Chapter 1
Functions and Graphs

■ Section 1.1 Modeling and Equation Solving

Exploration 1

1. $k = \frac{d}{m} = \frac{100 - 25}{100} = \frac{75}{100} = 0.75$
2. $t = 6.5\% + 0.5\% = 7\%$ or 0.07
3. $m = \frac{d}{k}, s = d + td$

$s = pm$

$p = \frac{s}{m} = \frac{d + td}{\frac{d}{k}} = \frac{d + td}{1} \cdot \frac{k}{d} = \frac{d(1 + t)}{1} \cdot \frac{k}{d}$

$= \frac{k(1 + t)}{1} = (0.75)(1.07) = 0.8025$

4. Yes, because $\$36.99 \times 0.8025 = \$29.68$.
5. $\$100 \div 0.8025 = \$124.61$

Exploration 2

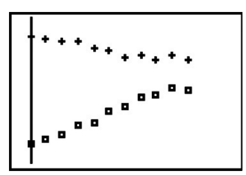
1. Because the linear model maintains a constant positive slope, it will eventually reach the point where 100% of the prisoners are female. It will then continue to rise, giving percentages above 100%, which are impossible.
2. Yes, because 2009 is still close to the data we are modeling. We would have much less confidence in the linear model for predicting the percentage 25 years from 2000.
3. One possible answer: Males are heavily dominant in violent crime statistics, while female crimes tend to be property crimes like burglary or shoplifting. Since property crimes rates are senstive to economic conditions, a statistician might look for adverse economic factors in 1990, especially those that would affect people near or below the poverty level.
4. Yes. Table 1.1 shows that the minimum wage worker had less purchasing power in 1990 than in any other year since 1955, which gives some evidence of adverse economic conditions among lower-income Americans that year. Nonetheless, a careful sociologist would certainly want to look at other data before claiming a connection between this statistic and the female crime rate.

Quick Review 1.1

1. $(x + 4)(x - 4)$
2. $(x + 5)(x + 5)$
3. $(9y + 2)(9y - 2)$
4. $3x(x^2 - 5x + 6) = 3x(x - 2)(x - 3)$
5. $(4h^2 + 9)(4h^2 - 9) = (4h^2 + 9)(2h + 3)(2h - 3)$
6. $(x + h)(x + h)$
7. $(x + 4)(x - 1)$
8. $x^2 - 3x + 4$
9. $(2x - 1)(x - 5)$
10. $(x^2 + 5)(x^2 - 4) = (x^2 + 5)(x + 2)(x - 2)$

Section 1.1 Exercises

1. (d) (q)
2. (f) (r)
3. (a) (p)
4. (h) (o)
5. (e) (l)
6. (b) (s)
7. (g) (t)
8. (j) (k)
9. (i) (m)
10. (c) (n)
11. **(a)** The percentage increased from 1954 to 1999 and then decreased slightly from 1999 to 2004.
(b) The greatest increase occurred between 1974 and 1979.
12. **(a)** Except for some minor fluctuations, the percentage has been decreasing overall.
(b) The greatest decrease occurred between 1979 and 1984.
13. Women (□), Men (+)



[–5, 55] by [23, 92]

14. Vice versa: The female percentages are increasing faster than the male percentages are decreasing.
15. To find the equation, first find the slope.

Women: Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{58.5 - 32.3}{1999 - 1954} = \frac{26.2}{45}$ $= 0.582$. The y-intercept is 32.3, so the equation of the line is $y = 0.582x + 32.3$.

Men: Slope $= \frac{74.0 - 83.5}{1999 - 1954} = \frac{-9.5}{45} = -0.211$. The y-intercept is 83.5, so the equation of the line is $y = -0.211x + 83.5$.

In both cases, x represents the number of years after 1954.

16. 2009 is 55 years since 1954, so $x = 55$.
Women: $y = (0.582)(55) + 32.3 \approx 64.3\%$
Men: $y = (-0.211)(55) + 83.5 \approx 71.9\%$

17. For the percentages to be the same, we need to set the two equations equal to each other.

$$0.582x + 32.3 = -0.211x + 83.5$$
$$0.793x = 51.2$$
$$x \approx 64.6$$

So, approximately 65 years after 1954 (2018), the models predict that the percentages will be about the same. To check:

Males: $y = (-0.211)(65) + 83.5 \approx 69.9\%$
Females: $y = (0.582)(65) + 32.3 \approx 69.9\%$

18. The linear equations will eventually give percentages above 100% for women and below 0% for men, neither of which is possible.

19.

L1	L2	L3 3
316	12	3.7975
480	21	4.375
740	41	5.5405
1085	64	5.8986
1382	92	6.657
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L3(1)=3.797468354...

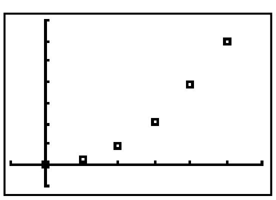
20. Let h be the height of the rectangular cake in inches.
The volume of the rectangular cake is
$V_1 = 9 \cdot 13 \cdot h = 117h$ in.^{3}
The volume of the round cake is
$V_2 = \pi(4)^2(2h) \approx 3.14 \cdot 16 \cdot 2h = 100.48h$ in.^{3}
The rectangular cake gives a greater amount of cake for the same price.

21. Because all stepping stones have the same thickness, what matters is area.
The area of a square stepping stone is
$A_1 = 12 \cdot 12 = 144$ in.^{2}
The area of a round stepping stone is
$$A_2 = \pi\left(\frac{13}{2}\right)^2 \approx 3.14(6.5)^2 = 132.665 \text{ in.}^2$$
The square stones give a greater amount of rock for the same price.

22. (a) $t = \frac{1}{4}\sqrt{180} \approx 3.35$ sec

(b) $d = 16(12.5)^2 = 2500$ ft

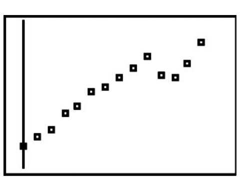
23. A scatter plot of the data suggests a parabola with its vertex at the origin.



[–1, 6] by [–5, 35]

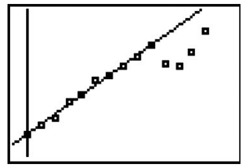
The model $y = 1.2t^2$ fits the data.

24. (a)



[–1, 15] by [400, 750]

(b) To find the equation, first find the slope:
$$\text{Slope} = \frac{666.1 - 452.2}{2000 - 1991} = \frac{213.9}{9} \approx 23.77.$$
The y-intercept is 452.3, so the equation of the line is $y = 23.77x + 452.3$.



[–1, 15] by [400, 750]

(c) To find the year the number of passengers should reach 900, let $y = 900$, and solve the equation for x. $900 = 23.77x + 452.3$; $x \approx 19$, so by the model, the number of passengers should reach 900 million by 2010 (1991 + 19).

(d) The terrorist attacks on September 11, 2001, caused a major disruption in American air traffic from which the airline industry was slow to recover.

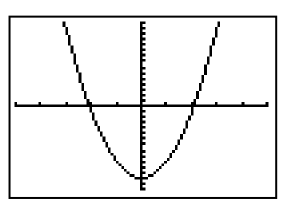
25. The lower line shows the minimum salaries, since they are lower than the average salaries.

26. The points that show the 1990 salaries are the Year 10 points. Both graphs show unprecedented increases in that year. Note: At year 10 the minimum salary jumps, but at year 11 the average salary jumps.

27. The 1995 points are third from the right, Year 15, on both graphs. There is a clear drop in the average salary right after the 1994 strike.

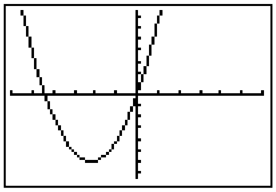
28. One possible answer: **(a)** The players will be happy to see the average salary continue to rise at this rate. The discrepancy between the minimum salary and the average salary will not bother baseball players like it would factory workers, because they are happy to be in the major leagues with the chance to become a star. **(b)** The team owners are not happy with this graph because it shows that their top players are being paid more and more money, forcing them to pay higher salaries to be competitive. This benefits the wealthiest owners. **(c)** Fans are unhappy with the higher ticket prices and with the emphasis on money in baseball rather than team loyalty. Fans of less wealthy teams are unhappy that rich owners are able to pay high salaries to build super-teams filled with talented free agents.

29. Adding $2v^2 + 5$ to both sides gives $3v^2 = 13$. Divide both sides by 3 to get $v^2 = \frac{13}{3}$, so $v = \pm\sqrt{\frac{13}{3}}$.
$3v^2 = 13$ is equivalent to $3v^2 - 13 = 0$. The graph of $y = 3v^2 - 13$ is zero for $v \approx -2.08$ and for $v \approx 2.08$.



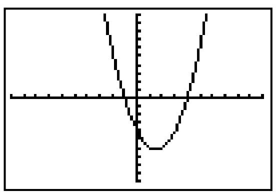
[–5, 5] by [–15, 15]

30. $x + 11 = \pm 11$ so $x = -11 \pm 11$, which gives $x = -22$ or $x = 0$.
$(x + 11)^2 = 121$ is equivalent to $(x + 11)^2 - 121 = 0$. The graph of $y = (x + 11)^2 - 121$ is zero for $x = -22$ and for $x = 0$.



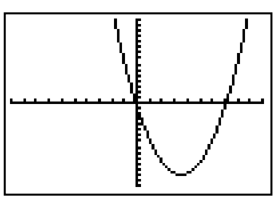
[–30, 30] by [–150, 150]

31.
$$\begin{aligned} 2x^2 - 5x + 2 &= x^2 - 5x + 6 + 3x \\ x^2 - 3x - 4 &= 0 \\ (x - 4)(x + 1) &= 0 \\ x - 4 = 0 \quad &\text{or} \quad x + 1 = 0 \\ x = 4 \quad &\text{or} \quad x = -1 \end{aligned}$$
$2x^2 - 5x + 2 = (x - 3)(x - 2) + 3x$ is equivalent to $2x^2 - 8x + 2 - (x - 3)(x - 2) = 0$. The graph of $y = 2x^2 - 8x + 2 - (x - 3)(x - 2)$ is zero for $x = -1$ and for $x = 4$.



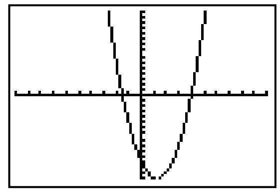
[–10, 10] by [–10, 10]

32.
$$\begin{aligned} x^2 - 7x &= \frac{3}{4} \\ x^2 - 7x + \left(-\frac{7}{2}\right)^2 &= 0.75 + \left(-\frac{7}{2}\right)^2 \\ (x - 3.5)^2 &= 0.75 + 12.25 \\ x - 3.5 &= \pm\sqrt{13} \\ x &= 3.5 \pm \sqrt{13} \end{aligned}$$
The graph of $y = x^2 - 7x - \frac{3}{4}$ is zero for $x \approx -0.11$ and for $x \approx 7.11$.



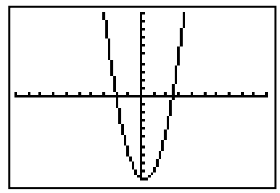
[–10, 10] by [–15, 15]

33. Rewrite as $2x^2 - 5x - 12 = 0$; the left side factors to $(2x + 3)(x - 4) = 0$:
$$\begin{aligned} 2x + 3 = 0 \quad &\text{or} \quad x - 4 = 0 \\ 2x = -3 \quad & \quad\quad x = 4 \\ x = -1.5 \quad & \end{aligned}$$
The graph of $y = 2x^2 - 5x - 12$ is zero for $x = -1.5$ and for $x = 4$.



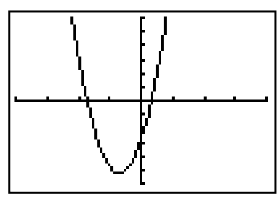
[–10, 10] by [–15, 15]

34. Rewrite as $2x^2 - x - 10 = 0$; the left side factors to $(x + 2)(2x - 5) = 0$:
$$\begin{aligned} x + 2 = 0 \quad &\text{or} \quad 2x - 5 = 0 \\ x = -2 \quad & \quad\quad 2x = 5 \\ & \quad\quad x = 2.5 \end{aligned}$$
The graph of $y = 2x^2 - x - 10$ is zero for $x = -2$ and for $x = 2.5$.



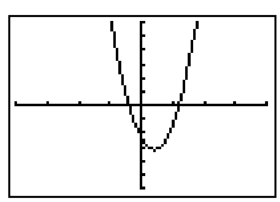
[–10, 10] by [–10, 10]

35. $x^2 + 7x - 14 = 0$, so $a = 1, b = 7$, and $c = -14$:
$$\begin{aligned} x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-14)}}{2(1)} &= \frac{-7 \pm \sqrt{105}}{2} \\ &= -\frac{7}{2} \pm \frac{1}{2}\sqrt{105} \end{aligned}$$
The graph of $y = x^2 + 7x - 14$ is zero for $x \approx -8.62$ and for $x \approx 1.62$.



[–20, 20] by [–30, 30]

36. $x^2 - 4x - 12 = 0$, so $a = 1, b = -4$, and $c = -12$:
$$\begin{aligned} x &= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)} = \frac{4 \pm \sqrt{64}}{2} \\ &= 2 \pm \frac{8}{2} = 2 \pm 4 \end{aligned}$$
$x = -2$ or $x = 6$
The graph of $y = x^2 - 4x - 12$ is zero for $x = -2$ and for $x = 6$.

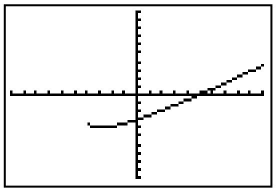


[–20, 20] by [–30, 30]

37. Change to $x^2 - 2x - 15 = 0$ (see below); this factors to $(x + 3)(x - 5) = 0$, so $x = -3$ or $x = 5$. Substituting the first of these shows that it is extraneous.

$$x + 1 = 2\sqrt{x + 4}$$
$$(x + 1)^2 = 2^2(\sqrt{x + 4})^2$$
$$x^2 + 2x + 1 = 4x + 16$$
$$x^2 - 2x - 15 = 0$$

The graph of $y = x + 1 - 2\sqrt{x + 4}$ is zero for $x = 5$.



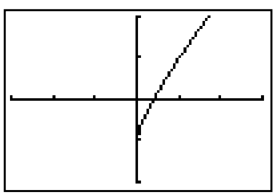
[–10, 10] by [–10, 10]

38. Change to $x^2 - 3x + 1 = 0$ (see below); then $x = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3}{2} \pm \frac{1}{2}\sqrt{5}$, so $x = \frac{3}{2} - \frac{\sqrt{5}}{2}$. Substituting the second of these shows that it is extraneous.

$$\sqrt{x} = 1 - x$$
$$(\sqrt{x})^2 = (1 - x)^2$$
$$x = 1 - 2x + x^2$$
$$0 = x^2 - 3x + 1$$

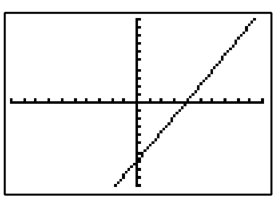
$\sqrt{x} + x = 1$ is equivalent to $x + \sqrt{x} - 1 = 0$.

The graph of $y = x + \sqrt{x} - 1$ is zero for $x \approx 0.38$.



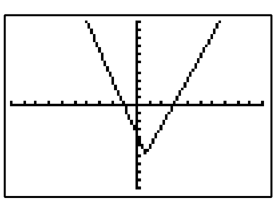
[–3, 3] by [–2, 2]

39. $x \approx 3.91$



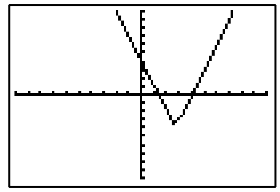
[−10, 10] by [−10, 10]

40. $x \approx -1.09$ or $x \approx 2.86$



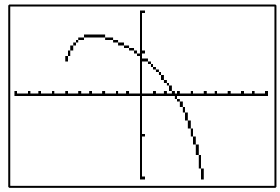
[−10, 10] by [−10, 10]

41. $x \approx 1.33$ or $x = 4$



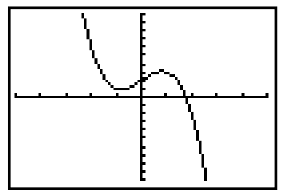
[−10, 10] by [−10, 10]

42. $x \approx 2.66$



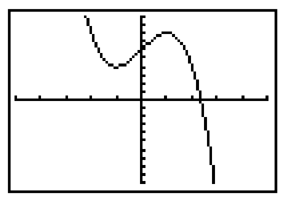
[−10, 10] by [−2, 2]

43. $x \approx 1.77$



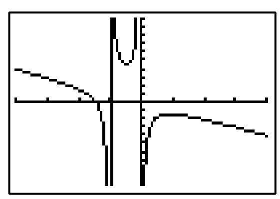
[–5, 5] by [–10, 10]

44. $x \approx 2.36$



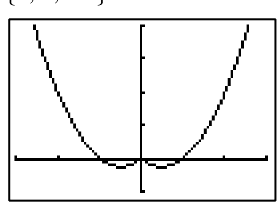
[–5, 5] by [–10, 10]

45. $x \approx -1.47$



[−4, 4] by [−10, 10]

46. $\{0, 1, -1\}$



[−3, 3] by [−1, 4]

47. Model the situation using $C = 0.18x + 32$, where x is the number of miles driven and C is the cost of a day's rental.

(a) Elaine's cost is $0.18(83) + 32 = \$46.94$.

(b) If for Ramon $C = \$69.80$, then

$$x = \frac{69.80 - 32}{0.18} = 210 \text{ miles.}$$

48. **(a)** $4x + 5 - (x^3 + 2x^2 - x + 3) = 0$ or
$-x^3 - 2x^2 + 5x + 2 = 0$

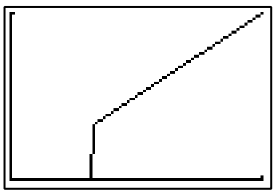
(b) $-x^3 - 2x^2 + 5x + 2 = 0$

(c) A vertical line through the x-intercept of y_3 passes through the point of intersection of y_1 and y_2.

(d) At $x = 1.6813306$, $y_1 = y_2 = 11.725322$.
At $x = -0.3579264$, $y_1 = y_2 = 3.5682944$.
At $x = -3.323404$, $y_1 = y_2 = -8.293616$.

49. **(a)** $y = (x^{200})^{1/200} = x^{200/200} = x^1 = x$ for all $x \geq 0$.

(b) The graph looks like this:



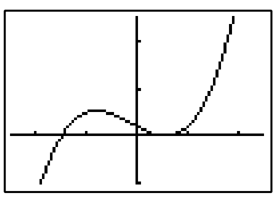
[0, 1] by [0, 1]

(c) Yes, this is different from the graph of $y = x$.

(d) For values of x close to 0, x^{200} is so small that the calculator is unable to distinguish it from zero. It returns a value of $0^{1/200} = 0$ rather than x.

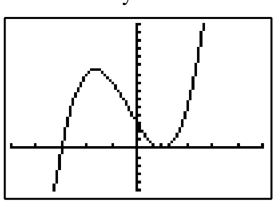
50. The length of each side of the square is $x + b$, so the area of the whole square is $(x + b)^2$. The square is made up of one square with area $x \cdot x = x^2$, one square with area $b \cdot b = b^2$, and two rectangles, each with area $b \cdot x = bx$. Using these four figures, the area of the square is $x^2 + 2bx + b^2$.

51. **(a)** $x = -3$ or $x = 1.1$ or $x = 1.15$.



[−5, 5] by [−200, 500]

(b) $x = -3$ only.



[−10, 10] by [−5, 5]

52. **(a)** Area: $x^2 + x\left(\frac{b}{2}\right) + x\left(\frac{b}{2}\right) = x^2 + bx$

(b) $\frac{b}{2} \cdot \frac{b}{2} = \left(\frac{b}{2}\right)^2$

(c) $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$ is the algebraic formula for completing the square, just as the area $\left(\frac{b}{2}\right)^2$ completes the area $x^2 + bx$ to form the area $\left(x + \frac{b}{2}\right)^2$.

53. Let n be any integer.
$n^2 + 2n = n(n + 2)$, which is either the product of two odd integers or the product of two even integers.

The product of two odd integers is odd.

The product of two even integers is a multiple of 4, since each even integer in the product contributes a factor of 2 to the product.

Therefore, $n^2 + 2n$ is either odd or a multiple of 4.

54. One possible story: The jogger travels at an approximately constant speed throughout her workout. She jogs to the far end of the course, turns around and returns to her starting point, then goes out again for a second trip.

55. False. A product is zero if *any* factor is zero. That is, it takes only one zero factor to make the product zero.

56. False. Predictions are always fallible, and in particular an algebraic model that fits the data well for a certain range of input values may not work for other input values.

57. This is a line with a negative slope and a y-intercept of 12. The answer is C. (The graph checks.)

58. This is the graph of a square root function, but flipped left-over-right. The answer is E. (The graph checks.)

59. As x increases by ones, the y-values get farther and farther apart, which implies an increasing slope and suggests a quadratic equation. The answer is B. (The equation checks.)

60. As x increases by 2's, y increases by 4's, which implies a constant slope of 2. The answer is A. (The equation checks.)

