies directly with the distance x from its resting position, with constant of variation k.

20. The area of a circle A varies directly with the square of its radius.

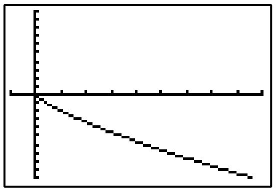
21. $k = 4$, $a = \dfrac{1}{3}$. In Quadrant I, $f(x)$ is increasing and concave down since $0 < a < 1$.



$[-10, 10]$ by $[-10, 10]$

$f(-x) = 4(-x)^{1/3} = -4x^{1/3} = -f(x)$, so f is odd.

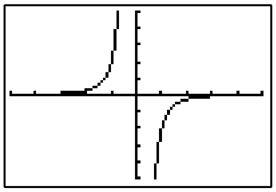
22. $k = -2$, $a = \dfrac{3}{4}$. In Quadrant IV, $f(x)$ is decreasing and concave up since $0 < a < 1$. f is not defined for $x < 0$.



$[-1, 9]$ by $[-10, 10]$

23. $k = -2$, $a = -3$. In Quadrant IV, f is increasing and concave down. $f(-x) = -2(-x)^{-3} = \dfrac{-2}{(-x)^3} = \dfrac{-2}{-x^3}$

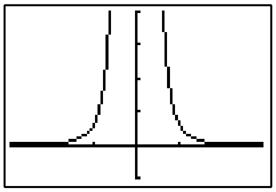
$= \dfrac{2}{x^3} = 2x^{-3} = -f(x)$, so f is odd.



$[-5, 5]$ by $[-5, 5]$

24. $k = \dfrac{2}{3}$, $a = -4$. In Quadrant I, $f(x)$ is decreasing and concave up. $f(-x) = \dfrac{2}{3}(-x)^{-4} = \dfrac{2}{3} \cdot \dfrac{1}{(-x)^4}$

$= \dfrac{2}{3x^4} = \dfrac{2}{3}x^{-4} = f(x)$, so f is even.



$[-3, 3]$ by $[-1, 4]$

25. $\dfrac{2x^3 - 7x^2 + 4x - 5}{x - 3} = 2x^2 - x + 1 - \dfrac{2}{x - 3}$

$$\begin{array}{r}
2x^2 - x + 1\phantom{{}-5} \\
x - 3\,\overline{)\,2x^3 - 7x^2 + 4x - 5} \\
\underline{2x^3 - 6x^2\phantom{{}+4x-5}} \\
-x^2 + 4x\phantom{{}-5} \\
\underline{-x^2 + 3x\phantom{{}-5}} \\
x - 5 \\
\underline{x - 3} \\
-2
\end{array}$$

26. $\dfrac{x^4 + 3x^3 + x^2 - 3x + 3}{x + 2} = x^3 + x^2 - x - 1 + \dfrac{5}{x + 2}$

$$\begin{array}{rl} & x^3 + x^2 - x - 1 \\ \cline{2-2} x + 2 \big) & x^4 + 3x^3 + x^2 - 3x + 3 \\ & x^4 + 2x^3 \\ \cline{2-2} & \quad x^3 + x^2 \\ & \quad x^3 + 2x^2 \\ \cline{2-2} & \qquad -x^2 - 3x \\ & \qquad -x^2 - 2x \\ \cline{2-2} & \qquad\qquad -x + 3 \\ & \qquad\qquad -x - 2 \\ \cline{2-2} & \qquad\qquad\qquad 5 \end{array}$$

27. $\dfrac{2x^4 - 3x^3 + 9x^2 - 14x + 7}{x^2 + 4}$

$= 2x^2 - 3x + 1 + \dfrac{-2x + 3}{x^2 + 4}$

$$\begin{array}{rl} & 2x^2 - 3x + 1 \\ \cline{2-2} x^2 + 4 \big) & 2x^4 - 3x^3 + 9x^2 - 14x + 7 \\ & 2x^4 \qquad + 8x^2 \\ \cline{2-2} & \quad -3x^3 + x^2 - 14x + 7 \\ & \quad -3x^3 \qquad - 12x \\ \cline{2-2} & \qquad\quad x^2 - 2x + 7 \\ & \qquad\quad x^2 \qquad + 4 \\ \cline{2-2} & \qquad\qquad -2x + 3 \end{array}$$