61. **(a)** March

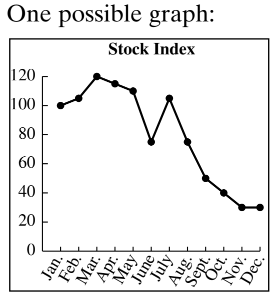
(b) \$120

(c) June, after three months of poor performance

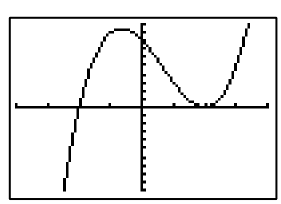
(d) Ahmad paid (100)(\$120) = \$12,000 for the stock and sold it for (100)(\$100) = \$10,000. He lost \$2,000 on the stock.

(e) After reaching a low in June, the stock climbed back to a price near \$140 by December. LaToya's shares had gained \$2000 by that point.

(f) One possible graph:



62. **(a)**



[−4, 4] by [−10, 10]

(b) Factoring, we find $y = (x + 2)(x - 2)(x - 2)$. There is a double zero at $x = 2$, a zero at $x = -2$, and no other zeros (since it is a cubic).

(c) Same visually as the graph in (a).

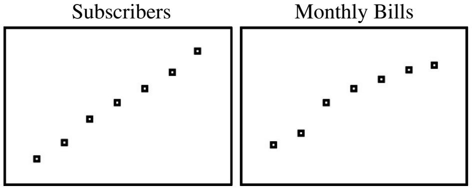
(d) $b^2 - 4ac$ is the discriminant. In this case, $b^2 - 4ac = (-4)^2 - 4(1)(4.01) = -0.04$, which is negative. So the only real zero of the product $y = (x + 2)(x^2 - 4x + 4.01)$ is at $x = -2$.

(e) Same visually as the graph in (a).

(f) $b^2 - 4ac = (-4)^2 - 4(1)(3.99) = 0.04$, which is positive. The discriminant will provide two real zeros of the quadratic, and $(x + 2)$ provides the third. A cubic equation can have no more than three real roots.

63. (a)

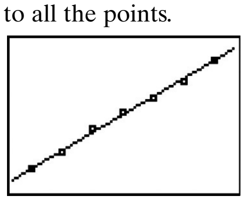
Subscribers   Monthly Bills



[7, 15] by [50, 200]   [7, 15] by [35, 55]

(b) The graph for subscribers appears to be linear. Since time $t =$ the number of years after 1990, $t = 8$ for 1998 and $t = 14$ for 2004. The slope of the line is $\frac{180.4 - 69.2}{14 - 8} = \frac{111.2}{6} \approx 18.53$.
Use the point-slope form to write the equation: $y - 69.2 = 18.53(x - 8)$.
Solve for y: $y - 69.2 = 18.53x - 148.24$
$y = 18.53x - 79.04$
The linear model for subscribers as a function of years is $y = 18.53x - 79.04$.

(c) The fit is very good. The line goes through or is close to all the points.

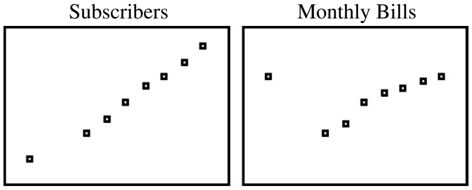


[7, 15] by [50, 200]

(d) The monthly bill scatter plot has a curved shape that could be modeled more effectively by a function with a curved graph. Some possibilities include a quadratic function (parabola), a logarithmic function, a power function (e.g., square root), a logistic function, or a sine function.

(e)

Subscribers   Monthly Bills



[4, 15] by [10, 200]   [4, 15] by [30, 60]

(f) In 1995, cellular phone technology was still emerging, so the growth rate was not as fast as it was in more recent years. Thus, the slope from 1995 ($t = 5$) to 1998 ($t = 8$) is lower than the slope from 1998 to 2004. Cellular technology was more expensive before competition brought prices down. This explains the anomaly on the monthly bill scatter plot.

64. One possible answer: The number of cell phone users is increasing steadily (as the linear model shows), and the average monthly bill is climbing more slowly as more people share the industry cost. The model shows that the number of users will continue to rise, although the linear model cannot hold up indefinitely.

■ Section 1.2 Functions and Their Properties

Exploration 1

1. From left to right, the tables are (c) constant, (b) decreasing, and (a) increasing.

2.

X moves from	ΔX	ΔY1	X moves from	ΔX	ΔY2	X moves from	ΔX	ΔY3
−2 to −1	1	0	−2 to −1	1	−2	−2 to −1	1	2
−1 to 0	1	0	−1 to 0	1	−1	−1 to 0	1	2
0 to 1	1	0	0 to 1	1	−2	0 to 1	1	2
1 to 3	2	0	1 to 3	2	−4	1 to 3	2	3
3 to 7	4	0	3 to 7	4	−6	3 to 7	4	6

3. For an increasing function, ΔY/ΔX is positive. For a decreasing function, ΔY/ΔX is negative. For a constant function, ΔY/ΔX is 0.

4. For lines, ΔY/ΔX is the slope. Lines with positive slope are increasing, lines with negative slope are decreasing, and lines with 0 slope are constant, so this supports our answers to part 3.

Quick Review 1.2

1. $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm 4$

2. $9 - x^2 = 0$
$9 = x^2$
$\pm 3 = x$

3. $x - 10 < 0$
$x < 10$

4. $5 - x \le 0$
$-x \le -5$
$x \ge 5$

5. As we have seen, the denominator of a function cannot be zero.
We need $x - 16 = 0$
$x = 16$

6. We need $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm 4$

7. We need $x - 16 < 0$
$x < 16$

8. We need $x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

9. We need $3 - x \leq 0$ and $x + 2 < 0$
$3 \leq x$ $\quad x < -2$
$x < -2$ and $x \geq 3$

10. We need $x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

Section 1.2 Exercises

1. Yes, $y = \sqrt{x - 4}$ is a function of x, because when a number is substituted for x, there is at most one value produced for $\sqrt{x - 4}$.

2. No, $y = x^2 \pm 3$ is not a function of x, because when a number is substituted for x, y can be either 3 more or 3 less than x^2.

3. No, $x = 2y^2$ does not determine y as a function of x, because when a positive number is substituted for x, y can be either $\sqrt{\frac{x}{2}}$ or $-\sqrt{\frac{x}{2}}$.

4. Yes, $x = 12 - y$ determines y as a function of x, because when a number is substituted for x, there is exactly one number y which, when subtracted from 12, produces x.

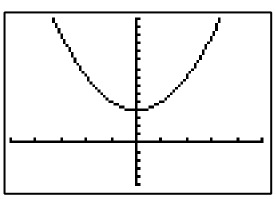
5. Yes

6. No

7. No

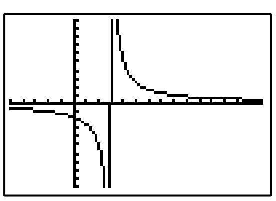
8. Yes

9. We need $x^2 + 4 \geq 0$; this is true for all real x. Domain: $(-\infty, \infty)$.



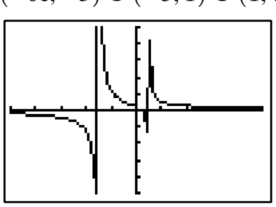
$[-5, 5]$ by $[-5, 15]$

10. We need $x - 3 \neq 0$. Domain: $(-\infty, 3) \cup (3, \infty)$.



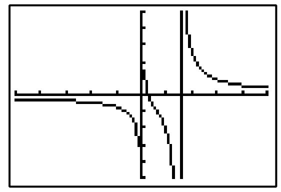
$[-5, 15]$ by $[-10, 10]$

11. We need $x + 3 \neq 0$ and $x - 1 \neq 0$. Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.



$[-10, 10]$ by $[-10, 10]$

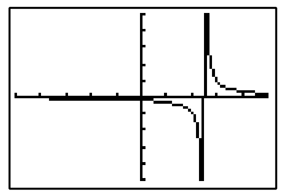
12. We need $x \neq 0$ and $x - 3 \neq 0$. Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.



$[-10, 10]$ by $[-10, 10]$

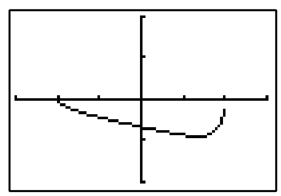
13. We notice that $g(x) = \dfrac{x}{x^2 - 5x} = \dfrac{x}{x(x - 5)}$.

As a result, $x - 5 \neq 0$ and $x \neq 0$.
Domain: $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.



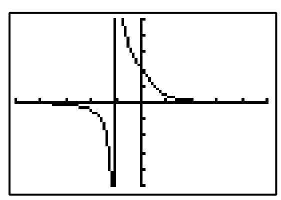
$[-10, 10]$ by $[-5, 5]$

14. We need $x - 3 \neq 0$ and $4 - x^2 \geq 0$. This means $x \neq 3$ and $x^2 \leq 4$; the latter implies that $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.



$[-3, 3]$ by $[-2, 2]$

15. We need $x + 1 \neq 0$, $x^2 + 1 \neq 0$, and $4 - x \geq 0$. The first requirement means $x \neq -1$, the second is true for all x, and the last means $x \leq 4$. The domain is therefore $(-\infty, -1) \cup (-1, 4]$.

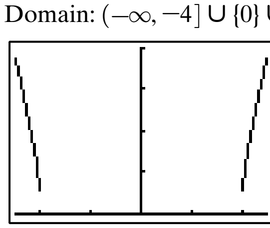


$[-5, 5]$ by $[-5, 5]$

16. We need $x^4 - 16x^2 \geq 0$
$x^2(x^2 - 16) \geq 0$
$x^2 = 0$ or $x^2 - 16 \geq 0$
$x^2 \geq 16$

$x = 0$ or $x \geq 4$, $x \leq -4$

Domain: $(-\infty, -4] \cup \{0\} \cup [4, \infty)$

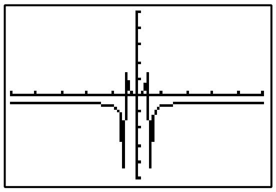


$[-5, 5]$ by $[0, 16]$

17. $f(x) = 10 - x^2$ can take on any negative value. Because x^2 is nonnegative, $f(x)$ cannot be greater than 10. The range is $(-\infty, 10]$.

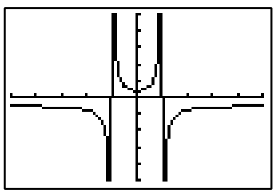
18. $g(x) = 5 + \sqrt{4 - x}$ can take on any value ≥ 5, but because $\sqrt{4 - x}$ is nonnegative, $g(x)$ cannot be less than 5. The range is $[5, \infty)$.

19. The range of a function is most simply found by graphing it. As our graph shows, the range of $f(x)$ is $(-\infty, -1) \cup [0, \infty)$.



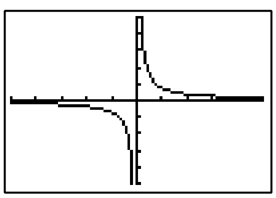
$[-10, 10]$ by $[-10, 10]$

20. As our graph illustrates, the range of $g(x)$ is $(-\infty, -1) \cup [0.75, \infty)$.



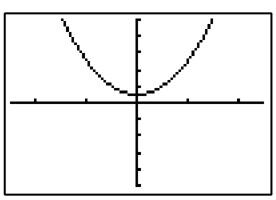
$[-10, 10]$ by $[-10, 10]$

21. Yes, non-removable



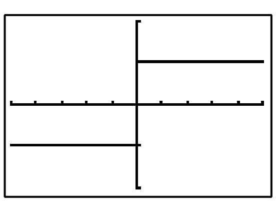
$[-10, 10]$ by $[-10, 10]$

22. Yes, removable



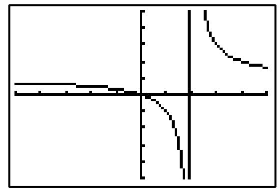
$[-5, 5]$ by $[-10, 10]$

23. Yes, non-removable



$[-10, 10]$ by $[-2, 2]$

24. Yes, non-removable



$[-5, 5]$ by $[-5, 5]$

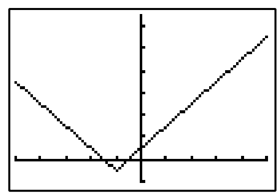
25. Local maxima at $(-1, 4)$ and $(5, 5)$, local minimum at $(2, 2)$. The function increases on $(-\infty, -1]$, decreases on $[-1, 2]$, increases on $[2, 5]$, and decreases on $[5, \infty)$.

26. Local minimum at $(1, 2)$, $(3, 3)$ is neither, and $(5, 7)$ is a local maximum. The function decreases on $(-\infty, 1]$, increases on $[1, 5]$, and decreases on $[5, \infty)$.

27. $(-1, 3)$ and $(3, 3)$ are neither. $(1, 5)$ is a local maximum, and $(5, 1)$ is a local minimum. The function increases on $(-\infty, 1]$, decreases on $[1, 5]$, and increases on $[5, \infty)$.

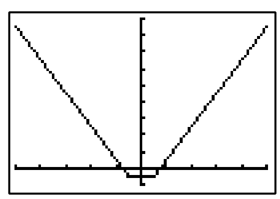
28. $(-1, 1)$ and $(3, 1)$ are local minima, while $(1, 6)$ and $(5, 4)$ are local maxima. The function decreases on $(-\infty, -1]$, increases on $[-1, 1]$, decreases on $(1, 3]$, increases on $[3, 5]$, and decreases on $[5, \infty)$.

29. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



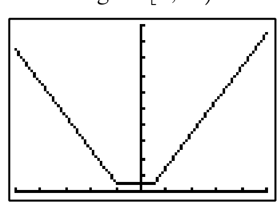
$[-10, 10]$ by $[-2, 18]$

30. Decreasing on $(-\infty, -1]$; constant on $[-1, 1]$; increasing on $[1, \infty)$



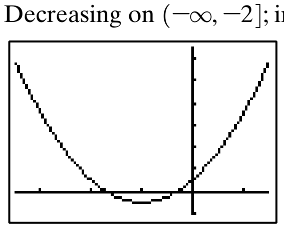
$[-10, 10]$ by $[-2, 18]$

31. Decreasing on $(-\infty, -2]$; constant on $[-2, 1]$; increasing on $[1, \infty)$



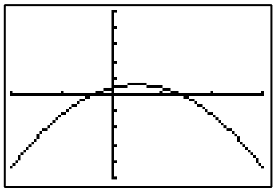
$[-10, 10]$ by $[0, 20]$

32. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



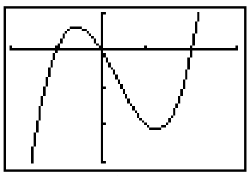
$[-7, 3]$ by $[-2, 13]$

33. Increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



$[-4, 6]$ by $[-25, 25]$

34. Increasing on $(-\infty, -0.5]$; decreasing on $[-0.5, 1.2]$, increasing on $[1.2, \infty)$. The middle values are approximate —they are actually at about -0.549 and 1.215. The values given are what might be observed on the decimal window.



$[-2, 3]$ by $[-3, 1]$

35. Constant functions are always bounded.

36.
$$x^2 > 0$$
$$-x^2 < 0$$
$$2 - x^2 < 2$$
y is bounded above by $y = 2$.

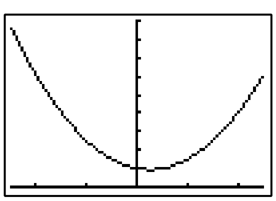
37. $2^x > 0$ for all x, so y is bounded below by $y = 0$.

38. $2^{-x} = \dfrac{1}{2^x} > 0$ for all x, so y is bounded below by $y = 0$.

39. Since $y = \sqrt{1 - x^2}$ is always positive, we know that $y \geq 0$ for all x. We must also check for an upper bound:
$$x^2 > 0$$
$$-x^2 < 0$$
$$1 - x^2 < 1$$
$$\sqrt{1 - x^2} < \sqrt{1}$$
$$\sqrt{1 - x^2} < 1$$
Thus, y is bounded.

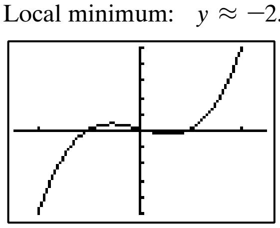
40. There are no restrictions on either x or x^3, so y is not bounded above or below.

41. f has a local minimum when $x = 0.5$, where $y = 3.75$. It has no maximum.



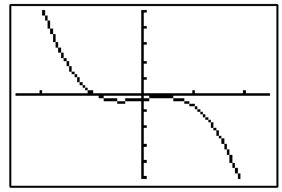
$[-5, 5]$ by $[0, 36]$

42. Local maximum: $y \approx 4.08$ at $x \approx -1.15$.
Local minimum: $y \approx -2.08$ at $x \approx 1.15$.



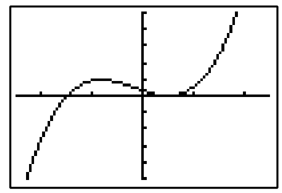
$[-5, 5]$ by $[-50, 50]$

43. Local minimum: $y \approx -4.09$ at $x \approx -0.82$.
Local maximum: $y \approx -1.91$ at $x \approx 0.82$.



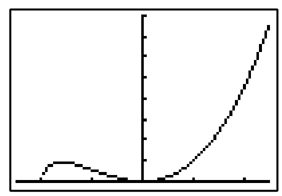
$[-5, 5]$ by $[-50, 50]$

44. Local maximum: $y \approx 9.48$ at $x \approx -1.67$.
Local minimum: $y = 0$ when $x = 1$.



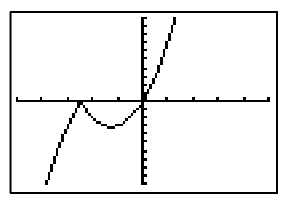
$[-5, 5]$ by $[-50, 50]$

45. Local maximum: $y \approx 9.16$ at $x \approx -3.20$.
Local minima: $y = 0$ at $x = 0$ and $y = 0$ at $x = -4$.



$[-5, 5]$ by $[0, 80]$

46. Local maximum: $y = 0$ at $x = -2.5$.
Local minimum: $y \approx -3.13$ at $x = -1.25$.



$[-5, 5]$ by $[-10, 10]$

47. Even: $f(-x) = 2(-x)^4 = 2x^4 = f(x)$

48. Odd: $g(-x) = (-x)^3 = -x^3 = -g(x)$

49. Even: $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

50. Even: $g(-x) = \dfrac{3}{1 + (-x)^2} = \dfrac{3}{1 + x^2} = g(x)$

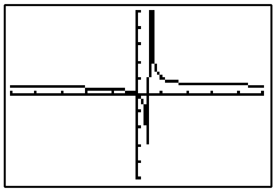
51. Neither: $f(-x) = -(-x)^2 + 0.03(-x) + 5 = -x^2 - 0.03x + 5$, which is neither $f(x)$ nor $-f(x)$.

52. Neither: $f(-x) = (-x)^3 + 0.04(-x)^2 + 3 = -x^3 + 0.04x^2 + 3$, which is neither $f(x)$ nor $-f(x)$.

53. Odd: $g(-x) = 2(-x)^3 - 3(-x)$
$= -2x^3 + 3x = -g(x)$

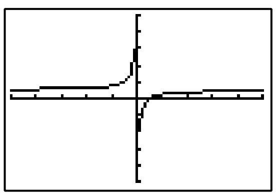
54. Odd: $h(-x) = \dfrac{1}{-x} = -\dfrac{1}{x} = -h(x)$

55. The quotient $\frac{x}{x-1}$ is undefined at $x = 1$, indicating that $x = 1$ is a vertical asymptote. Similarly, $\lim_{x\to\infty} \frac{x}{x-1} = 1$, indicating a horizontal asymptote at $y = 1$. The graph confirms these.



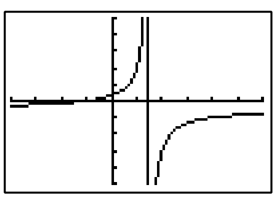
$[-10, 10]$ by $[-10, 10]$

56. The quotient $\frac{x-1}{x}$ is undefined at $x = 0$, indicating a possible vertical asymptote at $x = 0$. Similarly, $\lim_{x\to\infty} \frac{x-1}{x} = 1$, indicating a possible horizontal asymptote at $y = 1$. The graph confirms those asymptotes.



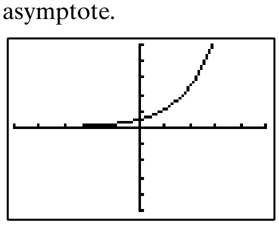
$[-10, 10]$ by $[-10, 10]$

57. The quotient $\frac{x+2}{3-x}$ is undefined at $x = 3$, indicating a possible vertical asymptote at $x = 3$. Similarly, $\lim_{x\to\infty} \frac{x+2}{3-x} = -1$, indicating a possible horizontal asymptote at $y = -1$. The graph confirms these asymptotes.



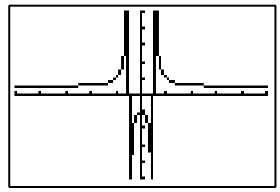
$[-8, 12]$ by $[-10, 10]$

58. Since $g(x)$ is continuous over $-\infty < x < \infty$, we do not expect a vertical asymptote. However, $\lim_{x\to-\infty} 1.5^x = \lim_{x\to\infty} 1.5^{-x} = \lim_{x\to\infty} \frac{1}{1.5^x} = 0$, so we expect a horizontal asymptote $y = 0$. The graph confirms this asymptote.



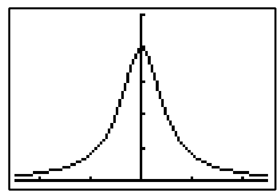
$[-10, 10]$ by $[-10, 10]$

59. The quotient $\frac{x^2+2}{x^2-1}$ is undefined at $x = 1$ and $x = -1$. So we expect two vertical asymptotes. Similarly, the $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = 1$, so we expect a horizontal asymptote at $y = 1$. The graph confirms these asymptotes.



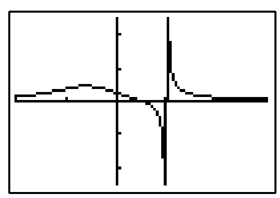
$[-10, 10]$ by $[-10, 10]$

60. We note that $x^2 + 1 > 0$ for $-\infty < x < \infty$, so we do not expect a vertical asymptote. However, $\lim_{x\to\infty} \frac{4}{x^2+1} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms this.



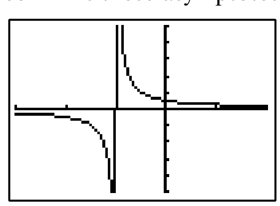
$[-5, 5]$ by $[0, 5]$

61. The quotient $\frac{4x-4}{x^3-8}$ does not exist at $x = 2$, so we expect a vertical asymptote there. Similarly, $\lim_{x\to\infty} \frac{4x-4}{x^3+8} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



$[-4, 6]$ by $[-5, 5]$

62. The quotient $\frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x-2)(x+2)} = \frac{2}{x+2}$. Since $x = 2$ is a removable discontinuity, we expect a vertical asymptote at only $x = -2$. Similarly, $\lim_{x\to\infty} \frac{2}{x-2} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



$[-6, 4]$ by $[-10, 10]$

63. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x+2}{2x+1}$ behaves much like $\frac{x}{2x} = \frac{1}{2}$, so there is a horizontal asymptote at $y = \frac{1}{2}$. The graph matching this description is (b).

64. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x^2+2}{2x+1}$ behaves much like $\frac{x^2}{2x} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (c).

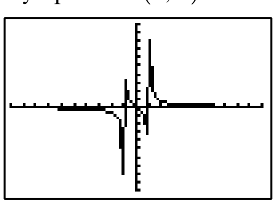
65. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x+2}{2x^2+1}$ behaves much like $\frac{x}{2x^2} = \frac{1}{2x}$, which for large x is close to zero. So there is a horizontal asymptote at $y = 0$. The graph matching this description is (a).

66. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x^3+2}{2x^2+1}$ behaves much like $\frac{x^3}{2x^2} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (d).

67. **(a)** Since, $\lim_{x\to\infty} \frac{x}{x^2-1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation

$$\frac{x}{x^2-1} = 0$$
$$x = 0 \cdot (x^2 - 1)$$
$$x = 0$$

The graph confirms that $f(x)$ crosses the horizontal asymptote at $(0, 0)$.

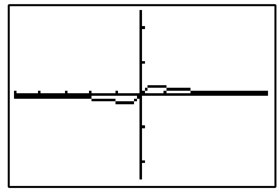


$[-10, 10]$ by $[-10, 10]$

(b) Since $\lim_{x\to\infty} \frac{x}{x^2+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation:

$$\frac{x}{x^2+1} = 0$$
$$x = 0 \cdot (x^2 + 1)$$
$$x = 0$$

The graph confirms that $g(x)$ crosses the horizontal asymptote at $(0, 0)$.

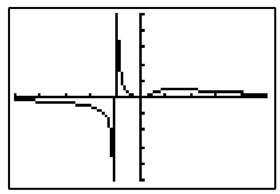


$[-10, 10]$ by $[-5, 5]$

(c) Since $\lim_{x\to\infty} \frac{x^2}{x^3+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where $h(x)$ crosses $y = 0$, we solve the equation

$$\frac{x^2}{x^3+1} = 0$$
$$x^2 = 0 \cdot (x^3 + 1)$$
$$x^2 = 0$$
$$x = 0$$

The graph confirms that $h(x)$ intersects the horizontal asymptote at $(0, 0)$.



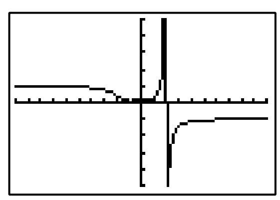
$[-5, 5]$ by $[-5, 5]$

68. We find **(a)** and **(c)** have graphs with more than one horizontal asymptote as follows:

(a) To find horizontal asymptotes, we check limits, at $x \to \infty$ and $x \to -\infty$. We also know that our numerator $|x^3 + 1|$, is positive for all x, and that our denominator, $8 - x^3$, is positive for $x < 2$ and negative for $x > 2$. Considering these two statements, we find

$$\lim_{x\to\infty} \frac{|x^3+1|}{8-x^3} = -1 \text{ and } \lim_{x\to-\infty} \frac{|x^3+1|}{8-x^3} = 1.$$

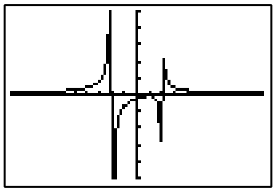
The graph confirms that we have horizontal asymptotes at $y = 1$ and $y = -1$.



$[-10, 10]$ by $[-5, 5]$

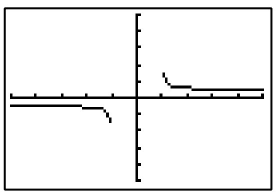
(b) Again, we see that our numerator, $|x - 1|$, is positive for all x. As a result, $g(x)$ can be negative only when $x^2 - 4 < 0$, and $g(x)$ can be positive only when $x^2 - 4 > 0$. This means that $g(x)$ can be negative only when $-2 < x < 2$; if $x < -2$ or $x > 2$, $g(x)$ will be positive. As a result, we know that

$$\lim_{x\to\infty} \frac{|x-1|}{x^2-4} = \lim_{x\to-\infty} \frac{|x-1|}{x^2-4} = 0,$$ giving just one horizontal asymptote at $y = 0$. Our graph confirms this asymptote.



$[-5, 5]$ by $[-5, 15]$

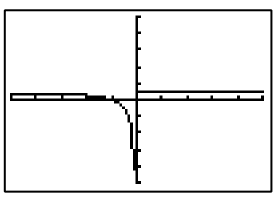
(c) As we demonstrated earlier, we need $x^2 - 4 > 0$ otherwise our function is not defined within the real numbers. As a result, we know that our denominator, $\sqrt{x^2 - 4}$, is always positive [and that $h(x)$ is defined only in the domain $(-\infty, -2) \cup (2, \infty)$]. Checking limits, we find $\lim_{x\to\infty} \frac{x}{\sqrt{x^2 - 4}} = 1$ and $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2 - 4}} = -1$. The graph confirms that we have horizontal asymptotes at $y = 1$ and $y = -1$.



$[-10, 10]$ by $[-10, 10]$

69. (a) The vertical asymptote is $x = 0$, and this function is undefined at $x = 0$ (because a denominator can't be zero).

(b)



$[-10, 10]$ by $[-10, 10]$

Add the point $(0, 0)$.

(c) Yes. It passes the vertical line test.

70. The horizontal asymptotes are determined by the two limits, $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to+\infty} f(x)$. These are at most two different numbers.

71. True. This is what it means for a set of points to be the graph of a function.

72. False. There are many function graphs that are symmetric with respect to the x-axis. One example is $f(x) = 0$.

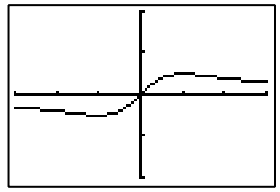
73. Temperature is a continuous variable, whereas the other quantities all vary in steps. The answer is B.

74. "Number of balls" represents a whole number, so that the quantity changes in jumps as the ball radius is altered. The answer is C.

75. Air pressure drops with increasing height. All the other functions either steadily increase or else go both up and down. The answer is C.

76. The height of a swinging pendulum goes up and down over time as the pendulum swings back and forth. The answer is E.

77. (a)



$[-3, 3]$ by $[-2, 2]$

$k = 1$

(b) $\frac{x}{1 + x^2} < 1 \Leftrightarrow x < 1 + x^2 \Leftrightarrow x^2 - x + 1 > 0$

But the discriminant of $x^2 - x + 1$ is negative (-3), so the graph never crosses the x-axis on the interval $(0, \infty)$.

(c) $k = -1$

(d) $\frac{x}{1 + x^2} > -1 \Leftrightarrow x > -1 - x^2 \Leftrightarrow x^2 + x + 1 > 0$

But the discriminant of $x^2 + x + 1$ is negative (-3), so the graph never crosses the x-axis on the interval $(-\infty, 0)$.

78. (a) Increasing

(b)

Δy
1
1.05
0.52
0.43
0.36
0.33
0.31
0.28

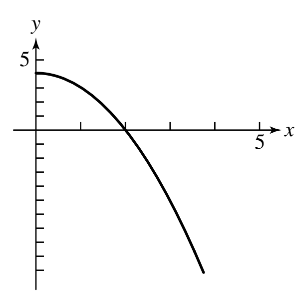
(c)

$\Delta\Delta y$
0.05
−0.53
−0.09
−0.07
−0.03
−0.02
−0.03

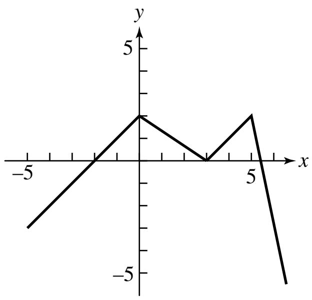
Δy is none of these since it first increases from 1 to 1.05 and then decreases.

(d) The graph rises, but bends downward as it rises.

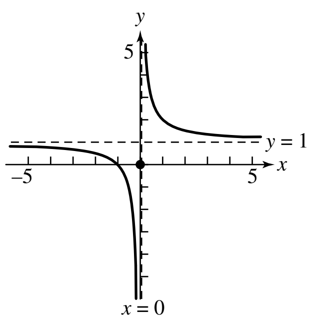
(e) An example:



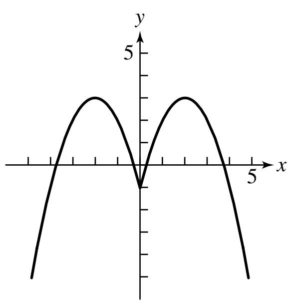
79. One possible graph:



80. One possible graph:



81. One possible graph:



82. Answers vary.

83. (a)

$x^2 > 0$

$-0.8x^2 < 0$

$2 - 0.8x^2 < 2$

$f(x)$ is bounded above by $y = 2$. To determine if $y = 2$ is in the range, we must solve the equation for x: $2 = 2 - 0.8x^2$

$0 = -0.8x^2$

$0 = x^2$

$0 = x$

Since $f(x)$ exists at $x = 0$, $y = 2$ is in the range.

(b) $\lim_{x\to\infty} \dfrac{3x^2}{3 + x^2} = \lim_{x\to\infty} \dfrac{3x^2}{x^2} = \lim_{x\to\infty} 3 = 3$. Thus, $g(x)$ is bounded by $y = 3$. However, when we solve for x, we get

$$3 = \frac{3x^2}{3 + x^2}$$

$$3(3 + x^2) = 3x^2$$

$$9 + 3x^2 = 3x^2$$

$$9 = 0$$

Since $9 \neq 0$, $y = 3$ is not in the range of $g(x)$.

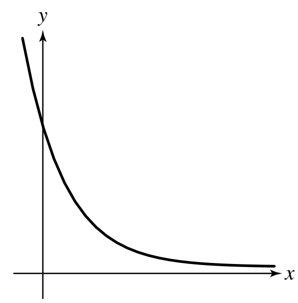
(c) $h(x)$ is not bounded above.

(d) For all values of x, we know that $\sin(x)$ is bounded above by $y = 1$. Similarly, $2\sin(x)$ is bounded above by $y = 2 \cdot 1 = 2$. It is in the range.

(e) $\lim_{x\to\infty} \dfrac{4x}{x^2 + 2x + 1} = \lim_{x\to\infty} \dfrac{4x}{(x + 1)^2} =$
$\lim_{x\to\infty} 4\left(\dfrac{x}{x + 1}\right)\left(\dfrac{1}{x + 1}\right) = \lim_{x\to\infty} \dfrac{4}{x + 1}$
(since $x + 1 \approx x$ for very large x) $= 0$.

[Similarly, $\lim_{x\to-\infty} \dfrac{4x}{x^2 + x + 1} = 0$] As a result, we know that $g(x)$ is bounded by $y = 0$ as x goes to ∞ and $-\infty$.

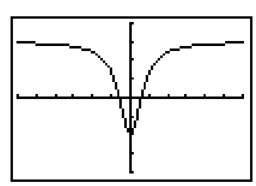
However, $g(x) > 0$ for all $x > 0$ (since $(x + 1)^2 > 0$ always and $4x > 0$ when $x > 0$), so we must check points near $x = 0$ to determine where the function is at its maximum. [Since $g(x) < 0$ for all $x < 0$ (since $(x + 1)^2 > 0$ always and $4x < 0$ when $x < 0$) we can ignore those values of x since we are concerned only with the upper bound of $g(x)$.] Examining our graph, we see that $g(x)$ has an upper bound at $y = 1$, which occurs when $x = 1$. The least upper bound of $g(x) = 1$, and it is in the range of $g(x)$.



84. As the graph moves continuously from the point $(-1, 5)$ down to the point $(1, -5)$, it must cross the x-axis somewhere along the way. That x-intercept will be a zero of the function in the interval $[-1, 1]$.

85. Since f is odd, $f(-x) = -f(x)$ for all x. In particular, $f(-0) = -f(0)$. This is equivalent to saying that $f(0) = -f(0)$, and the only number which equals its opposite is 0. Therefore, $f(0) = 0$, which means the graph must pass through the origin.

86.



[–6, 6] by [–2, 2]

(a) $y = 1.5$

(b) $[-1, 1.5]$

(c) $-1 \leq \dfrac{3x^2 - 1}{2x^2 + 1} \leq 1.5$

$0 \leq 1 + \dfrac{3x^2 - 1}{2x^2 + 1} \leq 2.5$

$0 \leq 2x^2 + 1 + 3x^2 - 1 \leq 5x^2 + 2.5$

$0 \leq 5x^2 \leq 5x^2 + 2.5$

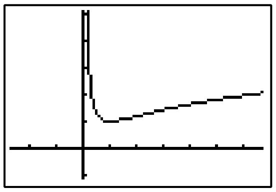
True for all x.

■ Section 1.3 Twelve Basic Functions

Exploration 1

1. The graphs of $f(x) = \frac{1}{x}$ and $f(x) = \ln x$ have vertical asymptotes at $x = 0$.

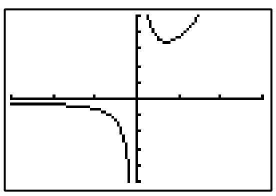
2. The graph of $g(x) = \frac{1}{x} + \ln x$ (shown below) does have a vertical asymptote at $x = 0$.



$[-2.7, 6.7]$ by $[-1.1, 5.1]$

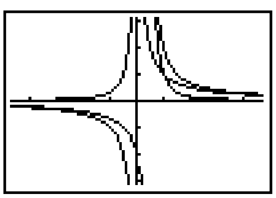
3. The graphs of $f(x) = \frac{1}{x}$, $f(x) = e^x$, and $f(x) = \frac{1}{1 + e^{-x}}$ have horizontal asymptotes at $y = 0$.

4. The graph of $g(x) = \frac{1}{x} + e^x$ (shown below) does have a horizontal asymptote at $y = 0$.



$[-3, 3]$ by $[-5, 5]$

5.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

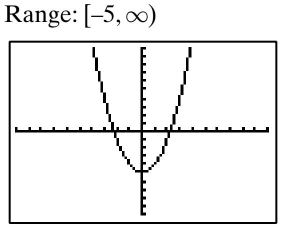
Both $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{2x^2 - x} = \frac{1}{x(2x - 1)}$ have vertical asymptotes at $x = 0$, but $h(x) = f(x) + g(x)$ does not; it has a removable discontinuity.

Quick Review 1.3

1. 59.34
2. $5 - \pi$
3. $7 - \pi$
4. 3
5. 0
6. 1
7. 3
8. -15
9. -4
10. $|1 - \pi| - \pi = (\pi - 1) - \pi = \pi - 1 - \pi = -1$

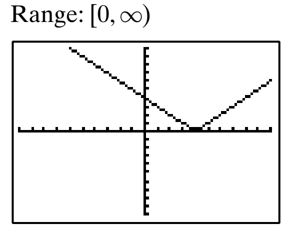
Section 1.3 Exercises

1. $y = x^3 + 1$; (e)
2. $y = |x| - 2$; (g)
3. $y = -\sqrt{x}$; (j)
4. $y = -\sin x$ or $y = \sin(-x)$; (a)
5. $y = -x$; (i)
6. $y = (x - 1)^2$; (f)
7. $y = \text{int}(x + 1)$; (k)
8. $y = -\frac{1}{x}$; (h)
9. $y = (x + 2)^3$; (d)
10. $y = e^x - 2$; (c)
11. $2 - \frac{4}{1 + e^{-x}}$; (l)
12. $y = \cos x + 1$; (b)
13. Exercise 8
14. Exercise 3
15. Exercises 7, 8
16. Exercise 7 (Remember that a continuous function is one that is continuous at every point *in its domain.*)
17. Exercises 2, 4, 6, 10, 11, 12
18. Exercises 3, 4, 11, 12
19. $y = x, y = x^3, y = \frac{1}{x}, y = \sin x$
20. $y = x, y = x^3, y = \sqrt{x}, y = e^x, y = \ln x, y = \frac{1}{1 + e^{-x}}$
21. $y = x^2, y = \frac{1}{x}, y = |x|$
22. $y = \sin x, y = \cos x, y = \text{int}(x)$
23. $y = \frac{1}{x}, y = e^x, y = \frac{1}{1 + e^{-x}}$
24. $y = x, y = x^3, y = \ln x$
25. $y = \frac{1}{x}, y = \sin x, y = \cos x, y = \frac{1}{1 + e^{-x}}$
26. $y = x, y = x^3, y = \text{int}(x)$
27. $y = x, y = x^3, y = \frac{1}{x}, y = \sin x$
28. $y = \sin x, y = \cos x$
29. Domain: All reals
Range: $[-5, \infty)$



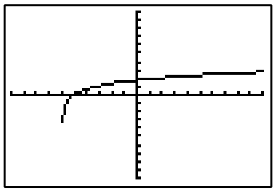
$[-10, 10]$ by $[-10, 10]$

30. Domain: All reals
Range: $[0, \infty)$



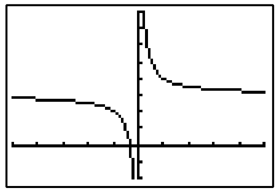
$[-10, 10]$ by $[-10, 10]$

31. Domain: $(-6, \infty)$
Range: All reals



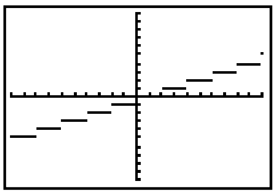
$[-10, 10]$ by $[-10, 10]$

32. Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 3) \cup (3, \infty)$



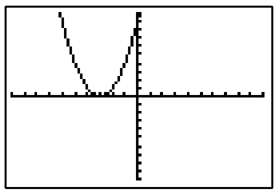
$[-5, 5]$ by $[-2, 8]$

33. Domain: All reals
Range: All integers



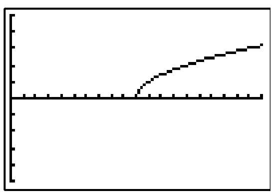
$[-10, 10]$ by $[-10, 10]$

34. Domain: All reals
Range: $[0, \infty)$



$[-10, 10]$ by $[-10, 10]$

35.



$[0, 20]$ by $[-5, 5]$

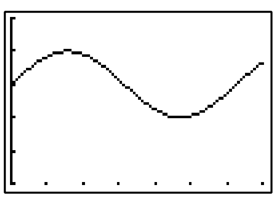
(a) $r(x)$ is increasing on $[10, \infty)$.

(b) $r(x)$ is neither odd nor even.

(c) The one extreme is a minimum value of 0 at $x = 10$.

(d) $r(x) = \sqrt{x - 10}$ is the square root function, shifted 10 units right.

36.



$[0, 7]$ by $[2, 7]$

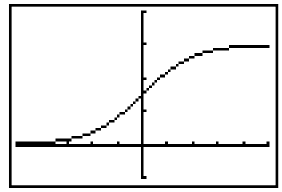
(a) $f(x)$ is increasing on $\left[(2k - 1)\frac{\pi}{2}, (2k + 1)\frac{\pi}{2}\right]$ and decreasing on $\left[(2k + 1)\frac{\pi}{2}, (2k + 3)\frac{\pi}{2}\right]$, where k is an even integer.

(b) $f(x)$ is neither odd nor even.

(c) There are minimum values of 4 at $x = (2k - 1)\frac{\pi}{2}$ and maximum values of 6 at $x = (2k + 1)\frac{\pi}{2}$, where k is an even integer.

(d) $f(x) = \sin(x) + 5$ is the sine function, $\sin x$, shifted 5 units up.

37.



$[-5, 5]$ by $[-1, 4]$

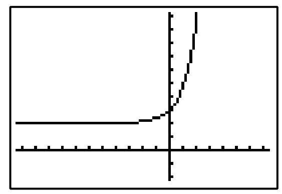
(a) $f(x)$ is increasing on $(-\infty, \infty)$.