28. $\dfrac{3x^4 - 5x^3 - 2x^2 + 3x - 6}{3x + 1} = x^3 - 2x^2 + 1 + \dfrac{-7}{3x + 1}$

$$\begin{array}{rl} & x^3 - 2x^2 \qquad + 1 \\ \cline{2-2} 3x + 1 \big) & 3x^4 - 5x^3 - 2x^2 + 3x - 6 \\ & 3x^4 + x^3 \\ \cline{2-2} & \quad -6x^3 - 2x^2 + 3x - 6 \\ & \quad -6x^3 - 2x^2 \\ \cline{2-2} & \qquad\qquad 3x - 6 \\ & \qquad\qquad 3x + 1 \\ \cline{2-2} & \qquad\qquad\quad -7 \end{array}$$

29. Remainder: $f(-2) = -39$

30. Remainder: $f(3) = -2$

31. Yes: 2 is a zero of the second polynomial.

32. No: $x = -3$ yields 1 from the second polynomial.

33.
$$\begin{array}{r|rrrr} 5 & 1 & -5 & 3 & 4 \\ & & 5 & 0 & 15 \\ \hline & 1 & 0 & 3 & 19 \end{array}$$

Yes, $x = 5$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

34.
$$\begin{array}{r|rrrrr} 4 & 4 & -16 & 8 & 16 & -12 \\ & & 16 & 0 & 32 & 192 \\ \hline & 4 & 0 & 8 & 48 & 180 \end{array}$$

Yes, $x = 4$ is an upper bound for the zeros of $f(x)$ because all entries on the bottom row are nonnegative.

35.
$$\begin{array}{r|rrrrr} -3 & 4 & 4 & -15 & -17 & -2 \\ & & -12 & 24 & -27 & 132 \\ \hline & 4 & -8 & 9 & -44 & 130 \end{array}$$

Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs.

36.
$$\begin{array}{r|rrrr} -3 & 2 & 6 & 1 & -6 \\ & & -6 & 0 & -3 \\ \hline & 2 & 0 & 1 & -9 \end{array}$$

Yes, $x = -3$ is a lower bound for the zeros of $f(x)$ because all entries on the bottom row alternate signs (remember that $0 = -0$).

37. Possible rational zeros: $\dfrac{\pm1, \pm2, \pm3, \pm6}{\pm1, \pm2}$,

or $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$; $-\frac{3}{2}$ and 2 are zeros.

38. Possible rational zeros: $\dfrac{\pm1, \pm7}{\pm1, \pm2, \pm3, \pm6}$,

or $\pm1, \pm7, \pm\frac{1}{2}, \pm\frac{7}{2}, \pm\frac{1}{3}, \pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{7}{6}$; $\frac{7}{3}$ is a zero.

39. $(1 + i)^3 = (1 + 2i + i^2)(1 + i) = (2i)(1 + i)$
$= -2 + 2i$

40. $(1 + 2i)^2(1 - 2i)^2 = [(1 + 2i)(1 - 2i)]^2 = (1 + 2^2)^2$
$= 25$

41. $i^{29} = i$

42. $\sqrt{-16} = 4i$

For #43–44, use the quadratic formula.

43. $x = \dfrac{6 \pm \sqrt{36 - 52}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

44. $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = \dfrac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$

45. (c) $f(x) = (x - 2)^2$ is a quadratic polynomial that has vertex $(2, 0)$ and y-intercept $(0, 4)$, so its graph must be graph (c).

46. (d) $f(x) = (x - 2)^3$ is a cubic polynomial that passes through $(2, 0)$ and $(0, -8)$, so its graph must be graph (d).

47. (b) $f(x) = (x - 2)^4$ is a quartic polynomial that passes through $(2, 0)$ and $(0, 16)$, so its graph must be graph (b).

48. (a) $f(x) = (x - 2)^5$ is a quintic polynomial that passes through $(2, 0)$ and $(0, -32)$, so its graph must be graph (a).