(b) $f(x)$ is neither odd nor even.

(c) There are no extrema.

(d) $f(x) = \dfrac{3}{1 + e^{-x}}$ is the logistic function, $\dfrac{1}{1 + e^{-x}}$, stretched vertically by a factor of 3.

38.



$[-11.4, 7.4]$ by $[-2.2, 10.2]$

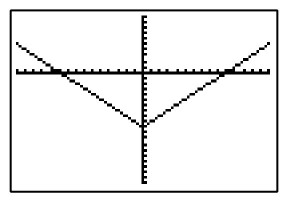
(a) $q(x)$ is increasing on $(-\infty, \infty)$.

(b) $q(x)$ is neither odd nor even.

(c) There are no extrema.

(d) $q(x) = e^x + 2$ is the exponential function, e^x, shifted 2 units up.

39.



$[-15, 15]$ by $[-20, 10]$

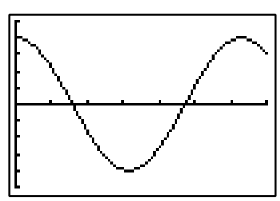
(a) $h(x)$ is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$.

(b) $h(x)$ is even, because it is symmetric about the y-axis.

(c) The one extremum is a minimum value of -10 at $x = 0$.

(d) $h(x) = |x| - 10$ is the absolute value function, $|x|$, shifted 10 units down.

40.



$[0, 7]$ by $[-5, 5]$

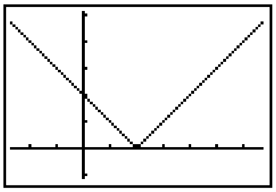
(a) $g(x)$ is increasing on $[(2k - 1)\pi, 2k\pi]$ and decreasing on $[2k\pi, (2k + 1)\pi]$, where k is an integer.

(b) $g(x)$ is even, because it is symmetric about the y-axis.

(c) There are minimum values of -4 at $x = (2k - 1)\pi$ and maximum values of 4 at $x = 2k\pi$, where k is an integer.

(d) $g(x) = 4\cos(x)$ is the cosine function, $\cos x$, stretched vertically by a factor of 4.

41.



[–2.7, 6.7] by [–1.1, 5.1]

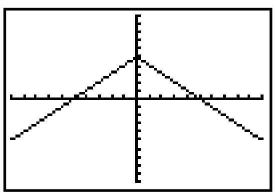
(a) $s(x)$ is increasing on $[2, \infty)$ and decreasing on $(-\infty, 2]$.

(b) $s(x)$ is neither odd nor even.

(c) The one extremum is a minimum value of 0 at $x = 2$.

(d) $s(x) = |x - 2|$ is the absolute value function, $|x|$, shifted 2 units to the right.

42.



[–10, 10] by [–10, 10]

(a) $f(x)$ is increasing on $(-\infty, 0]$ and decreasing on $[0, \infty)$.

(b) $f(x)$ is even, because it is symmetric about the y-axis.

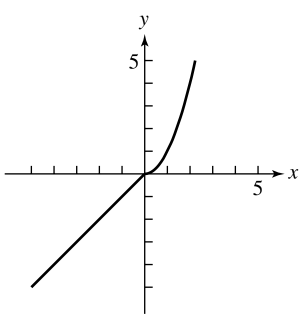
(c) The one extremum is a maximum value of 5 at $x = 0$.

(d) $f(x) = 5 - \text{abs}(x)$ is the absolute value function, $\text{abs}(x)$, reflected across the x-axis and then shifted 5 units up.

43. The end behavior approaches the horizontal asymptotes $y = 2$ and $y = -2$.

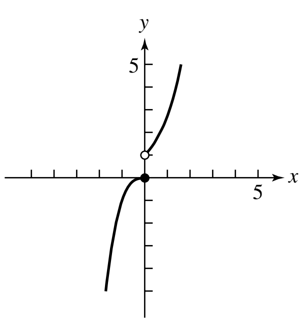
44. The end behavior approaches the horizontal asymptotes $y = 0$ and $y = 3$.

45.



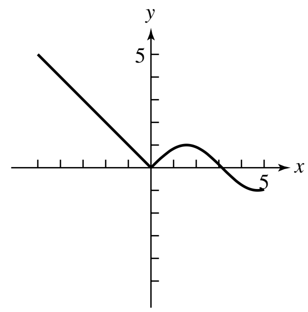
There are no points of discontinuity.

46.



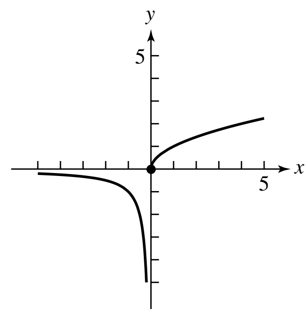
There is a point of discontinuity at $x = 0$.

47.



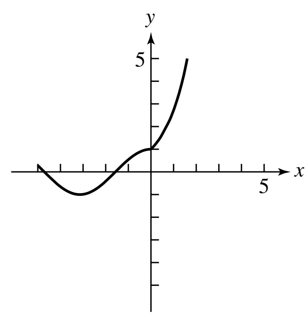
There are no points of discontinuity.

48.



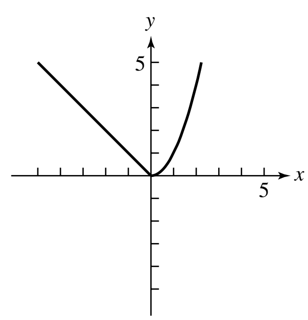
There is a point of discontinuity at $x = 0$.

49.



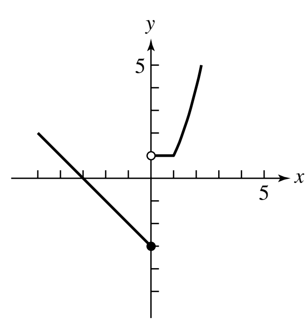
There are no points of discontinuity.

50.



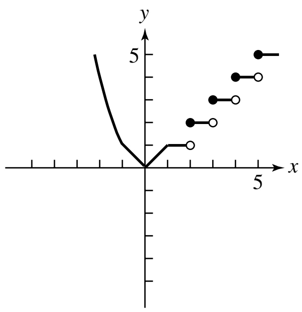
There are no points of discontinuity.

51.



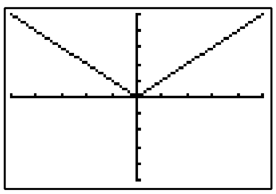
There is a point of discontinuity at $x = 0$.

52.



There are points of discontinuity at $x = 2, 3, 4, 5, \ldots$.

53. (a)

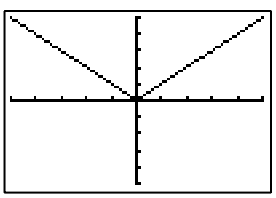


$[-5, 5]$ by $[-5, 5]$

This is $g(x) = |x|$.

(b) Squaring x and taking the (positive) square root has the same effect as the absolute value function.
$f(x) = \sqrt{x^2} = \sqrt{|x|^2} = |x| = g(x)$

54. (a)

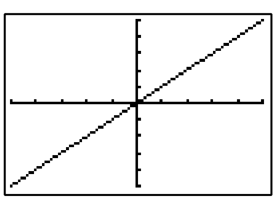


$[-5, 5]$ by $[-5, 5]$

This appears to be $f(x) = |x|$.

(b) For example, $g(1) \approx 0.99 \neq f(1) = 1$.

55. (a)

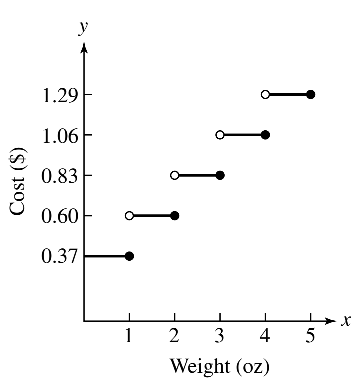


$[-5, 5]$ by $[-5, 5]$

This is the function $f(x) = x$.

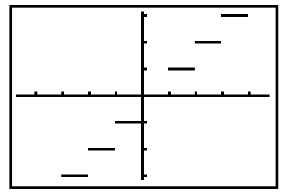
(b) The fact that $\ln(e^x) = x$ shows that the natural logarithm function takes on arbitrarily large values. In particular, it takes on the value L when $x = e^L$.

56. (a)



(b) One possible answer: It is similar because it has discontinuities spaced at regular intervals. It is different because its domain is the set of positive real numbers, and because it is constant on intervals of the form $(k, k + 1]$ instead of $[k, k + 1)$, where k is an integer.

57. The Greatest Integer Function $f(x) = \text{int}(x)$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: all real numbers
Range: all integers
Continuity: There is a discontinuity at each integer value of x.
Increasing/decreasing behavior: constant on intervals of the form $[k, k + 1)$, where k is an integer
Symmetry: none
Boundedness: not bounded
Local extrema: every non-integer is both a local minimum and local maximum
Horizontal asymptotes: none
Vertical asymptotes: none
End behavior: $\text{int}(x) \to -\infty$ as $x \to -\infty$ and $\text{int}(x) \to \infty$ as $x \to \infty$.

58. False. Because the greatest integer function is not one-to-one, its inverse relation is not a function.

59. True. The asymptotes are $x = 0$ and $x = 1$.

60. Because $3 - \dfrac{1}{x} \neq 3, 0 < \dfrac{5}{1 + e^{-x}} < 5, -4 \leq 4 \cos x \leq 4$, and $\text{int}(x - 2)$ takes only integer values. The answer is A.

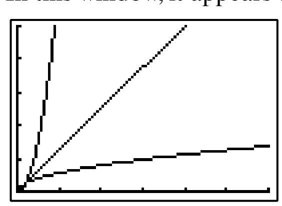
61. $3 < 3 + \dfrac{1}{1 + e^{-x}} < 4$. The answer is D.

62. By comparison of the graphs, the answer is C.

63. The answer is E. The others all have either a restricted domain or intervals where the function is decreasing or constant.

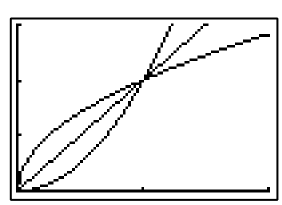
64. (a) Answers will vary.

(b) In this window, it appears that $\sqrt{x} < x < x^2$:



$[0, 30]$ by $[0, 20]$

(c)



$[0, 2]$ by $[0, 1.5]$

(d) On the interval $(0, 1)$, $x^2 < x < \sqrt{x}$.
On the interval $(1, \infty)$, $\sqrt{x} < x < x^2$.
All three functions equal 1 when $x = 1$.

65. (a) A product of two odd functions is even.
(b) A product of two even functions is even.
(c) A product of an odd function and an even function is odd.

66. Answers vary.

67. (a) Pepperoni count ought to be proportional to the area of the pizza, which is proportional to the square of the radius.
(b) $12 = k(4)^2$
$$k = \frac{12}{16} = \frac{3}{4} = 0.75$$
(c) Yes, very well.
(d) The fact that the pepperoni count fits the expected quadratic model so perfectly suggests that the pizzeria uses such a chart. If repeated observations produced the same results, there would be little doubt.

68. (a) $y = e^x$ and $y = \ln x$
(b) $y = x$ and $y = \dfrac{1}{x}$
(c) With domain $[0, \infty)$, $y = x^2$ becomes the inverse of $y = \sqrt{x}$.

69. (a) At $x = 0$, $\dfrac{1}{x}$ does not exist, $e^x = 1$, $\ln x$ is not defined, $\cos x = 1$, and $\dfrac{1}{1 + e^{-x}} = 1$.
(b) for $f(x) = x$, $f(x + y) = x + y = f(x) + f(y)$
(c) for $f(x) = e^x$, $f(xy) = e^{xy} = e^x e^y = f(x) \cdot f(y)$
(d) for $f(x) = \ln x$, $f(x + y) = \ln(xy) = \ln(x) + \ln(y) = f(x) + f(y)$
(e) The odd functions: $x, x^3, \dfrac{1}{x}, \sin x$

■ Section 1.4 Building Functions from Functions

Exploration 1

If $f = 2x - 3$ and $g = \dfrac{x + 3}{2}$, then
$$f \circ g = 2\left(\frac{x + 3}{2}\right) - 3 = x + 3 - 3 = x.$$

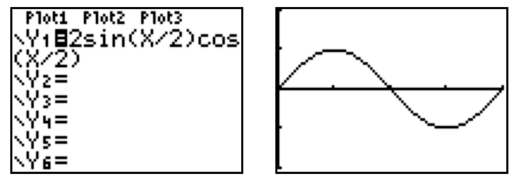
If $f = |2x + 4|$ and $g = \dfrac{(x - 2)(x + 2)}{2}$,
then $$f \circ g = 2\left(\frac{(x - 2)(x + 2)}{2}\right) + 4 = (x - 2)(x + 2) + 4 = x^2 - 4 + 4 = x^2.$$

If $f = \sqrt{x}$ and $g = x^2$, then $f \circ g = \sqrt{x^2} = |x|$. Note, we use the absolute value of x because g is defined for $-\infty < x < +\infty$, while f is defined only for positive values of x. The absolute value function is always positive.

If $f = x^5$ and $g = x^{0.6}$, then $f \circ g = (x^{0.6})^5 = x^3$.

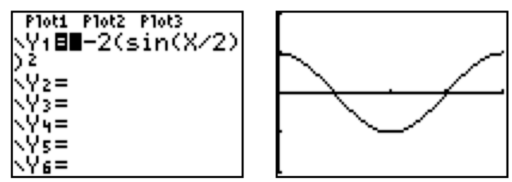
If $f = x - 3$ and $g = \ln(e^3 x)$, then $f \circ g = \ln(e^3 x) - 3 = \ln(e^3) + \ln x - 3 = 3 \ln e + \ln x - 3 = 3 + \ln x - 3 = \ln x$.

If $f = 2 \sin x \cos x$ and $g = \dfrac{x}{2}$, then $f \circ g = 2 \sin \dfrac{x}{2} \cos \dfrac{x}{2} = \sin\left(2\left(\dfrac{x}{2}\right)\right) = \sin x$. This is the double angle formula (*see Section 5.4*). You can see this graphically.



$[0, 2\pi]$ by $[-2, 2]$

If $f = 1 - 2x^2$ and $g = \sin\left(\dfrac{x}{2}\right)$,
then $f \circ g = 1 - 2\left(\sin^2\left(\dfrac{x}{2}\right)\right) = \cos\left(2\left(\dfrac{x}{2}\right)\right) = \cos x$.
(The double angle formula for $\cos 2x$ is $\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2 \sin^2 x$. *See Section 5.3*.) This can be seen graphically:



$[0, 2\pi]$ by $[-2, 2]$

f	g	$f \circ g$
$2x - 3$	$\dfrac{x + 3}{2}$	x
$\lvert 2x + 4 \rvert$	$\dfrac{(x - 2)(x + 2)}{2}$	x^2
$\sqrt{x}$	x^2	$\lvert x \rvert$
x^5	$x^{0.6}$	x^3
$x - 3$	$\ln(e^3 x)$	$\ln x$
$2 \sin x \cos x$	$\dfrac{x}{2}$	$\sin x$
$1 - 2x^2$	$\sin\left(\dfrac{x}{2}\right)$	$\cos x$

Quick Review 1.4

1. $(-\infty, -3) \cup (-3, \infty)$
2. $(1, \infty)$
3. $(-\infty, 5]$
4. $(1/2, \infty)$
5. $[1, \infty)$
6. $[-1, 1]$
7. $(-\infty, \infty)$
8. $(-\infty, 0) \cup (0, \infty)$
9. $(-1, 1)$
10. $(-\infty, \infty)$

Section 1.4 Exercises

1. $(f + g)(x) = 2x - 1 + x^2$; $(f - g)(x) = 2x - 1 - x^2$; $(fg)(x) = (2x - 1)(x^2) = 2x^3 - x^2$.

There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.

2. $(f + g)(x) = (x - 1)^2 + 3 - x = x^2 - 2x + 1 + 3 - x = x^2 - 3x + 4$;
$(f - g)(x) = (x - 1)^2 - 3 + x = x^2 - 2x + 1 - 3 + x = x^2 - x - 2$;
$(fg)(x) = (x - 1)^2(3 - x) = (x^2 - 2x + 1)(3 - x) = 3x^2 - x^3 - 6x + 2x^2 + 3 - x = -x^3 + 5x^2 - 7x + 3$.

There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.

3. $(f + g)(x) = \sqrt{x} + \sin x$; $(f - g)(x) = \sqrt{x} - \sin x$; $(fg)(x) = \sqrt{x}\sin x$.
Domain in each case is $[0, \infty)$. For $\sqrt{x}$, $x \ge 0$. For $\sin x$, $-\infty < x < \infty$.

4. $(f + g)(x) = \sqrt{x + 5} + |x + 3|$;
$(f - g)(x) = \sqrt{x + 5} - |x + 3|$;
$(fg)(x) = \sqrt{x + 5}\,|x + 3|$.
All three expressions contain $\sqrt{x + 5}$, so $x + 5 \ge 0$ and $x \ge -5$; all three domains are $[-5, \infty)$. For $|x + 3|$, $-\infty < x < \infty$.

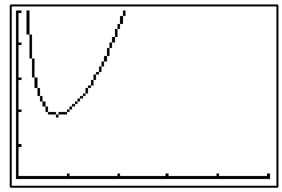
5. $(f/g)(x) = \dfrac{\sqrt{x + 3}}{x^2}$; $x + 3 \ge 0$ and $x \ne 0$, so the domain is $[-3, 0) \cup (0, \infty)$.
$(g/f)(x) = \dfrac{x^2}{\sqrt{x + 3}}$; $x + 3 > 0$, so the domain is $(-3, \infty)$.

6. $(f/g)(x) = \dfrac{\sqrt{x - 2}}{\sqrt{x + 4}} = \sqrt{\dfrac{x - 2}{x + 4}}$; $x - 2 \ge 0$ and $x + 4 > 0$, so $x \ge 2$ and $x > -4$; the domain is $[2, \infty)$.
$(g/f)(x) = \dfrac{\sqrt{x + 4}}{\sqrt{x - 2}} = \sqrt{\dfrac{x + 4}{x - 2}}$; $x + 4 \ge 0$ and $x - 2 > 0$, so $x \ge -4$ and $x > 2$; the domain is $(2, \infty)$.

7. $(f/g)(x) = \dfrac{x^2}{\sqrt{1 - x^2}}$. The denominator cannot be zero and the term under the square root must be positive, so $1 - x^2 > 0$. Therefore, $x^2 < 1$, which means that $-1 < x < 1$. The domain is $(-1, 1)$.
$(g/f)(x) = \dfrac{\sqrt{1 - x^2}}{x^2}$. The term under the square root must be nonnegative, so $1 - x^2 \ge 0$ (or $x^2 \le 1$). The denominator cannot be zero, so $x \ne 0$. Therefore, $-1 \le x < 0$ or $0 < x \le 1$. The domain is $[-1, 0) \cup (0, 1]$.

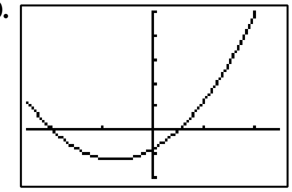
8. $(f/g)(x) = \dfrac{x^3}{\sqrt[3]{1 - x^3}}$. The denominator cannot be 0, so $1 - x^3 \ne 0$ and $x^3 \ne 1$. This means that $x \ne 1$. There are no restrictions on x in the numerator. The domain is $(-\infty, 1) \cup (1, \infty)$.
$(g/f)(x) = \dfrac{\sqrt[3]{1 - x^3}}{x^3}$. The denominator cannot be 0, so $x^3 \ne 0$ and $x \ne 0$. There are no restrictions on x in the numerator. The domain is $(-\infty, 0) \cup (0, \infty)$.

9.



$[0, 5]$ by $[0, 5]$

10.



$[-5, 5]$ by $[-10, 25]$

11. $(f \circ g)(3) = f(g(3)) = f(4) = 5$;
$(g \circ f)(-2) = g(f(-2)) = g(-7) = -6$

12. $(f \circ g)(3) = f(g(3)) = f(3) = 8$;
$(g \circ f)(-2) = g(f(-2)) = g(3) = 3$

13. $(f \circ g)(3) = f(g(3)) = f(\sqrt{3 + 1}) = f(2) = 2^2 + 4 = 8$;
$(g \circ f)(-2) = g(f(-2)) = g((-2)^2 + 4) = g(8) = \sqrt{8 + 1} = 3$

14. $(f \circ g)(3) = f(g(3)) = f(9 - 3^2) = f(0) = \dfrac{0}{0 + 1} = 0$;
$(g \circ f)(-2) = g(f(-2)) = g\left(\dfrac{-2}{-2 + 1}\right) = g(2) = 9 - 2^2 = 5$

15. $f(g(x)) = 3(x - 1) + 2 = 3x - 3 + 2 = 3x - 1$.
Because both f and g have domain $(-\infty, \infty)$, the domain of $f(g(x))$ is $(-\infty, \infty)$.
$g(f(x)) = (3x + 2) - 1 = 3x + 1$; again, the domain is $(-\infty, \infty)$.

16. $f(g(x)) = \left(\dfrac{1}{x - 1}\right)^2 - 1 = \dfrac{1}{(x - 1)^2} - 1$. The domain of g is $x \ne 1$, while the domain of f is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \ne 1$, or $(-\infty, 1) \cup (1, \infty)$.
$g(f(x)) = \dfrac{1}{(x^2 - 1) - 1} = \dfrac{1}{x^2 - 2}$.
The domain of f is $(-\infty, \infty)$, while the domain of g is $(-\infty, 1) \cup (1, \infty)$, so $g(f(x))$ requires that $f(x) \ne 1$. This means $x^2 - 1 \ne 1$, or $x^2 \ne 2$, so the domain of $g(f(x))$ is $x \ne \pm\sqrt{2}$, or $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

17. $f(g(x)) = (\sqrt{x + 1})^2 - 2 = x + 1 - 2 = x - 1$. The domain of g is $x \ge -1$, while the domain of f is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \ge -1$, or $[-1, \infty)$.
$g(f(x)) = \sqrt{(x^2 - 2) + 1} = \sqrt{x^2 - 1}$. The domain of f is $(-\infty, \infty)$, while the domain of g is $[-1, \infty)$, so $g(f(x))$ requires that $f(x) \ge -1$.
This means $x^2 - 2 \ge -1$, or $x^2 \ge 1$, which means $x \le -1$ or $x \ge 1$. Therefore the domain of $g(f(x))$ is $(-\infty, -1] \cup [1, \infty)$.

18. $f(g(x)) = \dfrac{1}{\sqrt{x} - 1}$. The domain of g is $x \ge 0$, while the domain of f is $(-\infty, 1) \cup (1, \infty)$, so $f(g(x))$ requires that $x \ge 0$ and $g(x) \ne 1$, or $x \ge 0$, and $x \ne 1$. The domain of $f(g(x))$ is $[0, 1) \cup (1, \infty)$.
$g(f(x)) = \sqrt{\dfrac{1}{x - 1}} = \dfrac{1}{\sqrt{x - 1}}$. The domain of f is $x \ne 1$, while the domain of g is $[0, \infty)$, so $g(f(x))$ requires that $x \ne 1$ and $f(x) \ge 0$, or $x \ne 1$ and $\dfrac{1}{x - 1} \ge 0$. The latter occurs if $x - 1 > 0$, so the domain of $g(f(x))$ is $(1, \infty)$.

19. $f(g(x)) = f(\sqrt{1-x^2}) = (\sqrt{1-x^2})^2 = 1 - x^2$; the domain is $[-1, 1]$.
$g(f(x)) = g(x^2) = \sqrt{1-(x^2)^2} = \sqrt{1-x^4}$; the domain is $[-1, 1]$.

20. $f(g(x)) = f(\sqrt[3]{1-x^3}) = (\sqrt[3]{1-x^3})^3 = 1 - x^3$; the domain is $(-\infty, \infty)$.
$g(f(x)) = g(x^3) = \sqrt[3]{1-(x^3)^3} = \sqrt[3]{1-x^9}$; the domain is $(-\infty, \infty)$.

21. $f(g(x)) = f\left(\frac{1}{3x}\right) = \frac{1}{2(1/3x)} = \frac{1}{2/3x} = \frac{3x}{2}$; the domain is $(-\infty, 0) \cup (0, \infty)$.
$g(f(x)) = g\left(\frac{1}{2x}\right) = \frac{1}{3(1/2x)} = \frac{1}{3/2x} = \frac{2x}{3}$; the domain is $(-\infty, 0) \cup (0, \infty)$.

22. $f(g(x)) = f\left(\frac{1}{x-1}\right) = \frac{1}{(1/(x-1))+1} = \frac{1}{(1+(x-1))/(x-1)} = \frac{1}{x/(x-1)} = \frac{x-1}{x}$; the domain is all reals except 0 and 1.
$g(f(x)) = g\left(\frac{1}{x+1}\right) = \frac{1}{(1/(x+1))-1} = \frac{1}{(1+(x-1))/(x+1)} = \frac{1}{x/(x+1)} = \frac{x+1}{x}$; the domain is all reals except 0 and 1.

23. One possibility: $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5x$

24. One possibility: $f(x) = (x+1)^2$ and $g(x) = x^3$

25. One possibility: $f(x) = |x|$ and $g(x) = 3x - 2$

26. One possibility: $f(x) = 1/x$ and $g(x) = x^3 - 5x + 3$

27. One possibility: $f(x) = x^5 - 2$ and $g(x) = x - 3$

28. One possibility: $f(x) = e^x$ and $g(x) = \sin x$

29. One possibility: $f(x) = \cos x$ and $g(x) = \sqrt{x}$.

30. One possibility: $f(x) = x^2 + 1$ and $g(x) = \tan x$.

31. $r = 48 + 0.03t$ in., so $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(48 + 0.03t)^3$; when $t = 300$,
$V = \frac{4}{3}\pi(48 + 9)^3 = 246{,}924\pi \approx 775{,}734.6$ in^3.

32. The original diameter of each snowball is 4 in., so the original radius is 2 in. and the original volume $V = \frac{4}{3}\pi r^3 \approx 33.5$ in^3. The new volume is $V = 33.5 - t$, where t is the number of 40-day periods. At the end of 360 days, the new volume is $V = 33.5 - 9 = 24.5$. Since $V = \frac{4}{3}\pi r^3$, we know that $r = \sqrt[3]{\frac{3V}{4\pi}} \approx 1.8$ in. The diameter, then, is 2 times r, or ≈ 3.6 in.

33. The initial area is $(5)(7) = 35$ km^2. The new length and width are $l = 5 + 2t$ and $w = 7 + 2t$, so $A = lw = (5 + 2t)(7 + 2t)$. Solve $(7 + 2t)(5 + 2t) = 175$ (5 times its original size), either graphically or algebraically: the positive solution is $t \approx 3.63$ seconds.

34. The initial volume is $(5)(7)(3) = 105$ cm^3. The new length, width, and height are $l = 5 + 2t$, $w = 7 + 2t$, and $h = 3 + 2t$, so the new volume is $V = (5 + 2t)(7 + 2t)(3 + 2t)$. Solve graphically $(5 + 2t)(7 + 2t)(3 + 2t) \geq 525$ (5 times the original volume): $t \approx 1.62$ sec.

35. $3(1) + 4(1) = 3 + 4 = 7 \neq 5$
$3(4) + 4(-2) = 12 - 8 = 4 \neq 5$
$3(3) + 4(-1) = 9 - 4 = 5$
The answer is $(3, -1)$.

36. $(5)^2 + (1)^2 = 25 + 1 = 26 \neq 25$
$(3)^2 + (4)^2 = 9 + 16 = 25$
$(0)^2 + (-5)^2 = 0 + 25 = 25$
The answer is $(3, 4)$ and $(0, -5)$.

37. $y^2 = 25 - x^2$, $y = \sqrt{25 - x^2}$ and $y = -\sqrt{25 - x^2}$

38. $y^2 = 25 - x$, $y = \sqrt{25 - x}$ and $y = -\sqrt{25 - x}$

39. $y^2 = x^2 - 25$, $y = \sqrt{x^2 - 25}$ and $y = -\sqrt{x^2 - 25}$

40. $y^2 = 3x^2 - 25$, $y = \sqrt{3x^2 - 25}$ and $y = -\sqrt{3x^2 - 25}$

41. $x + |y| = 1 \Rightarrow |y| = -x + 1 \Rightarrow y = -x + 1$ or $y = -(-x + 1)$. $y = 1 - x$ and $y = x - 1$

42. $x - |y| = 1 \Rightarrow |y| = x - 1 \Rightarrow y = x - 1$ or $y = -(x - 1) = -x + 1$. $y = x - 1$ and $y = 1 - x$

43. $y^2 = x^2 \Rightarrow y = x$ and $y = -x$ or $y = |x|$ and $y = -|x|$

44. $y^2 = x \Rightarrow y = \sqrt{x}$ and $y = -\sqrt{x}$

45. False. If $g(x) = 0$, then $\left(\frac{f}{g}\right)(x)$ is not defined and 0 is not in the domain of $\left(\frac{f}{g}\right)(x)$, even though 0 may be in the domains of both $f(x)$ and $g(x)$.

46. False. For a number to be in the domain of $(fg)(x)$, it must be in the domains of *both* $f(x)$ and $g(x)$, so that $f(x)$ and $g(x)$ are both defined.

47. Composition of functions isn't necessarily commutative. The answer is C.

48. $g(x) = \sqrt{4 - x}$ cannot equal zero and the term under the square root must be positive, so x can be any real number less than 4. The answer is A.

49. $(f \circ f)(x) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = (x^4 + 2x^2 + 1) + 1 = x^4 + 2x^2 + 2$. The answer is E.

50. $y = |x| \Rightarrow y = x, y = -x; y = -x \Rightarrow x = -y; x = -y$ or $x = y \Rightarrow x^2 = y^2$. The answer is B.

51. If $f(x) = e^x$ and $g(x) = 2 \ln x$, then $f(g(x)) = f(2 \ln x) = e^{2 \ln x} = (e^{\ln x})^2 = x^2$. The domain is $(0, \infty)$.
If $f(x) = (x^2 + 2)^2$ and $g(x) = \sqrt{x - 2}$, then $f(g(x)) = f(\sqrt{x - 2}) = ((\sqrt{x - 2})^2 + 2)^2 = (x - 2 + 2)^2 = x^2$. The domain is $[2, \infty)$.
If $f(x) = (x^2 - 2)^2$ and $g(x) = \sqrt{2 - x}$, then $f(g(x)) = f(\sqrt{2 - x}) = ((\sqrt{2 - x})^2 - 2)^2 = (2 - x - 2)^2 = x^2$. The domain is $(-\infty, 2]$.

If $f(x) = \frac{1}{(x-1)^2}$ and $g(x) = \frac{x+1}{x}$, then
$f(g(x)) = f\left(\frac{x+1}{x}\right) = \frac{1}{\left(\frac{x+1}{x} - 1\right)^2} =$

$\frac{1}{\left(\frac{x+1-x}{x}\right)^2} = \frac{1}{\frac{1}{x^2}} = x^2$. The domain is $x \neq 0$.

If $f(x) = x^2 - 2x + 1$ and $g(x) = x + 1$, then
$f(g(x)) = f(x+1) = (x+1)^2 - 2(x+1) + 1 =$
$((x+1) - 1)^2 = x^2$. The domain is $(-\infty, \infty)$.

If $f(x) = \left(\frac{x+1}{x}\right)^2$ and $g(x) = \frac{1}{x-1}$, then
$f(g(x)) = f\left(\frac{1}{x-1}\right) = \left(\frac{\frac{1}{x-1} + 1}{\frac{1}{x-1}}\right)^2 =$

$\left(\frac{\frac{1+x-1}{x-1}}{\frac{1}{x-1}}\right)^2 = x^2$. The domain is $x \neq 1$.

f	g	D
e^x	$2 \ln x$	$(0, \infty)$
$(x^2+2)^2$	$\sqrt{x-2}$	$[2, \infty)$
$(x^2-2)^2$	$\sqrt{2-x}$	$(-\infty, 2]$
$\frac{1}{(x-1)^2}$	$\frac{x+1}{x}$	$x \neq 0$
$x^2 - 2x + 1$	$x + 1$	$(-\infty, \infty)$
$\left(\frac{x+1}{x}\right)^2$	$\frac{1}{x-1}$	$x \neq 1$

52. (a) $(fg)(x) = x^4 - 1 = (x^2+1)(x^2-1) = f(x) \cdot (x^2-1)$, so $g(x) = x^2 - 1$.

(b) $(f+g)(x) = 3x^2 \Rightarrow 3x^2 - (x^2+1) = 2x^2 - 1 = g(x)$.

(c) $(f/g)(x) = 1 \Rightarrow f(x) = g(x)$. So $g(x) = x^2 + 1$.

(d) $f(g(x)) = 9x^4 + 1$ and $f(x) = x^2 + 1$. If $g(x) = 3x^2$, then $f(g(x)) = f(3x^2) = (3x^2)^2 + 1 = 9x^4 + 1$.

(e) $g(f(x)) = 9x^4 + 1$ and $f(x) = x^2 + 1$. Then $g(x^2+1) = 9x^4 + 1 = 9((x^2+1) - 1)^2 + 1$, so $g(x) = 9(x-1)^2 + 1$.

53. (a) $(f+g)(x) = (g+f)(x) = f(x)$ if $g(x) = 0$.

(b) $(fg)(x) = (gf)(x) = f(x)$ if $g(x) = 1$.

(c) $(f \circ g)(x) = (g \circ f)(x) = f(x)$ if $g(x) = x$.

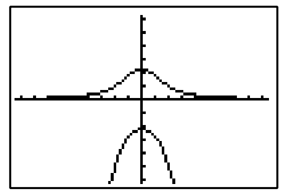
54. Yes, by definition, function composition is associative. That is, $(f \circ (g \circ h))(x) = f(g(h))(x)$ and $((f \circ g) \circ h)(x) = f(g(h))(x)$.

55. $y^2 + x^2y - 5 = 0$. Using the quadratic formula,

$$y = \frac{-x^2 \pm \sqrt{(x^2)^2 - 4(1)(-5)}}{2} = \frac{-x^2 \pm \sqrt{x^4 + 20}}{2}$$

so, $y_1 = \frac{-x^2 + \sqrt{x^4 + 20}}{2}$

and $y_2 = \frac{-x^2 - \sqrt{x^4 + 20}}{2}$.



[−9.4, 9.4] by [−6.2, 6.2]

■ Section 1.5 Parametric Relations and Inverses

Exploration 1

1. T starts at −4, at the point (8, −3). It stops at T = 2, at the point (8, 3). 61 points are computed.
2. The graph is smoother because the plotted points are closer together.
3. The graph is less smooth because the plotted points are further apart. In CONNECT mode, they are connected by straight lines.
4. The smaller the Tstep, the slower the graphing proceeds. This is because the calculator has to compute more X and Y values.
5. The grapher skips directly from the point (0, −1) to the point (0, 1), corresponding to the T-values T = −2 and T = 0. The two points are connected by a straight line, hidden by the Y-axis.
6. With the Tmin set at −1, the grapher begins at the point (−1, 0), missing the bottom of the curve entirely.
7. Leave everything else the same, but change Tmin back to −4 and Tmax to −1.

Quick Review 1.5

1. $3y = x + 6$, so $y = \frac{x+6}{3} = \frac{1}{3}x + 2$
2. $0.5y = x - 1$, so $y = \frac{x-1}{0.5} = 2x - 2$
3. $y^2 = x - 4$, so $y = \pm\sqrt{x-4}$
4. $y^2 = x + 6$, so $y = \pm\sqrt{x+6}$
5. $$\begin{aligned} x(y+3) &= y - 2 \\ xy + 3x &= y - 2 \\ xy - y &= -3x - 2 \\ y(x-1) &= -(3x+2) \\ y &= -\frac{3x+2}{x-1} = \frac{3x+2}{1-x} \end{aligned}$$

6. $x(y+2) = 3y - 1$
$xy + 2x = 3y - 1$
$xy - 3y = -2x - 1$
$y(x-3) = -(2x+1)$
$y = -\dfrac{2x+1}{x-3} = \dfrac{2x+1}{3-x}$

7. $x(y-4) = 2y + 1$
$xy - 4x = 2y + 1$
$xy - 2y = 4x + 1$
$y(x-2) = 4x + 1$
$y = \dfrac{4x+1}{x-2}$

8. $x(3y-1) = 4y + 3$
$3xy - x = 4y + 3$
$3xy - 4y = x + 3$
$y(3x-4) = x + 3$
$y = \dfrac{x+3}{3x-4}$

9. $x = \sqrt{y+3}$, $y \ge -3$ [and $x \ge 0$]
$x^2 = y + 3$, $y \ge -3$, and $x \ge 0$
$y = x^2 - 3$, $y \ge -3$, and $x \ge 0$

10. $x = \sqrt{y-2}$, $y \ge 2$ [and $x \ge 0$]
$x^2 = y - 2$, $y \ge 2$, and $x \ge 0$
$y = x^2 + 2$, $y \ge 2$, and $x \ge 0$

Section 1.5 Exercises

1. $x = 3(2) = 6$, $y = 2^2 + 5 = 9$. The answer is (6, 9).

2. $x = 5(-2) - 7 = -17$, $y = 17 - 3(-2) = 23$. The answer is $(-17, 23)$.

3. $x = 3^3 - 4(3) = 15$, $y = \sqrt{3+1} = 2$. The answer is (15, 2).

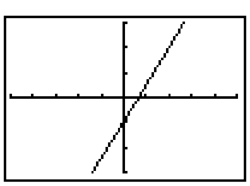
4. $x = |-8+3| = 5$, $y = \dfrac{1}{-8} = -\dfrac{1}{8}$

5. (a)

t	$(x, y) = (2t, 3t-1)$
−3	(−6, −10)
−2	(−4, −7)
−1	(−2, −4)
0	(0, −1)
1	(2, 2)
2	(4, 5)
3	(6, 8)

(b) $t = \dfrac{x}{2}$, $y = 3\left(\dfrac{x}{2}\right) - 1 = 1.5x - 1$. This is a function.

(c)

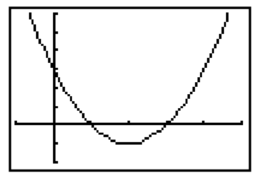

[–5, 5] by [–3, 3]

6. (a)

t	$(x, y) = (t+1, t^2 - 2t)$
−3	(−2, 15)
−2	(−1, 8)
−1	(0, 3)
0	(1, 0)
1	(2, −1)
2	(3, 0)
3	(4, 3)

(b) $t = x - 1$, $y = (x-1)^2 - 2(x-1)$
$= x^2 - 2x + 1 - 2x + 2$
$= x^2 - 4x + 3$
This is a function.

(c)

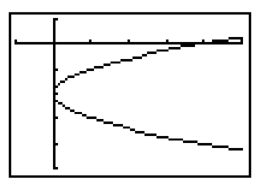

[–1, 5] by [–2, 6]

7. (a)

t	$(x, y) = (t^2, t-2)$
−3	(9, −5)
−2	(4, −4)
−1	(1, −3)
0	(0, −2)
1	(1, −1)
2	(4, 0)
3	(9, 1)

(b) $t = y + 2$, $x = (y+2)^2$. This is not a function.

(c)

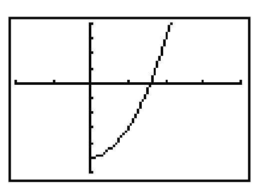

[–1, 5] by [–5, 1]

8. (a)

t	$(x, y) = (\sqrt{t}, 2t - 5)$
−3	$\sqrt{-3}$ not defined
−2	$\sqrt{-2}$ not defined
−1	$\sqrt{-1}$ not defined
0	(0, −5)
1	(1, −3)
2	$(\sqrt{2}, -1)$
3	$(\sqrt{3}, 1)$

(b) $t = x^2$, $y = 2x^2 - 5$. This is a function.

(c)


[–2, 4] by [–6, 4]

9. **(a)** By the vertical line test, the relation is not a function.

(b) By the horizontal line test, the relation's inverse is a function.

10. **(a)** By the vertical line test, the relation is a function.

(b) By the horizontal line test, the relation's inverse is not a function.

11. **(a)** By the vertical line test, the relation is a function.

(b) By the horizontal line test, the relation's inverse is a function.

12. **(a)** By the vertical line test, the relation is not a function.

(b) By the horizontal line test, the relation's inverse is a function.