In #49–52, use a graph and the Rational Zeros Test to determine zeros.

49. Rational: 0 (multiplicity 2) — easily seen by inspection. Irrational: $5 \pm \sqrt{2}$ (using the quadratic formula, after taking out a factor of x^2). No non-real zeros.

50. Rational: ±2. Irrational: $\pm\sqrt{3}$. No nonreal zeros. These zeros may be estimated from a graph, or by dividing $k(t)$ by $t - 2$ and $t + 2$ then applying the quadratic formula, or by using the quadratic formula on $k(t)$ to determine that $t^2 = \dfrac{7 \pm \sqrt{49 - 48}}{2}$, i.e., t^2 is 3 or 4.

51. Rational: none. Irrational: approximately −2.34, 0.57, 3.77. No non-real zeros.

52. Rational: none. Irrational: approximately −3.97, −0.19. Two non-real zeros.

53. The only rational zero is $-\frac{3}{2}$. Dividing by $x + \frac{3}{2}$ (below) leaves $2x^2 - 12x + 20$, which has zeros $\frac{12 \pm \sqrt{144 - 160}}{4} = 3 \pm i$. Therefore

$$\begin{aligned} f(x) &= (2x + 3)[x - (3 - i)][x - (3 + i)] \\ &= (2x + 3)(x - 3 + i)(x - 3 - i). \end{aligned}$$

$$\begin{array}{r|rrrr} -3/2 & 2 & -9 & 2 & 30 \\ & & -3 & 18 & -30 \\ \hline & 2 & -12 & 20 & 0 \end{array}$$

54. The only rational zero is $\frac{4}{5}$. Dividing by $x - \frac{4}{5}$ (below) leaves $5x^2 - 20x - 15$, which has zeros $\frac{20 \pm \sqrt{400 + 300}}{10} = 2 \pm \sqrt{7}$. Therefore

$$\begin{aligned} f(x) &= (5x - 4)\left[x - \left(2 + \sqrt{7}\right)\right]\left[x - \left(2 - \sqrt{7}\right)\right] \\ &= (5x - 4)\left(x - 2 - \sqrt{7}\right)\left(x - 2 + \sqrt{7}\right). \end{aligned}$$

$$\begin{array}{r|rrrr} 4/5 & 5 & -24 & 1 & 12 \\ & & 4 & -16 & -12 \\ \hline & 5 & -20 & -15 & 0 \end{array}$$

55. All zeros are rational: 1, −1, $\frac{2}{3}$, and $-\frac{5}{2}$. Therefore $f(x) = (3x - 2)(2x + 5)(x - 1)(x + 1)$; this can be confirmed by multiplying out the terms or graphing the original function and the factored form of the function.

56. Since all coefficients are real, $1 - 2i$ is also a zero. Dividing synthetically twice leaves the quadratic $x^2 - 6x + 10$, which has zeros $3 \pm i$.
$f(x) = [x - (1 + 2i)][x - (1 - 2i)][x - (3 + i)]$
$[x - (3 - i)] = (x - 1 - 2i)(x - 1 + 2i)$
$(x - 3 - i)(x - 3 + i)$

$$\begin{array}{r|rrrrr} 1 + 2i & 1 & -8 & 27 & -50 & 50 \\ & & 1 + 2i & -11 - 12i & 40 + 20i & -50 \\ \hline & 1 & -7 + 2i & 16 - 12i & -10 + 20i & 0 \\ 1 - 2i & 1 & -7 + 2i & 16 - 12i & -10 + 20i & \\ & & 1 - 2i & -6 + 12i & 10 - 20i & \\ \hline & 1 & -6 & 10 & 0 & \end{array}$$

In #57–60, determine rational zeros (graphically or otherwise) and divide synthetically until a quadratic remains. If more real zeros remain, use the quadratic formula.

57. The only real zero is 2; dividing by $x - 2$ leaves the quadratic factor $x^2 + x + 1$, so $f(x) = (x - 2)(x^2 + x + 1)$.