13. $y = 3x - 6 \Rightarrow \quad x = 3y - 6$

$$3y = x + 6$$
$$f^{-1}(x) = y = \frac{x+6}{3} = \frac{1}{3}x + 2; (-\infty, \infty)$$

14. $y = 2x + 5 \Rightarrow \quad x = 2y + 5$

$$2y = x - 5$$
$$f^{-1}(x) = y = \frac{x-5}{2} = \frac{1}{2}x - \frac{5}{2};$$
$$(-\infty, \infty)$$

15. $y = \dfrac{2x-3}{x+1} \Rightarrow \quad x = \dfrac{2y-3}{y+1}$

$$x(y+1) = 2y - 3$$
$$xy + x = 2y - 3$$
$$xy - 2y = -x - 3$$
$$y(x-2) = -(x+3)$$
$$f^{-1}(x) = y = -\frac{x+3}{x-2} = \frac{x+3}{2-x};$$
$$(-\infty, 2) \cup (2, \infty)$$

16. $y = \dfrac{x+3}{x-2} \Rightarrow \quad x = \dfrac{y+3}{y-2}$

$$x(y-2) = y + 3$$
$$xy - 2x = y + 3$$
$$xy - y = 2x + 3$$
$$y(x-1) = 2x + 3$$
$$f^{-1}(x) = y = \frac{2x+3}{x-1};$$
$$x \neq 1 \text{ or } (-\infty, 1) \cup (1, \infty)$$

17. $y = \sqrt{x-3}, x \geq 3, y \geq 0 \Rightarrow$

$$x = \sqrt{y-3}, \quad x \geq 0, y \geq 3$$
$$x^2 = y - 3, \quad x \geq 0, y \geq 3$$
$$f^{-1}(x) = y = x^2 + 3, \quad x \geq 0$$

18. $y = \sqrt{x+2}, x \geq -2, y \geq 0 \Rightarrow$

$$x = \sqrt{y+2}, \quad x \geq 0, y \geq -2$$
$$x^2 = y + 2, \quad x \geq 0, y \geq -2$$
$$f^{-1}(x) = y = x^2 - 2, \quad x \geq 0$$

19. $y = x^3 \Rightarrow \quad x = y^3$

$$f^{-1}(x) = y = \sqrt[3]{x}; (-\infty, \infty)$$

20. $y = x^3 + 5 \Rightarrow \quad x = y^3 + 5$

$$x - 5 = y^3$$
$$f^{-1}(x) = y = \sqrt[3]{x-5}; (-\infty, \infty)$$

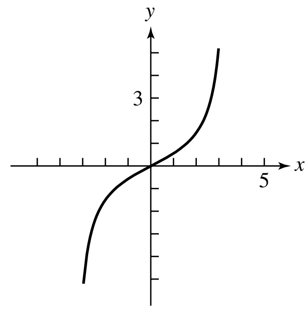
21. $y = \sqrt[3]{x+5} \Rightarrow \quad x = \sqrt[3]{y+5}$

$$x^3 = y + 5$$
$$f^{-1}(x) = y = x^3 - 5; (-\infty, \infty)$$

22. $y = \sqrt[3]{x-2} \Rightarrow \quad x = \sqrt[3]{y-2}$

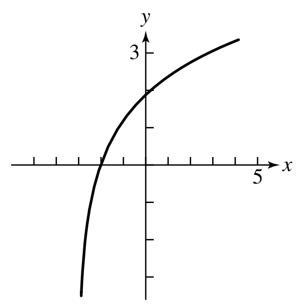
$$x^3 = y - 2$$
$$f^{-1}(x) = y = x^3 + 2; (-\infty, \infty)$$

23. One-to-one



24. Not one-to-one

25. One-to-one



26. Not one-to-one

27. $f(g(x)) = 3\left[\frac{1}{3}(x+2)\right] - 2 = x + 2 - 2 = x;$

$$g(f(x)) = \frac{1}{3}[(3x-2)+2] = \frac{1}{3}(3x) = x$$

28. $f(g(x)) = \frac{1}{4}[(4x-3)+3] = \frac{1}{4}(4x) = x;$

$$g(f(x)) = 4\left[\frac{1}{4}(x+3)\right] - 3 = x + 3 - 3 = x$$

29. $f(g(x)) = [(x-1)^{1/3}]^3 + 1 = (x-1)^1 + 1$

$$= x - 1 + 1 = x;$$
$$g(f(x)) = [(x^3+1)-1]^{1/3} = (x^3)^{1/3} = x^1 = x$$

30. $f(g(x)) = \dfrac{7}{\frac{7}{x}} = \dfrac{7}{1} \cdot \dfrac{x}{7} = x; g(f(x)) = \dfrac{7}{\frac{7}{x}} = \dfrac{7}{1} \cdot \dfrac{x}{7} = x$

31. $f(g(x)) = \dfrac{\frac{1}{x-1} + 1}{\frac{1}{x-1}} = (x-1)\left(\dfrac{1}{x-1} + 1\right)$

$$= 1 + x - 1 = x;$$
$$g(f(x)) = \frac{1}{\frac{x+1}{x} - 1} = \left(\frac{1}{\frac{x+1}{x} - 1}\right) \cdot \frac{x}{x}$$
$$= \frac{x}{x+1-x} = \frac{x}{1} = x$$

32. $f(g(x)) = \dfrac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2} = \left(\dfrac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2}\right)\cdot\left(\dfrac{x-1}{x-1}\right)$

$= \dfrac{2x+3+3(x-1)}{2x+3-2(x-1)} = \dfrac{5x}{5} = x;$

$g(f(x)) = \dfrac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1}$

$= \left[\dfrac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1}\right]\cdot\left(\dfrac{x-2}{x-2}\right)$

$= \dfrac{2(x+3)+3(x-2)}{x+3-(x-2)} = \dfrac{5x}{5} = x$

33. **(a)** $y = (1.08)(100) = 108$ euros

(b) $x = \dfrac{y}{1.08} = \dfrac{25}{27}y$. This converts euros ($x$) to dollars ($y$).

(c) $x = (0.9259)(48) = \$44.44$

34. **(a)** $9c(x) = 5(x-32)$

$\dfrac{9}{5}c(x) = x - 32$

$\dfrac{9}{5}c(x) + 32 = x$

In this case, $c(x)$ becomes x, and x becomes $c^{-1}(x)$ for the inverse. So, $c^{-1}(x) = \dfrac{9}{5}x + 32$. This converts Celsius temperature to Fahrenheit temperature.

(b) $(k \circ c)(x) = k(c(x)) = k\left(\dfrac{5}{9}(x-32)\right)$

$\dfrac{5}{9}(x-32) + 273.16 = \dfrac{5}{9}x + 255.38$. This is used to convert Fahrenheit temperature to Kelvin temperature.

35. $y = e^x$ and $y = \ln x$ are inverses. If we restrict the domain of the function $y = x^2$ to the interval $[0, \infty)$, then the restricted function and $y = \sqrt{x}$ are inverses.

36. $y = x$ and $y = 1/x$ are their own inverses.

37. $y = |x|$

38. $y = x$

39. True. All the ordered pairs swap domain and range values.

40. True. This is a parametrization of the line $y = 2x + 1$.

41. The inverse of the relation given by $x^2y + 5y = 9$ is the relation given by $y^2x + 5x = 9$.

$(1)^2(2) + 5(2) = 2 + 10 = 12 \neq 9$
$(1)^2(-2) + 5(-2) = -2 - 10 = -12 \neq 9$
$(2)^2(-1) + 5(-1) = -4 - 5 = -9 \neq 9$
$(-1)^2(2) + 5(2) = 2 + 10 = 12 \neq 9$
$(-2)^2(1) + 5(1) = 4 + 5 = 9$

The answer is E.

42. The inverse of the relation given by $xy^2 - 3x = 12$ is the relation given by $yx^2 - 3y = 12$.

$(-4)(0)^2 - 3(-4) = 0 + 12 = 12$
$(1)(4)^2 - 3(1) = 16 - 3 = 13 \neq 12$
$(2)(3)^2 - 3(2) = 18 - 6 = 12$
$(12)(2)^2 - 3(12) = 48 - 36 = 12$
$(-6)(1)^2 - 3(-6) = -6 + 18 = 12$

The answer is B.

43. $f(x) = 3x - 2$
$y = 3x - 2$

The inverse relation is

$x = 3y - 2$
$x + 2 = 3y$
$\dfrac{x+2}{3} = y$
$f^{-1}(x) = \dfrac{x+2}{3}$

The answer is C.

44. $f(x) = x^3 + 1$
$y = x^3 + 1$

The inverse relation is

$x = y^3 + 1$
$x - 1 = y^3$
$\sqrt[3]{x-1} = y$
$f^{-1}(x) = \sqrt[3]{x-1}$

The answer is A.

45. (Answers may vary.)

(a) If the graph of f is unbroken, its reflection in the line $y = x$ will be also.

(b) Both f and its inverse must be one-to-one in order to be inverse functions.

(c) Since f is odd, $(-x, -y)$ is on the graph whenever (x, y) is. This implies that $(-y, -x)$ is on the graph of f^{-1} whenever (x, y) is. That implies that f^{-1} is odd.

(d) Let $y = f(x)$. Since the ratio of Δy to Δx is positive, the ratio of Δx to Δy is positive. Any ratio of Δy to Δx on the graph of f^{-1} is the same as some ratio of Δx to Δy on the graph of f, hence positive. This implies that f^{-1} is increasing.

46. (Answers may vary.)

(a) $f(x) = e^x$ has a horizontal asymptote; $f^{-1}(x) = \ln x$ does not.

(b) $f(x) = e^x$ has domain all real numbers; $f^{-1}(x) = \ln x$ does not.

(c) $f(x) = e^x$ has a graph that is bounded below; $f^{-1}(x) = \ln x$ does not.

(d) $f(x) = \dfrac{x^2 - 25}{x - 5}$ has a removable discontinuity at $x = 5$ because its graph is the line $y = x + 5$ with the point $(5, 10)$ removed. The inverse function is the line $y = x - 5$ with the point $(10, 5)$ removed. This function has a removable discontinuity, but not at $x = 5$.

47. (a) $\dfrac{\Delta y}{\Delta x} = \dfrac{97 - 70}{88 - 52} = \dfrac{27}{36} = 0.75$, which gives us the slope of the equation. To find the rest of the equation, we use one of the initial points

$$y - 70 = 0.75(x - 52)$$
$$y = 0.75x - 39 + 70$$
$$y = 0.75x + 31$$

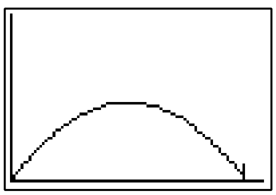
(b) To find the inverse, we substitute y for x and x for y, and then solve for y:

$$x = 0.75y + 31$$
$$x - 31 = 0.75y$$
$$y = \frac{4}{3}(x - 31)$$

The inverse function converts scaled scores to raw scores.

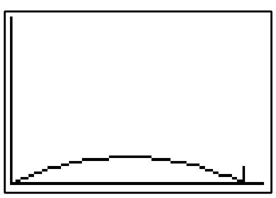
48. The function must be increasing so that the *order* of the students' grades, top to bottom, will remain the same after scaling as it is before scaling. A student with a raw score of 136 gets dropped to 133, but that will still be higher than the scaled score for a student with 134.

49. (a) It does not clear the fence.



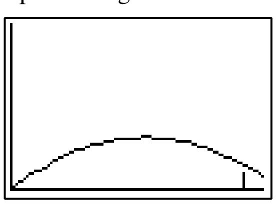
[0, 350] by [0, 300]

(b) It still does not clear the fence.



[0, 350] by [0, 300]

(c) Optimal angle is 45°. It clears the fence.



[0, 350] by [0, 300]

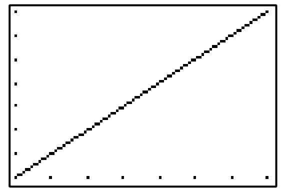
50. (a)

$$x = \left(\frac{3^{1.7}}{30}(y - 65)\right)^{\frac{1}{1.7}} + 1$$
$$x - 1 = \left(\frac{3^{1.7}}{30}(y - 65)\right)^{\frac{1}{1.7}}$$
$$(x - 1)^{1.7} = \left(\frac{3^{1.7}}{30}(y - 65)\right)$$
$$\frac{30}{3^{1.7}}(x - 1)^{1.7} = y - 65$$
$$y = \frac{30}{3^{1.7}}(x - 1)^{1.7} + 65$$

This can be use to convert GPA's to percentage grades.

(b) Yes; x is restricted to the domain [1, 4.28].

(c)



[65, 100] by [65, 100]

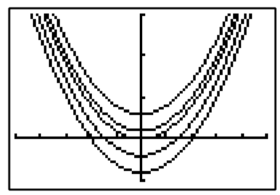
The composition function of $(y \circ y^{-1})(x)$ is $y = x$, so they are inverses.

51. When $k = 1$, the scaling function is linear. Opinions will vary as to which is the best value of k.

■ Section 1.6 Graphical Transformations

Exploration 1

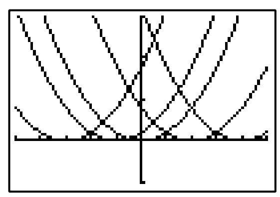
1.



[−5, 5] by [−5, 15]

They raise or lower the parabola along the y-axis.

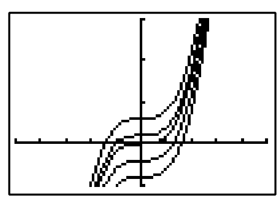
2.



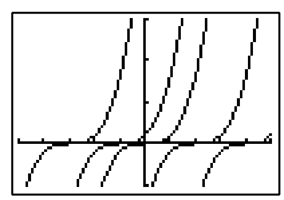
[−5, 5] by [−5, 15]

They move the parabola left or right along the x-axis.

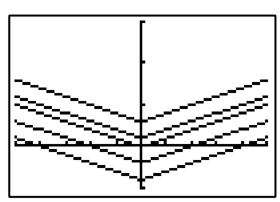
3.



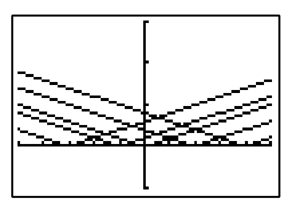
[−5, 5] by [−5, 15]



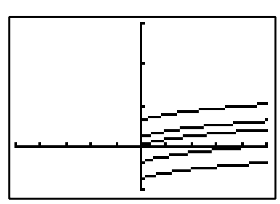
[−5, 5] by [−5, 15]



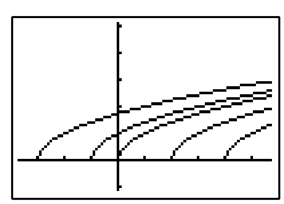
[−5, 5] by [−5, 15]



[−5, 5] by [−5, 15]



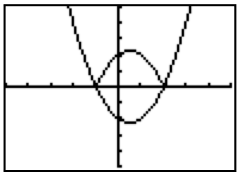
[−5, 5] by [−5, 15]



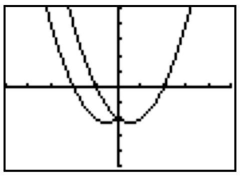
[−3.7, 5.7] by [−1.1, 5.1]

Yes

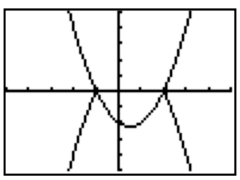
Exploration 2

1. 

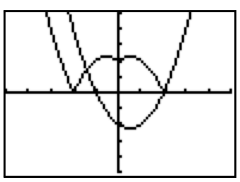
Graph C. Points with positive y-coordinates remain unchanged, while points with negative y-coordinates are reflected across the x-axis.

2. 

Graph A. Points with positive x-coordinates remain unchanged. Since the new function is even, the graph for negative x-values will be a reflection of the graph for positive x-values.

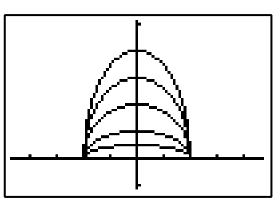
3. 

Graph F. The graph will be a reflection across the x-axis of graph C.

4. 

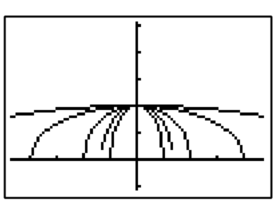
Graph D. The points with negative y-coordinates in graph A are reflected across the x-axis.

Exploration 3

1. 

$[-4.7, 4.7]$ by $[-1.1, 5.1]$

The 1.5 and the 2 stretch the graph vertically; the 0.5 and the 0.25 shrink the graph vertically.

2. 

$[-4.7, 4.7]$ by $[-1.1, 5.1]$

The 1.5 and the 2 shrink the graph horizontally; the 0.5 and the 0.25 stretch the graph horizontally.

Quick Review 1.6

1. $(x + 1)^2$
2. $(x - 3)^2$
3. $(x + 6)^2$
4. $(2x + 1)^2$
5. $(x - 5/2)^2$
6. $(2x - 5)^2$
7. $x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$
8. $2(x^2 + 6x + 9) - 5x - 15 - 2 =$ $2x^2 + 12x + 18 - 5x - 17 = 2x^2 + 7x + 1$
9. $(x^3 - 3x^2 + 3x - 1) + 3(x^2 - 2x + 1) - 3x + 3$ $= x^3 - 3x^2 + 2 + 3x^2 - 6x + 3 = x^3 - 6x + 5$
10. $2(x^3 + 3x^2 + 3x + 1) - 6(x^2 + 2x + 1) +$ $6x + 6 - 2 = 2x^3 + 6x^2 + 6x + 2 - 6x^2 - 12x$ $- 6 + 6x + 6 - 2 = 2x^3$

Section 1.6 Exercises

1. Vertical translation down 3 units
2. Vertical translation up 5.2 units
3. Horizontal translation left 4 units
4. Horizontal translation right 3 units
5. Horizontal translation to the right 100 units
6. Vertical translation down 100 units
7. Horizontal translation to the right 1 unit, and vertical translation up 3 units
8. Horizontal translation to the left 50 units and vertical translation down 279 units
9. Reflection across x-axis
10. Horizontal translation right 5 units
11. Reflection across y-axis
12. This can be written as $y = \sqrt{-(x - 3)}$ or $y = \sqrt{-x + 3}$. The first of these can be interpreted as reflection across the y-axis followed by a horizontal translation to the right 3 units. The second may be viewed as a horizontal translation left 3 units followed by a reflection across the y-axis.
    Note that when combining horizontal changes (horizontal translations and reflections across the y-axis), the order is "backwards" from what one may first expect: With $y = \sqrt{-(x - 3)}$, although we first subtract 3 from x then negate, the order of transformations is reflect then translate. With $y = \sqrt{-x + 3}$, although we negate x then add 3, the order of transformations is translate then reflect.

For #13–20, recognize $y = c \cdot x^3$ $(c > 0)$ as a vertical stretch (if $c > 1$) or shrink (if $0 < c < 1$) of factor c, and $y = (c \cdot x)^3$ as a horizontal shrink (if $c > 1$) or stretch (if $0 < c < 1$) of factor $1/c$. Note also that $y = (c \cdot x)^3 = c^3x^3$, so that for this function, any horizontal stretch/shrink can be interpreted as an equivalent vertical shrink/stretch (and vice versa).

13. Vertically stretch by 2
14. Horizontally shrink by 1/2, or vertically stretch by $2^3 = 8$
15. Horizontally stretch by $1/0.2 = 5$, or vertically shrink by $0.2^3 = 0.008$

16. Vertically shrink by 0.3

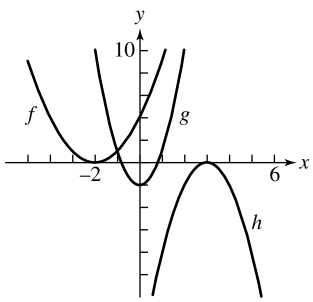
17. $g(x) = \sqrt{x - 6 + 2} = f(x - 6)$; starting with f, translate right 6 units to get g.

18. $g(x) = -(x + 4 - 1)^2 = -f(x + 4)$; starting with f, translate left 4 units, and reflect across the x-axis to get g.

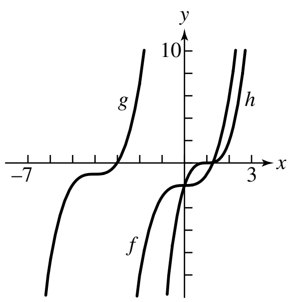
19. $g(x) = -(x + 4 - 2)^3 = -f(x + 4)$; starting with f, translate left 4 units, and reflect across the x-axis to get g.

20. $g(x) = 2|2x| = 2f(x)$; starting with f, vertically stretch by 2 to get g.

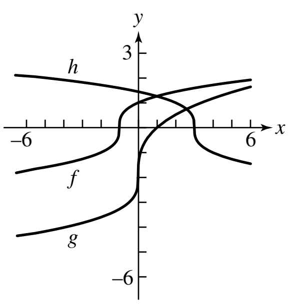
21.



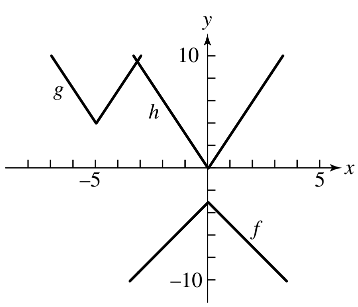
22.



23.



24.



25. Since the graph is translated left 5 units, $f(x) = \sqrt{x + 5}$.

26. The graph is reflected across the y-axis and translated right 3 units. $y = \sqrt{-x}$ would be reflected across the y-axis; the horizontal translation gives $f(x) = \sqrt{-(x - 3)} = \sqrt{3 - x}$. See also Exercise 12 in this section, and note accompanying that solution.

27. The graph is reflected across the x-axis, translated left 2 units, and translated up 3 units. $y = -\sqrt{x}$ would be reflected across the x-axis, $y = -\sqrt{x + 2}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = -\sqrt{x + 2} + 3 = 3 - \sqrt{x + 2}$.

28. The graph is vertically stretched by 2, translated left 5 units, and translated down 3 units. $y = 2\sqrt{x}$ would be vertically stretched, $y = 2\sqrt{x + 5}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = 2\sqrt{x + 5} - 3$.

29. **(a)** $y = -f(x) = -(x^3 - 5x^2 - 3x + 2)$
$= -x^3 + 5x^2 + 3x - 2$

(b) $y = f(-x) = (-x)^3 - 5(-x)^2 - 3(-x) + 2$
$= -x^3 - 5x^2 + 3x + 2$

30. **(a)** $y = -f(x) = -(2\sqrt{x + 3} - 4) = -2\sqrt{x + 3} + 4$

(b) $y = f(-x) = 2\sqrt{-x + 3} - 4 = 2\sqrt{3 - x} - 4$

31. **(a)** $y = -f(x) = -(\sqrt[3]{8x}) = -2\sqrt[3]{x}$

(b) $y = f(-x) = \sqrt[3]{8(-x)} = \sqrt[3]{-8x} = -2\sqrt[3]{x}$

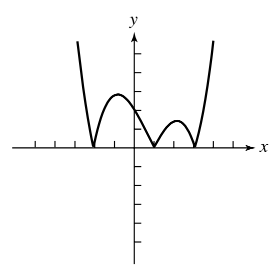
32. **(a)** $y = -f(x) = -3|x + 5|$

(b) $y = f(-x) = 3|-x + 5| = 3|5 - x|$

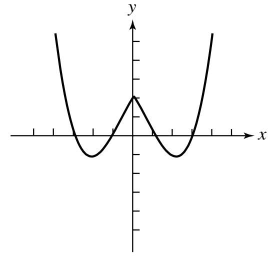
33. Let f be an odd function; that is, $f(-x) = -f(x)$ for all x in the domain of f. To reflect the graph of $y = f(x)$ across the y-axis, we make the transformation $y = f(-x)$. But $f(-x) = -f(x)$ for all x in the domain of f, so this transformation results in $y = -f(x)$. That is exactly the translation that reflects the graph of f across the x-axis, so the two reflections yield the same graph.