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

58. The only rational zero is −1; dividing by $x + 1$ leaves the quadratic factor $9x^2 - 12x - 1$, which has zeros $\frac{12 \pm \sqrt{144 + 36}}{18} = \frac{2}{3} \pm \frac{1}{3}\sqrt{5}$. Then $f(x) = (x + 1)(9x^2 - 12x - 1)$.

$$\begin{array}{r|rrrr} -1 & 9 & -3 & -13 & -1 \\ & & -9 & 12 & 1 \\ \hline & 9 & -12 & -1 & 0 \end{array}$$

59. The two real zeros are 1 and $\frac{3}{2}$; dividing by $x - 1$ and $x - \frac{3}{2}$ leaves the quadratic factor $2x^2 - 4x + 10$, so $f(x) = (2x - 3)(x - 1)(x^2 - 2x + 5)$.

$$\begin{array}{r|rrrrr} 1 & 2 & -9 & 23 & -31 & 15 \\ & & 2 & -7 & 16 & -15 \\ \hline & 2 & -7 & 16 & -15 & 0 \end{array} \qquad \begin{array}{r|rrrr} 3/2 & 2 & -7 & 16 & -15 \\ & & 3 & -6 & 15 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

60. The two real zeros are −1 and $-\frac{2}{3}$; dividing by $x + 1$ and $x + \frac{2}{3}$ leaves the quadratic factor $3x^2 - 12x + 15$, so $f(x) = (3x + 2)(x + 1)(x^2 - 4x + 5)$.

$$\begin{array}{r|rrrrr} -1 & 3 & -7 & -3 & 17 & 10 \\ & & -3 & 10 & -7 & -10 \\ \hline & 3 & -10 & 7 & 10 & 0 \end{array} \qquad \begin{array}{r|rrrr} -2/3 & 3 & -10 & 7 & 10 \\ & & -2 & 8 & -10 \\ \hline & 3 & -12 & 15 & 0 \end{array}$$

61. $(x - \sqrt{5})(x + \sqrt{5})(x - 3) = x^3 - 3x^2 - 5x + 15$. Other answers may be found by multiplying this polynomial by any real number.

62. $(x + 3)^2 = x^2 + 6x + 9$ (This may be multiplied by any real number.)

63. $(x - 3)(x + 2)(3x - 1)(2x + 1)$
$= 6x^4 - 5x^3 - 38x^2 - 5x + 6$ (This may be multiplied by any real number.)

64. The third zero must be $1 - i$:
$(x - 2)(x - 1 - i)(x - 1 + i) = x^3 - 4x^2 + 6x - 4$
(This may be multiplied by any real number.)

65. $(x + 2)^2(x - 4)^2 = x^4 - 4x^3 - 12x^2 + 32x + 64$
(This may be multiplied by any real number.)

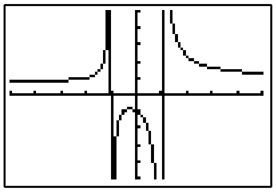
66. The third zero must be $2 + i$, so
$f(x) = a(x + 1)(x - 2 - i)(x - 2 + i)$.
Since $f(2) = 6$, $a = 2$:

$$\begin{aligned} f(x) &= 2(x + 1)(x - 2 - i)(x - 2 + i) \\ &= 2x^3 - 6x^2 + 2x + 10. \end{aligned}$$

67. $f(x) = -1 + \frac{2}{x - 5}$; translate right 5 units and vertically stretch by 2 (either order), then translate down 1 unit. Horizontal asymptote: $y = -1$; vertical asymptote: $x = 5$.

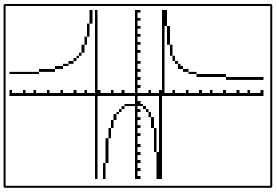
68. $f(x) = 3 - \frac{1}{x - 2}$; translate left 2 units and reflect across x-axis (either order), then translate up 3 units. Horizontal asymptote: $y = 3$; vertical asymptote: $x = -2$.