34. Let f be an odd function; that is, $f(-x) = -f(x)$ for all x in the domain of f. To reflect the graph of $y = f(x)$ across the y-axis, we make the transformation $y = f(-x)$. Then, reflecting across the x-axis yields $y = -f(-x)$. But $f(-x) = -f(x)$ for all x in the domain of f, so we have $y = -f(-x) = -[-f(x)] = f(x)$; that is, the original function.

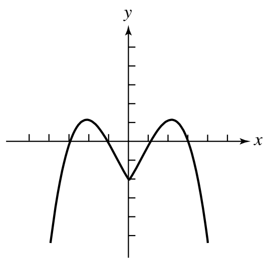
35.



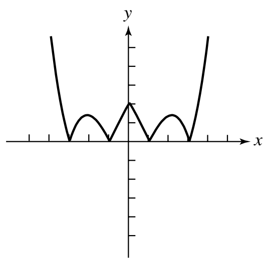
36.



37.



38.



39. **(a)** $y_1 = 2y = 2(x^3 - 4x) = 2x^3 - 8x$

(b) $y_2 = f\left(\frac{x}{\frac{1}{3}}\right) = f(3x) = (3x)^3 - 4(3x) = 27x^3 - 12x$

40. **(a)** $y_1 = 2y = 2|x + 2|$

(b) $y_2 = f(3x) = |3x + 2|$

41. **(a)** $y_1 = 2y = 2(x^2 + x - 2) = 2x^2 + 2x - 4$

(b) $y_2 = f(3x) = (3x)^2 + 3x - 2 = 9x^2 + 3x - 2$

42. **(a)** $y_1 = 2y = 2\left(\frac{1}{x + 2}\right) = \frac{2}{x + 2}$

(b) $y_2 = f(3x) = \frac{1}{3x + 2}$

43. Starting with $y = x^2$, translate right 3 units, vertically stretch by 2, and translate down 4 units.

44. Starting with $y = \sqrt{x}$, translate left 1 unit, vertically stretch by 3, and reflect across x-axis.

45. Starting with $y = x^2$, horizontally shrink by $\frac{1}{3}$ and translate down 4 units.

46. Starting with $y = |x|$, translate left 4 units, vertically stretch by 2, reflect across x-axis, and translate up 1 unit.

47. First stretch (multiply right side by 3): $y = 3x^2$, then translate (replace x with $x - 4$): $y = 3(x - 4)^2$.

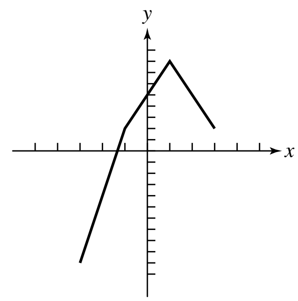
48. First translate (replace x with $x - 4$): $y = (x - 4)^2$, then stretch (multiply right side by 3): $y = 3(x - 4)^2$.

49. First translate left (replace x with $x + 2$): $y = |x + 2|$, then stretch (multiply right side by 2): $y = 2|x + 2|$, then translate down (subtract 4 from the right side): $y = 2|x + 2| - 4$.

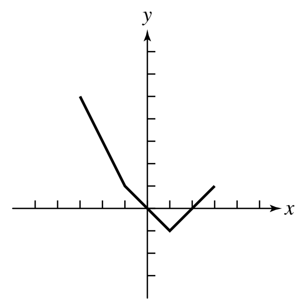
50. First translate left (replace x with $x + 2$): $y = |x + 2|$, then shrink (replace x with $2x$): $y = |2x + 2|$, then translate down (subtract 4 from the right side): $y = |2x + 2| - 4$. This can be simplified to $y = |2(x + 1)| - 4 = 2|x + 1| - 4$.

To make the sketches for #51–54, it is useful to apply the described transformations to several selected points on the graph. The original graph here has vertices $(-2, -4)$, $(0, 0)$, $(2, 2)$, and $(4, 0)$; in the solutions below, the images of these four points are listed.

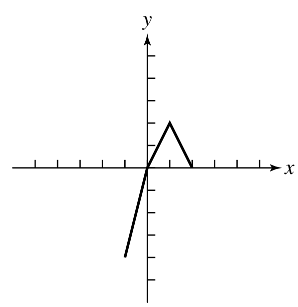
51. Translate left 1 unit, then vertically stretch by 3, and finally translate up 2 units. The four vertices are transformed to $(-3, -10)$, $(-1, 2)$, $(1, 8)$, and $(3, 2)$.



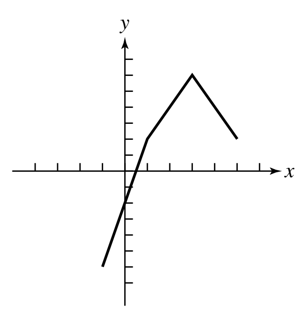
52. Translate left 1 unit, then reflect across the x-axis, and finally translate up 1 unit. The four vertices are transformed to $(-3, 5)$, $(-1, 1)$, $(1, -1)$, and $(3, 1)$.



53. Horizontally shrink by $\frac{1}{2}$. The four vertices are transformed to $(-1, -4)$, $(0, 0)$, $(1, 2)$, $(2, 0)$.



54. Translate right 1 unit, then vertically stretch by 2, and finally translate up 2 units. The four vertices are transformed to $(-1, -6)$, $(1, 2)$, $(3, 6)$, and $(5, 2)$.



55. Reflections have more effect on points that are farther away from the line of reflection. Translations affect the distance of points from the axes, and hence change the effect of the reflections.

56. The x-intercepts are the values at which the function equals zero. The stretching (or shrinking) factors have no effect on the number zero, so those y-coordinates do not change.

57. First vertically stretch by $\frac{9}{5}$, then translate up 32 units.

58. Solve for C: $F = \frac{9}{5}C + 32$, so $C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$. First vertically shrink by $\frac{5}{9}$, then translate down $\frac{160}{9} = 17.\bar{7}$ units.

59. False. $y = f(x + 3)$ is $y = f(x)$ translated 3 units to the *left.*

60. True. $y = f(x) - c$ represents a translation down by c units. (The translation is up when $c < 0$.)

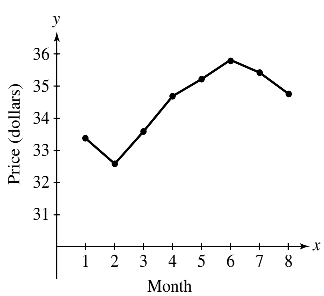
61. To vertically stretch $y = f(x)$ by a factor of 3, multiply the $f(x)$ by 3. The answer is C.

62. To translate $y = f(x)$ 4 units to the right, subtract 4 from x inside the $f(x)$. The answer is D.

63. To translate $y = f(x)$ 2 units up, add 2 to $f(x)$: $y = f(x) + 2$. To reflect the result across the y-axis, replace x with $-x$. The answer is A.

64. To reflect $y = f(x)$ across the x-axis, multiply $f(x)$ by -1: $y = -f(x)$. To shrink the result horizontally by a factor of $\frac{1}{2}$, replace x with $2x$. The answer is E.

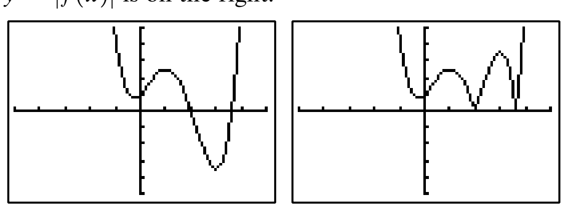
65. (a)



(b) Change the y-value by multiplying by the conversion rate from dollars to yen, a number that changes according to international market conditions. This results in a vertical stretch by the conversion rate.

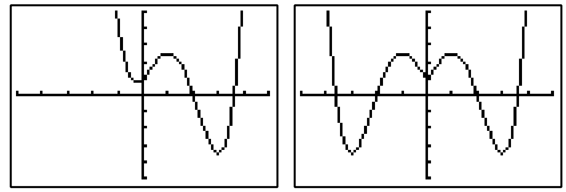
66. Apply the same transformation to the Ymin, Ymax, and Yscl as you apply to transform the function.

67. (a) The original graph is on the left; the graph of $y = |f(x)|$ is on the right.



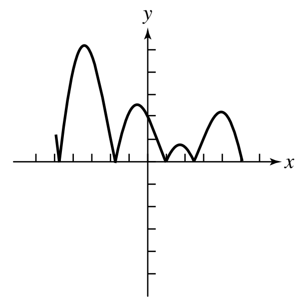
[−5, 5] by [−10, 10]   [−5, 5] by [−10, 10]

(b) The original graph is on the left; the graph of $y = f(|x|)$ is on the right.

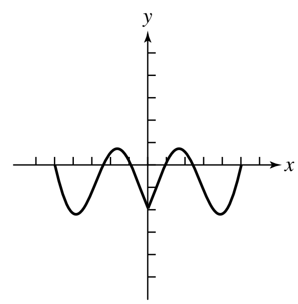


[−5, 5] by [−10, 10]   [−5, 5] by [−10, 10]

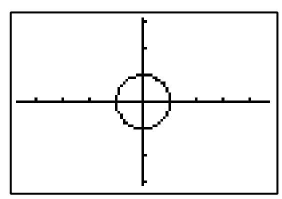
(c)



(d)

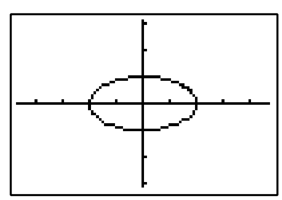


68. (a)



[−4.7, 4.7] by [−3.1, 3.1]

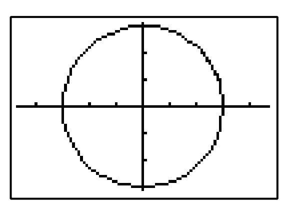
(b)



$x = 2 \cos t$
$y = \sin t$

[−4.7, 4.7] by [−3.1, 3.1]

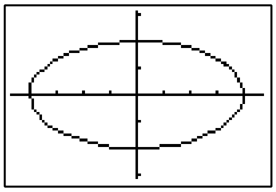
(c)



$x = 3 \cos t$
$y = 3 \sin t$

[−4.7, 4.7] by [−3.1, 3.1]

(d)



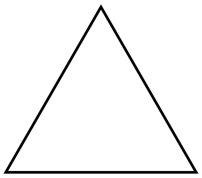
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$x = 4 \cos t$
$y = 2 \sin t$

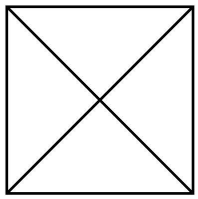
■ Section 1.7 Modeling with Functions

Exploration 1

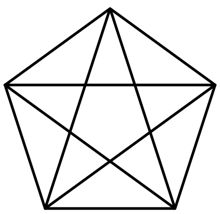
1.



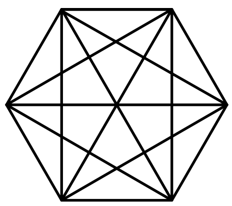
$n = 3;\ d = 0$



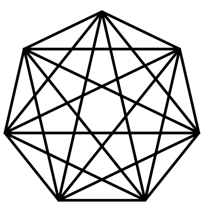
$n = 4;\ d = 2$



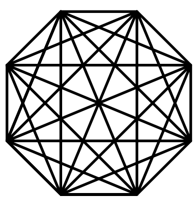
$n = 5;\ d = 5$



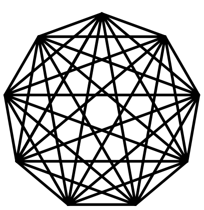
$n = 6;\ d = 9$



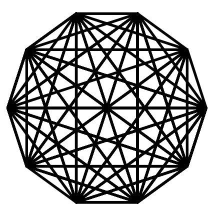
$n = 7;\ d = 14$



$n = 8;\ d = 20$

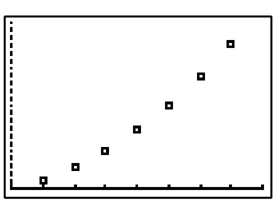


$n = 9;\ d = 27$



$n = 10;\ d = 35$

2.



$[3, 11]$ by $[0, 40]$

3. Linear: $r^2 = 0.9758$
Power: $r^2 = 0.9903$
Quadratic: $R^2 = 1$
Cubic: $R^2 = 1$
Quartic: $R^2 = 1$

4. The best-fit curve is quadratic: $y = 0.5x^2 - 1.5x$. The cubic and quartic regressions give this same curve.

5. Since the quadratic curve fits the points perfectly, there is nothing to be gained by adding a cubic term or a quartic term. The coefficients of these terms in the regressions are zero.

6. $y = 0.5x^2 - 1.5x$. At $x = 128$,
$y = 0.5(128)^2 - 1.5(128) = 8000$

Quick Review 1.7

1. $h = 2(A/b)$

2. $h = 2A/(b_1 + b_2)$

3. $h = V/(\pi r^2)$

4. $h = 3V/(\pi r^2)$

5. $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

6. $r = \sqrt{\dfrac{A}{4\pi}}$

7. $h = \dfrac{A - 2\pi r^2}{2\pi r} = \dfrac{A}{2\pi r} - r$

8. $t = I/(Pr)$

9. $P = \dfrac{A}{(1 + r/n)^{nt}} = A\left(1 + \dfrac{r}{n}\right)^{-nt}$

10. $t = \sqrt{\dfrac{2(H - s)}{g}}$

Section 1.7 Exercises

1. $3x + 5$

2. $3(x + 5)$

3. $0.17x$

4. $0.05x + 4$

5. $A = \ell w = (x + 12)(x)$

6. $A = \dfrac{1}{2}bh = \dfrac{1}{2}(x)(x + 2)$

7. $x + 0.045x = (1 + 0.045)x = 1.045x$

8. $x - 0.03x = (1 - 0.03)x = 0.97x$

9. $x - 0.40x = 0.60x$

10. $x + 0.0875x = 1.0875x$

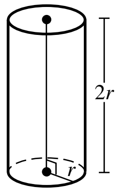
11. Let C be the total cost and n be the number of items produced; $C = 34{,}500 + 5.75n$.

12. Let C be the total cost and n be the number of items produced; $C = (1.09)28{,}000 + 19.85n$.

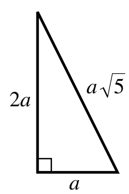
13. Let R be the revenue and n be the number of items sold; $R = 3.75n$.

14. Let P be the profit, and s be the amount of sales; then $P = 200{,}000 + 0.12s$.

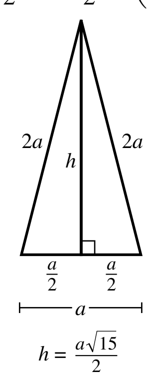
15. The basic formula for the volume of a right circular cylinder is $V = \pi r^2 h$, where r is the radius and h is height. Since height equals diameter $(h = d)$ and the diameter is two times r $(d = 2r)$, we know $h = 2r$. Then, $V = \pi r^2(2r) = 2\pi r^3$.



16. Let c = hypotenuse, a = "short" side, and b = "long" side. Then $c^2 = a^2 + b^2 = a^2 + (2a)^2 = a^2 + 4a^2 = 5a^2$, so $c = \sqrt{5a^2} = a\sqrt{5}$.



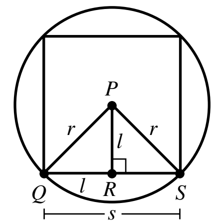
17. Let a be the length of the base. Then the other two sides of the triangle have length two times the base, or $2a$. Since the triangle is isoceles, a perpendicular dropped from the "top" vertex to the base is perpendicular. As a result, $h^2 + \left(\frac{a}{2}\right)^2 = (2a)^2$, or $h^2 = 4a^2 - \frac{a^2}{4} = \frac{16a^2 - a^2}{4}$ $= \frac{15a^2}{4}$, so $h = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2}$. The triangle's area is $A = \frac{1}{2}bh = \frac{1}{2}(a)\left(\frac{a\sqrt{15}}{2}\right) = \frac{a^2\sqrt{15}}{4}$.



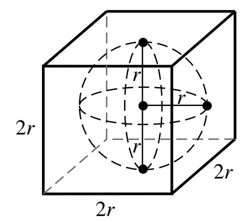
18. Since P lies at the center of the square and the circle, we know that segment $\overline{PR} = \overline{QR} = \overline{RS}$. Let ℓ be the length of these segments. Then, $\ell^2 + \ell^2 = r^2$, $2\ell^2 = r^2$, $\ell^2 = \frac{r^2}{2}$, $\ell = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$.

Since each side of the square is two times ℓ, we know that $s = 2\ell = \left(\frac{r\sqrt{2}}{2}\right)2 = r\sqrt{2}$.

As a result, $A = s^2 = (r\sqrt{2})^2 = r^2 \cdot 2 = 2r^2$.

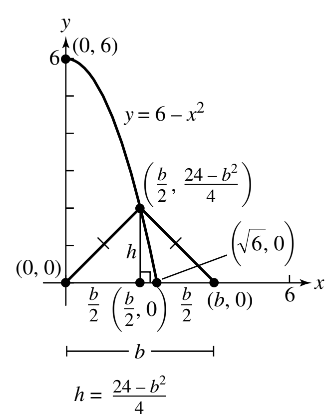


19. Let r be the radius of the sphere. Since the sphere is tangent to all six faces of the cube, we know that the height (and width, and depth) of the cube is equal to the sphere's diameter, which is two times r $(2r)$. The surface area of the cube is the sum of the area of all six faces, which equals $2r \cdot 2r = 4r^2$. Thus, $A = 6 \cdot 4r^2 = 24r^2$.



20. From our graph, we see that y provides the height of our triangle, i.e., $h = y$ when $x = \frac{b}{2}$. Since $y = 6 - x^2$ $= 6 - \left(\frac{b}{2}\right)^2 = 6 - \frac{b^2}{4} = \frac{24 - b^2}{4}$, $h = \frac{24 - b^2}{4}$.

The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}b\left(\frac{24 - b^2}{4}\right)$ $= \frac{24b - b^3}{8}$.



21. Solving $x + 4x = 620$ gives $x = 124$, so $4x = 496$. The two numbers are 124 and 496.

22. $x + 2x + 3x = 714$, so $x = 119$; the second and third numbers are 238 and 357.

23. $1.035x = 36{,}432$, so $x = 35{,}200$

24. $1.023x = 184.0$, so $x = 179.9$.

25. $182 = 52t$, so $t = 3.5$ hr.

26. $560 = 45t + 55(t + 2)$, so $t = 4.5$ hours on local highways.

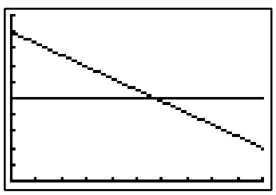
27. $0.60(33) = 19.8$; $0.75(27) = 20.25$. The \$33 shirt sells for \$19.80. The \$27 shirt sells for \$20.25. The \$33 shirt is a better bargain, because the sale price is cheaper.

28. Let x be gross sales. For the second job to be more attractive than the first, we need
$20{,}000 + 0.07x > 25{,}000 + 0.05x$, $0.02x > 5000$,
$x > \frac{5000}{0.02} = \$250{,}000$.
Gross sales would have to exceed \$250,000.

29. $71\,065\,000(1 + x) = 82\,400\,000$
$71\,065\,000x = 82\,400\,000 - 71\,065\,000$
$x = \frac{82\,400\,000 - 71\,065\,000}{71\,065\,000} \approx 0.1595$
There was a 15.95% increase in sales.

30. $26\,650\,000(1 + x) = 30\,989\,000$
$26\,650\,000x = 30\,989\,000 - 26\,650\,000$
$x = \frac{30\,989\,000 - 26\,650\,000}{26\,650\,000} \approx 0.1628$
Shipments of personal computers grew 16.28%.

31. **(a)** $0.10x + 0.45(100 - x) = 0.25(100)$.
(b) Graph $y_1 = 0.1x + 0.45(100 - x)$ and $y_2 = 25$.
Use $x \approx 57.14$ gallons of the 10% solution and about 42.86 gal of the 45% solution.



[0, 100] by [0, 50]

32. Solve $0.20x + 0.35(25 - x) = 0.26(25)$. Use $x = 15$ liters of the 20% solution and 10 liters of the 35% solution.

33. **(a)** The height of the box is x, and the base measures $10 - 2x$ by $18 - 2x$.
$V(x) = x(10 - 2x)(18 - 2x)$
(b) Because one side of the original piece of cardboard measures 10 in., $2x$ must be greater than 0 but less than 10, so that $0 < x < 5$. The domain of $V(x)$ is $(0, 5)$.
(c) Graphing $V(x)$ produces a cubic-function curve that between $x = 0$ and $x = 5$ has a maximum at approximately (2.06, 168.1). The cut-out squares should measure approximately 2.06 in. by 2.06 in.