69. Asymptotes: $y = 1, x = -1$, and $x = 1$.
Intercept: $(0, -1)$.



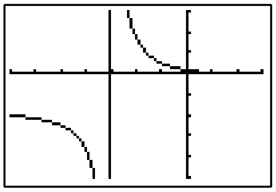
$[-5, 5]$ by $[-5, 5]$

70. Asymptotes: $y = 2, x = -3$, and $x = 2$.
Intercept: $\left(0, -\frac{7}{6}\right)$.



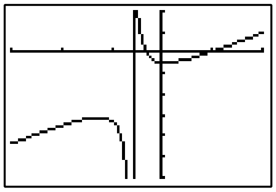
$[-10, 10]$ by $[-10, 10]$

71. End-behavior asymptote: $y = x - 7$.
Vertical asymptote: $x = -3$. Intercept: $\left(0, \frac{5}{3}\right)$.



$[-7, 3]$ by $[-50, 30]$

72. End-behavior asymptote: $y = x - 6$.
Vertical asymptote: $x = -3$. Intercepts: approx. $(-1.54, 0)$, $(4.54, 0)$, and $\left(0, -\frac{7}{3}\right)$.



$[-15, 10]$ by $[-30, 10]$

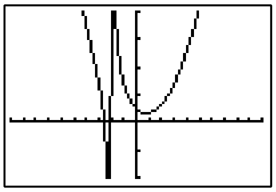
73. $f(x) = \dfrac{x^3 + x^2 - 2x + 5}{x + 2}$ has only one x-intercept, and we can use the graph to show that it is about -2.552. The y-intercept is $f(0) = 5/2$. The denominator is zero when $x = -2$, so the vertical asymptote is $x = -2$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{x^3 + x^2 - 2x + 5}{x + 2} = x^2 - x + \frac{5}{x + 2},$$

we know that the end-behavior asymptote is $y = x^2 - x$. The graph supports this information and allows us to conclude that

$\lim_{x \to -2^-} = -\infty, \lim_{x \to -2^+} = \infty$.

The graph also shows a local minimum of about 1.63 at about $x = 0.82$.



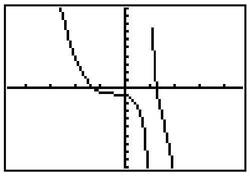
$[-10, 10]$ by $[-10, 20]$

y-intercept: $\left(0, \frac{5}{2}\right)$
x-intercept: $(-2.55, 0)$
Domain: All $x \neq -2$
Range: $(-\infty, \infty)$
Continuity: All $x \neq -2$
Increasing on $[0.82, \infty)$
Decreasing on $(-\infty, -2)$, $(-2, 0.82]$
Not symmetric.
Unbounded.
Local minimum: $(0.82, 1.63)$
No horizontal asymptote. End-behavior asymptote: $y = x^2 - x$
Vertical asymptote: $x = -2$.
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = \infty$

74. $f(x) = \dfrac{-x^4 + x^2 + 1}{x - 1}$ has two x-intercepts, and we can use the graph to show that they are about -1.27 and 1.27. The y-intercept is $f(0) = -1$. The denominator is zero when $x = 1$, so the vertical asymptote is $x = 1$. Because we can rewrite $f(x)$ as

$$f(x) = \frac{-x^4 + x^2 + 1}{x - 1} = -x^3 - x^2 + \frac{1}{x - 1},$$

we know that the end-behavior asymptote is $y = -x^3 - x^2$. The graph supports this information and allows us to conclude that $\lim_{x \to 1^-} = -\infty$ and $\lim_{x \to 1^+} = \infty$. The graph shows no local extrema.



$[-4.7, 4.7]$ by $[-10, 10]$

y-intercept: $(0, 1)$
x-intercepts: $(-1.27, 0)$, $(1.27, 0)$
Domain: All $x \neq 1$
Range: $(-\infty, \infty)$
Continuity: All $x \neq 1$
Never increasing
Decreasing on $(-\infty, 1)$, $(1, \infty)$
No symmetry.
Unbounded.
No local extrema.
No horizontal asymptote. End-behavior asymptote: $y = -x^3 - x^2$
Vertical asymptote: $x = 1$
End behavior: $\lim_{x \to -\infty} f(x) = \infty$; $\lim_{x \to \infty} f(x) = -\infty$

75. Multiply by x: $2x^2 - 11x + 12 = 0$, so $x = \frac{3}{2}$ or $x = 4$.

76. Multiply by $(x + 2)(x - 3) = x^2 - x - 6$: $x(x - 3) + 5(x + 2) = 25$, or $x^2 + 2x - 15 = 0$, so $x = -5$ or $x = 3$. The latter is extraneous; the only solution is $x = -5$.

For #77–78, find the zeros of $f(x)$ and then determine where the function is positive or negative by creating a sign chart.

77. $f(x) = (x - 3)(2x + 5)(x + 2)$, so the zeros of $f(x)$ are $x = \left\{-\frac{5}{2}, -2, 3\right\}$.

(−)(−)(−) | (−)(+)(−) | (−)(+)(+) | (+)(+)(+) x
Negative | Positive | Negative | Positive
$-\frac{5}{2}$ -2 3

As our sign chart indicates, $f(x) < 0$ on the interval $\left(-\infty, -\frac{5}{2}\right) \cup (-2, 3)$.

78. $f(x) = (x - 2)^2(x + 4)(3x + 1)$, so the zeros of $f(x)$ are $x = \left\{-4, -\frac{1}{3}, 2\right\}$.

(+)(−)(−) | (+)(+)(−) | (+)(+)(+) | (+)(+)(+) x
Positive | Negative | Positive | Positive
-4 $-\frac{1}{3}$ 2

As our sign chart indicates, $f(x) \geq 0$ on the interval $(-\infty, -4] \cup \left[-\frac{1}{3}, \infty\right)$.

79. Zeros of numerator and denominator: −3, −2, and 2. Choose −4, −2.5, 0, and 3; $\frac{x + 3}{x^2 - 4}$ is positive at −2.5 and 3, and equals 0 at −3, so the solution is $[-3, -2) \cup (2, \infty)$.

80. $\frac{x^2 - 7}{x^2 - x - 6} - 1 = \frac{x - 1}{x^2 - x - 6}$. Zeros of numerator and denominator: −2, 1, and 3. Choose −3, 0, 2, and 4; $\frac{x - 1}{x^2 - x - 6}$ is negative at −3 and 2, so the solution is $(-\infty, -2) \cup (1, 3)$.

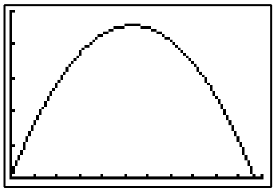
81. Since the function is always positive, we need only worry about the equality $(2x - 1)^2|x + 3| = 0$. By inspection, we see this holds true only when $x = \left\{-3, \frac{1}{2}\right\}$.

82. $\sqrt{x + 3}$ exists only when $x \geq -3$, so we are concerned only with the interval $(-3, \infty)$. Further $|x - 4|$ is always 0 or positive, so the only possible value for a sign change is $x = 1$. For $-3 < x < 1$, $\frac{(x - 1)|x - 4|}{\sqrt{x + 3}}$ is negative, and for $1 < x < 4$ or $4 < x < \infty$, $\frac{(x - 1)|x + 4|}{\sqrt{x + 3}}$ is positive. So the solution is $(1, 4) \cup (4, \infty)$.

83. Synthetic division reveals that we *cannot* conclude that 5 is an upper bound (since there are both positive and negative numbers on the bottom row), while −5 *is* a lower bound (because all numbers on the bottom row alternate signs). Yes, there is another zero (at $x \approx 10.0002$).

5	1	−10	−3	28	20	−2
		5	−25	−140	−560	−2700
	1	−5	−28	−112	−540	−2702
−5	1	−10	−3	28	20	−2
		−5	75	−360	1660	−8400
	1	−15	72	−332	1680	−8402

84. (a) $h = -16t^2 + 170t + 6$



[0, 11] by [0, 500]

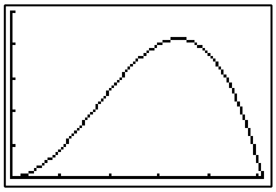
(b) When $t \approx 5.3125$, $h \approx 457.5625$.

(c) The rock will hit the ground after about 10.66 sec.

85. (a) $V = $ (height)(width)(length) $= x(30 - 2x)(70 - 2x)$ in.^{3}

(b) Either $x \approx 4.57$ or $x \approx 8.63$ in.

86. (a) & **(b)**



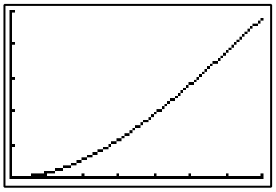
[0, 255] by [0, 2.5]

(c) When $d \approx 170$ ft, $s \approx 2.088$ ft.

(d) One possibility: The beam may taper off (become thinner) from west to east — e.g., perhaps it measures 8 in. by 8 in. at the west end, but only 7 in. by 7 in. on the east end. Then we would expect the beam to bend more easily closer to the east end (though not at the extreme east end, since it is anchored to the piling there). Another possibility: The two pilings are made of different materials.

87. (a) The tank is made up of a cylinder, with volume $\pi x^2(140 - 2x)$, and a sphere, with volume $\frac{4}{3}\pi x^3$. Thus, $V = \frac{4}{3}\pi x^3 + \pi x^2(140 - 2x)$.

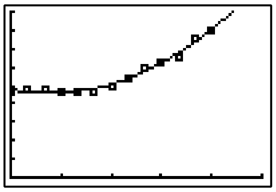
(b)



[0, 70] by [0, 1,500,000]

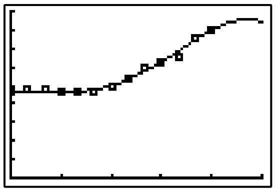
(c) The largest volume occurs when $x = 70$ (so it is actually a sphere). This volume is $\frac{4}{3}\pi(70)^3 \approx 1{,}436{,}755 \text{ ft}^3$.

88. (a) $y = 18.694x^2 - 88.144x + 2393.022$



[0, 15] by [0, 4500]

(b) $y = -0.291x^4 + 7.100x^3 - 35.865x^2 + 48.971x + 2336.634$

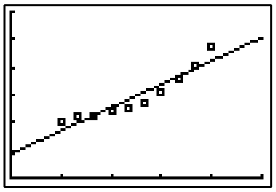


[0, 15] by [0, 4500]

(c) For $x = 16$, the quadratic model yields $y \approx \$5768$ and the quartic model yields $y \approx \$3949$.

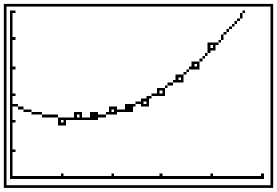
(d) The quadratic model, which has a positive leading coefficient, predicts that the amount of the Pell Grant will always increase. The quartic model, which has a negative leading coefficient, predicts that eventually the amount of the Pell Grant will decrease over time.

89. (a) $y = 1.401x + 4.331$



[0, 15] by [0, 30]

(b) $y = 0.188x^2 - 1.411x + 13.331$



[0, 15] by [0, 30]

(c) **Linear:** Solving $1.401x + 4.331 = 30$ graphically, we find that $y = 30$ when $x \approx 18.32$. The spending will exceed \$30 million in the year 2008.

Quadratic: Solving $0.188x^2 - 1.411x + 13.331 = 30$ graphically, we find that $y = 30$ when $x \approx 13.89$. The spending will exceed \$30 million in the year 2003.

90. (a) Each shinguard costs \$4.32 plus a fraction of the overhead: $C = 4.32 + 4000/x$.

(b) Solve $x(5.25 - 4.32 - 4000/x) = 8000$: $0.93x = 12{,}000$, so $x \approx 12{,}903.23$ — round up to 12,904.

91. (a) $P(15) = 325$, $P(70) = 600$, $P(100) = 648$

(b) $y = \frac{640}{0.8} = 800$

(c) The deer population approaches (but never equals) 800.

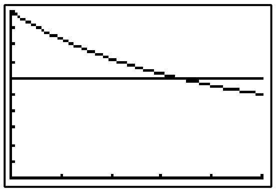
92. (a) $\frac{1}{1.2} = \frac{1}{x} + \frac{1}{R_2}$, so $\frac{1}{R_2} = \frac{1}{1.2} - \frac{1}{x} = \frac{x - 1.2}{1.2x}$.

Then, $R_2 = \frac{1.2x}{x - 1.2}$.

(b) When $x = 3$, $R_2 = \frac{3.6}{3 - 1.2} = \frac{3.6}{1.8} = 2$ ohms.

93. (a) $C(x) = \frac{50}{50 + x}$

(b) Shown is the window $[0, 50]$ by $[0, 1]$, with the graphs of $y = C(x)$ and $y = 0.6$. The two graphs cross when $x \approx 33.33$ ounces of distilled water.



[0, 50] by [0, 1]

(c) Algebraic solution of $\frac{50}{50 + x} = 0.6$ leads to $50 = 0.6(50 + x)$, so that $0.6x = 20$, or $x = \frac{100}{3} \approx 33.33$.

94. (a) Let h be the height (in cm) of the can; we know the volume is $1 \text{ L} = 1000 \text{ cm}^3 = \pi x^2 h$, so $h = \frac{1000}{\pi x^2}$. Then $S = 2\pi x^2 + 2\pi x h = 2\pi x^2 + 2000/x$.

(b) Solve $2\pi x^2 + 2000/x = 900$, or equivalently, $2\pi x^3 - 900x + 2000 = 0$. Graphically we find that either $x \approx 2.31$ cm and $h \approx 59.75$ cm, or $x \approx 10.65$ and $h \approx 2.81$ cm.

(c) Approximately $2.31 < x < 10.65$ (graphically) and $2.81 < h < 59.75$.

95. (a) Let y be the height of the tank; $1000 = x^2 y$, so $y = 1000/x^2$. The surface area equals the area of the base plus 4 times the area of one side. Each side is a rectangle with dimensions $x \times y$, so $S = x^2 + 4xy = x^2 + 4000/x$.

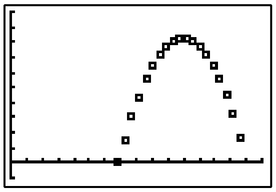
(b) Solve $x^2 + 4000/x = 600$, or $x^3 - 600x + 4000 = 0$ (a graphical solution is easiest): Either $x = 20$, giving dimensions 20 ft by 20 ft by 2.5 ft or $x \approx 7.32$, giving approximate dimensions 7.32 by 7.32 by 18.66.

(c) $7.32 < x < 20$ (lower bound approximate), so y must be between 2.5 ft and about 18.66 ft.

Chapter 2 Project

Answers are based on the sample data shown in the table.

1.



[0, 1.6] by [−0.1, 1]

2. We estimate the vertex to lie halfway between the two data points with the greatest height, so that h is the average of 1.075 and 1.118, or about 1.097. We estimate k to be 0.830, which is slightly greater than the greatest height in the data, 0.828.
Noting that $y = 0$ when $x = 0.688$, we solve $0 = a(0.688 - 1.097)^2 + 0.830$ to find $a \approx -4.962$. So the estimated quadratic model is $y = -4.962(x - 1.097)^2 + 0.830$.

3. The sign of a affects the direction the parabola opens. The magnitude of a affects the vertical stretch of the graph. Changes to h cause horizontal shifts to the graph, while changes to k cause vertical shifts.

4. $y \approx -4.962x^2 - 10.887x - 5.141$

5. $y \approx -4.968x^2 + 10.913x - 5.160$

6. $$\begin{aligned} y &\approx -4.968x^2 + 10.913x - 5.160 \\ &\approx -4.968\,(x^2 - 2.1967x + 1.0386) \\ &= -4.968\left[x^2 - 2.1967x + \left(\frac{2.1967}{2}\right)^2 - \left(\frac{2.1967}{2}\right)^2 + 1.0386\right] \\ &= -4.968\left[\left(x - \frac{2.1967}{2}\right)^2 - 0.1678\right] \\ &\approx -4.968\,(x - 1.098)^2 + 0.833 \end{aligned}$$