34. Solve $2x + 2(x + 16) = 136$. Two pieces that are $x = 26$ ft long are needed, along with two 42 ft pieces.

35. Equation of the parabola, to pass through $(-16, 8)$ and $(16, 8)$:
$y = kx^2$
$8 = k(\pm 16)^2$
$k = \frac{8}{256} = \frac{1}{32}$
$y = \frac{1}{32}x^2$
y-coordinate of parabola 8 in. from center:
$y = \frac{1}{32}(8)^2 = 2$
From that point to the top of the dish is $8 - 2 = 6$ in.

36. Solve $2x + 2(x + 3) = 54$. This gives $x = 12$; the room is 12 ft × 15 ft.

37. Original volume of water:
$V_0 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(9)^2(24) \approx 2035.75 \text{ in.}^3$
Volume lost through faucet:
$V_1 = \text{time} \times \text{rate} = (120 \text{ sec})(5 \text{ in.}^3/\text{sec}) = 600 \text{ in.}^3$
Find volume:
$V_f = V_0 - V_1 = 2035.75 - 600 = 1435.75$
Since the final cone-shaped volume of water has radius and height in a 9-to-24 ratio, or $r = \frac{3}{8}h$:
$$V_f = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h = \frac{3}{64}\pi h^3 = 1435.75$$
Solving, we obtain $h \approx 21.36$ in.

38. Solve $900 = 0.07x + 0.085(12{,}000 - x)$.
$x = 8000$ dollars was invested at 7%; the other \$4000 was invested at 8.5%.

39. Bicycle's speed in feet per second:
$(2 \times \pi \times 16 \text{ in./rot})(2 \text{ rot/sec}) = 64\pi \text{ in./sec}$
Unit conversion:
$(64\pi \text{ in./sec})\left(\frac{1}{12}\text{ ft/in.}\right)\left(\frac{1}{5280}\text{ mi/ft}\right)(3600 \text{ sec/hr})$
≈ 11.42 mi/hr

40. Solve $1571 = 0.055x + 0.083(25{,}000 - x)$.
$x = 18{,}000$ dollars was invested at 5.5%; the other \$7000 was invested at 8.3%.

41. True. The correlation coefficient is close to 1 (or −1) if there is a good fit. A correlation coefficient near 0 indicates a very poor fit.

42. False. The graph over time of the height of a freely falling object is a parabola. A quadratic regression is called for.

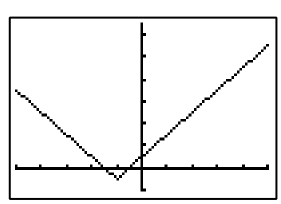
43. The pattern of points is S-shaped, which suggests a cubic model. The answer is C.

44. The points appear to lie along a straight line. The answer is A.

45. The points appear to lie along an upward-opening parabola. The answer is B.

46. The pattern of points looks sinusoidal. The answer is E.

47. **(a)** $C = 100{,}000 + 30x$
(b) $R = 50x$
(c) $100{,}000 + 30x = 50x$
$100{,}000 = 20x$
$x = 5000$ pairs of shoes
(d) Graph $y_1 = 100{,}000 + 30x$ and $y_2 = 50x$; these graphs cross when $x = 5000$ pairs of shoes. The point of intersection corresponds to the break-even point, where $C = R$.



[−10, 10] by [−2, 18]

48. Solve $48{,}814.20 = x + 0.12x + 0.03x + 0.004x$. Then $48{,}814.20 = 1.154x$, so $x = 42{,}300$ dollars.

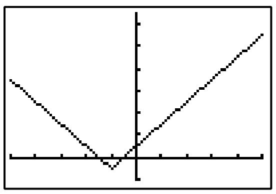
49. (a) $y_1 = u(x) = 125{,}000 + 23x$.

(b) $y_2 = s(x) = 125{,}000 + 23x + 8x = 125{,}000 + 31x$.

(c) $y_3 = r_u(x) = 56x$.

(d) $y_4 = R_s(x) = 79x$.

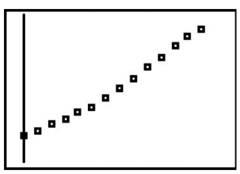
(e)



$[-10, 10]$ by $[-2, 18]$

(f) You should recommend stringing the rackets; fewer strung rackets need to be sold to begin making a profit (since the intersection of y_2 and y_4 occurs for smaller x than the intersection of y_1 and y_3).

50. (a)



[–1, 15] by [9, 16]

(b) $y = 0.409x + 9.861$

(c) $r = 0.993$, so the linear model is appropriate.

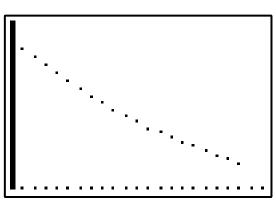
(d) $y = 0.012x^2 + 0.247x + 10.184$

(e) $r^2 = 0.998$, so a quadratic model is appropriate.

(f) The linear prediction is 18.04 and the quadratic prediction is 19.92. Despite the fact that both models look good for the data, the predictions differ by 1.88. One or both of them must be ineffective, as they both cannot be right.

(g) The linear regression and the quadratic regression are very close from $x = 0$ to $x = 13$. The quadratic regression begins to veer away from the linear regression at $x = 13$. Since there are no data points beyond $x = 13$, it is difficult to know which is accurate.

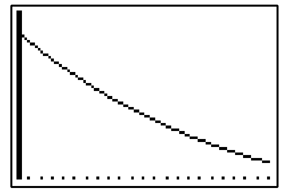
51. (a)



$[0, 22]$ by $[100, 200]$

(b) List L3 = {112.3, 106.5, 101.5, 96.6, 92.0, 87.2, 83.1, 79.8, 75.0, 71.7, 68, 64.1, 61.5, 58.5, 55.9, 53.0, 50.8, 47.9, 45.2, 43.2}

(c) The regression equation is $y = 118.07 \times 0.951^x$. It fits the data extremely well.



$[0, 22]$ by $[100, 200]$

52. Answers will vary in **(a)–(e)**, depending on the conditions of the experiment.

(f) Some possible answers: the thickness of the liquid, the darkness of the liquid, the type of cup it is in, the amount of surface exposed to the air, the specific heat of the substance (a technical term that may have been learned in physics), etc.

■ Chapter 1 Review

1. (d)

2. (f)

3. (i)

4. (h)

5. (b)

6. (j)

7. (g)

8. (c)

9. (a)

10. (e)

11. (a) All reals **(b)** All reals

12. (a) All reals **(b)** All reals

13. (a) All reals

(b) $g(x) = x^2 + 2x + 1 = (x + 1)^2$.
At $x = -1$, $g(x) = 0$, the function's minimum.
The range is $[0, \infty)$.

14. (a) All reals

(b) $(x - 2)^2 \geq 0$ for all x, so $(x - 2)^2 + 5 \geq 5$ for all x. The range is $[5, \infty)$.

15. (a) All reals

(b) $|x| \geq 0$ for all x, so $3|x| \geq 0$ and $3|x| + 8 \geq 8$ for all x. The range is $[8, \infty)$.

16. (a) We need $\sqrt{4 - x^2} \geq 0$ for all x, so $4 - x^2 \geq 0$, $4 \geq x^2$, $-2 \leq x \leq 2$. The domain is $[-2, 2]$.

(b) $0 \leq \sqrt{4 - x^2} \leq 2$ for all x, so $-2 \leq \sqrt{4 - x^2} - 2 \leq 0$ for all x. The range is $[-2, 0]$.

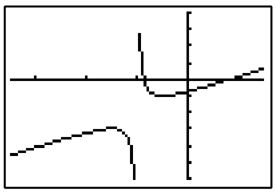
17. (a) $f(x) = \dfrac{x}{x^2 - 2x} = \dfrac{x}{x(x - 2)}$. $x \neq 0$ and $x - 2 \neq 0$, $x \neq 2$. The domain is all reals except 0 and 2.

(b) For $x > 2$, $f(x) > 0$ and for $x < 2$, $f(x) < 0$. $f(x)$ does not cross $y = 0$, so the range is all reals except $f(x) = 0$.

18. (a) We need
$\sqrt{9-x^2} > 0, 9 - x^2 > 0, 9 > x^2, -3 < x < 3.$
The domain is $(-3, 3)$.

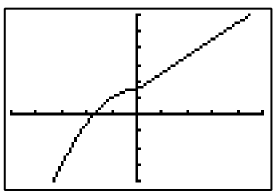
(b) Since $\sqrt{9-x^2} > 0, \dfrac{1}{\sqrt{9-x^2}} > 0$. On the domain $(-3, 3)$, $k(0) = \dfrac{1}{3}$, a minimum, while $k(x)$ approaches ∞ when x approaches both -3 and 3, maximums for $k(x)$. The range is $\left[\dfrac{1}{3}, \infty\right)$.

19. Continuous



$[-7, 3]$ by $[-12, 8]$

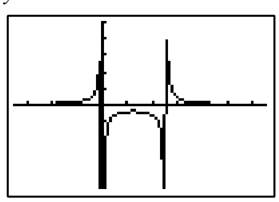
20. Continuous



$[-5, 5]$ by $[-8, 12]$

21. (a) $x^2 - 5x \neq 0$, $x(x-5) \neq 0$, so $x \neq 0$ and $x \neq 5$.
We expect vertical asymptotes at $x = 0$ and $x = 5$.

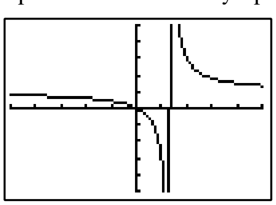
(b) $y = 0$



$[-7, 13]$ by $[-10, 10]$

22. (a) $x - 4 \neq 0$, $x \neq 4$, so we expect a vertical asymptote at $x = 4$.

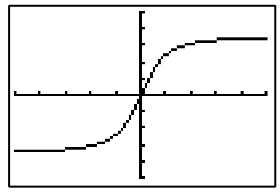
(b) Since $\lim\limits_{x\to\infty} \dfrac{3x}{x-4} = 3$ and $\lim\limits_{x\to-\infty} \dfrac{3x}{x-4} = 3$, we also expect a horizontal asymptote at $y = 3$.



$[-15, 15]$ by $[-15, 15]$

23. (a) None

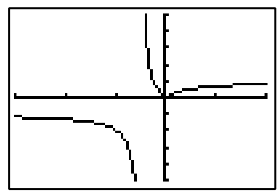
(b) Since $\lim\limits_{x\to\infty} \dfrac{7x}{\sqrt{x^2+10}} = 7$ and $\lim\limits_{x\to-\infty} \dfrac{7x}{\sqrt{x^2+10}} = -7$, we expect horizontal asymptotes at $y = 7$ and $y = -7$.



$[-15, 15]$ by $[-10, 10]$

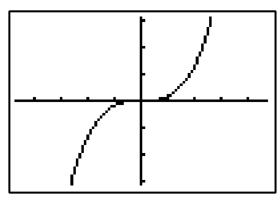
24. (a) $x + 1 \neq 0$, $x \neq -1$, so we expect a vertical asymptote at $x = -1$.

(b) $\lim\limits_{x\to\infty} \dfrac{|x|}{x+1} = 1$ and $\lim\limits_{x\to-\infty} \dfrac{|x|}{x+1} = -1$, so we can expect horizontal asymptotes at $y = 1$ and $y = -1$.



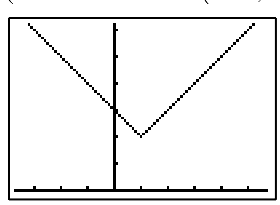
$[-6, 4]$ by $[-5, 5]$

25. $(-\infty, \infty)$



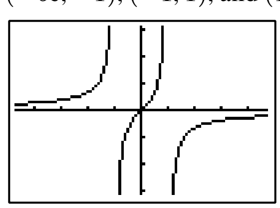
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

26. $|x - 1| = 0$ when $x = 1$, which is where the function's minimum occurs. y increases over the interval $[1, \infty)$. (Over the interval $(-\infty, 1]$, it is decreasing.)



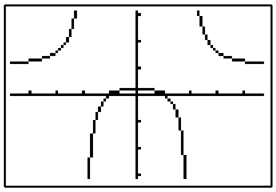
$[-3.7, 5.7]$ by $[0, 6.2]$

27. As the graph illustrates, y is increasing over the intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$.



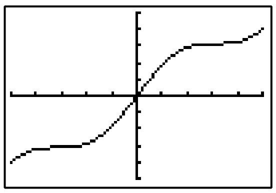
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

28. As the graph illustrates, y is increasing over the intervals $(-\infty, -2)$ and $(-2, 0]$.



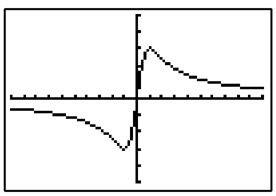
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

29. $-1 \leq \sin x \leq 1$, but $-\infty < x < \infty$, so $f(x)$ is not bounded.



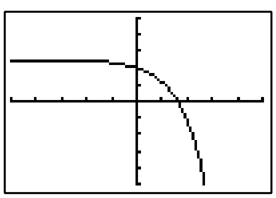
$[-5, 5]$ by $[-5, 5]$

30. $g(x) = 3$ at $x = 1$, a maximum and $g(x) = -3$, a minimum, at $x = -1$. It is bounded.



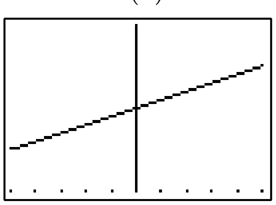
$[-10, 10]$ by $[-5, 5]$

31. $e^x > 0$ for all x, so $-e^x < 0$ and $5 - e^x < 5$ for all x. $h(x)$ is bounded above.



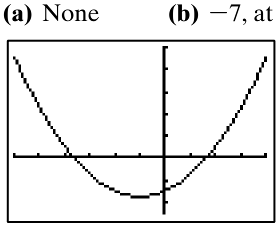
$[-5, 5]$ by $[-10, 10]$

32. The function is linear with slope $\frac{1}{1000}$ and y-intercept 1000. Thus $k(x)$ is not bounded.



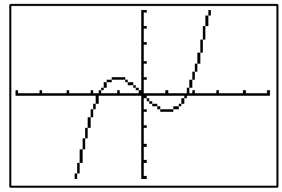
$[-5, 5]$ by $[-999.99, 1000.01]$

33. **(a)** None **(b)** -7, at $x = -1$



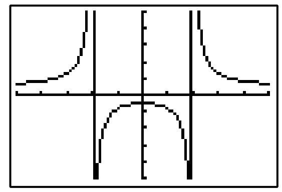
$[-6, 4]$ by $[-10, 20]$

34. **(a)** 2, at $x = -1$ **(b)** -2, at $x = 1$



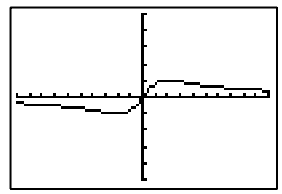
$[-5, 5]$ by $[-10, 10]$

35. **(a)** -1, at $x = 0$ **(b)** None



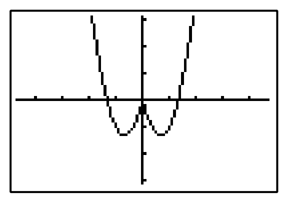
$[-5, 5]$ by $[-10, 10]$

36. **(a)** 1, at $x = 2$ **(b)** -1, at $x = -2$



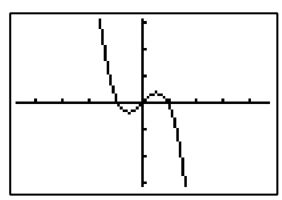
$[-10, 10]$ by $[-5, 5]$

37. The function is even since it is symmetrical about the y-axis.



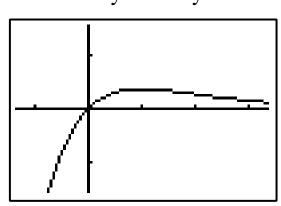
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

38. Since the function is symmetrical about the origin, it is odd.



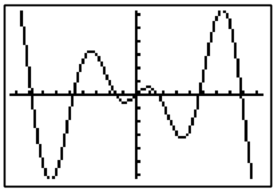
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

39. Since no symmetry is exhibited, the function is neither.



$[-1.35, 3.35]$ by $[-1.55, 1.55]$

40. Since the function is symmetrical about the origin, it is odd.



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

41. $x = 2y + 3, 2y = x - 3, y = \dfrac{x-3}{2}$, so

$f^{-1}(x) = \dfrac{x-3}{2}$.

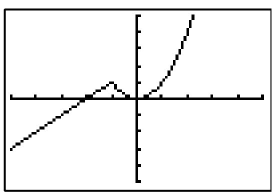
42. $x = \sqrt[3]{y-8}, x^3 = y - 8, y = x^3 + 8$, so $f^{-1}(x) = x^3 + 8$.

43. $x = \dfrac{2}{y}, xy = 2, y = \dfrac{2}{x}$, so $f^{-1}(x) = \dfrac{2}{x}$.

44. $x = \dfrac{6}{y+4}, (y+4)x = 6, xy + 4x = 6, xy = 6 - 4x,$

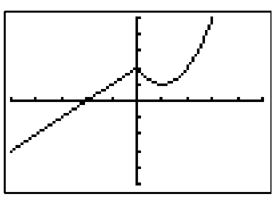
$y = \dfrac{6-4x}{x}$, so $f^{-1}(x) = \dfrac{6}{x} - 4$.

45.



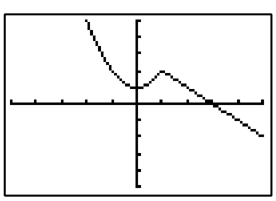
$[-5, 5]$ by $[-5, 5]$

46.



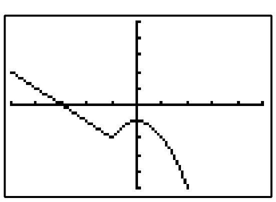
$[-5, 5]$ by $[-5, 5]$

47.



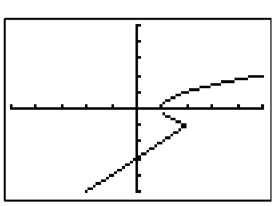
$[-5, 5]$ by $[-5, 5]$

48.



$[-5, 5]$ by $[-5, 5]$

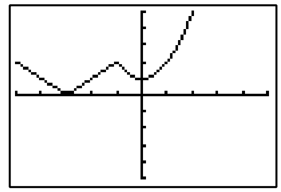
49.



$[-5, 5]$ by $[-5, 5]$

50. No

51.



$[-5, 5]$ by $[-5, 5]$

52. $f(x) = \begin{cases} x + 3 \text{ if } x \le -1 \\ x^2 + 1 \text{ if } x > -1 \end{cases}$

53. $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = \sqrt{x^2 - 4}$.
Since $x^2 - 4 \ge 0, x^2 \ge 4, x \le -2$ or $x \ge 2$.
The domain is $(-\infty, -2] \cup [2, \infty)$.

54. $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 4 = x - 4$. Since $\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, \infty)$.

55. $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot (x^2 - 4)$.
Since $\sqrt{x} \ge 0$, the domain is $[0, \infty)$.

56. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{x^2 - 4}$ Since $x^2 - 4 \ne 0$, $(x+2)(x-2) \ne 0, x \ne -2, x \ne 2$. Also since $\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, 2) \cup (2, \infty)$.

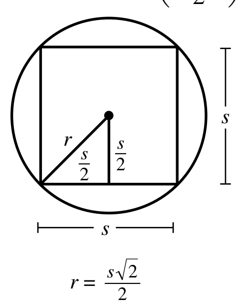
57. $\lim_{x\to\infty} \sqrt{x} = \infty$. (Large negative values are not in the domain.)

58. $\lim_{x\to\pm\infty} \sqrt{x^2 - 4} = \infty$. (The graph resembles the line $y = x$.)

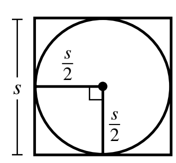
59. $r^2 = \left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right)^2 = \dfrac{2s^2}{4}, \ r = \sqrt{\dfrac{2s^2}{4}} = \dfrac{s\sqrt{2}}{2}$.

The area of the circle is

$A = \pi r^2 = \pi\left(\dfrac{s\sqrt{2}}{2}\right)^2 = \dfrac{2\pi s^2}{4} = \dfrac{\pi s^2}{2}$

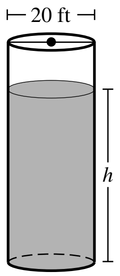


60. $A = \pi r^2 = \pi\left(\dfrac{s}{2}\right)^2 = \dfrac{\pi s^2}{4}$

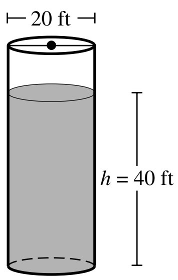


61. $d = 2r,\ r = \dfrac{d}{2}$, so the radius of the tank is 10 feet.

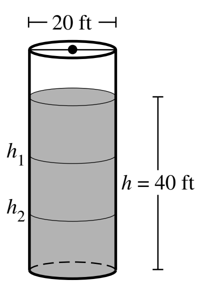
Volume is $V = \pi r^2 \cdot h = \pi(10)^2 \cdot h = 100\pi h$



62. The volume of oil in the tank is the amount of original oil $(\pi r^2 h)$ minus the amount of oil drained.
$V = \pi r^2 h - 2t = \pi(10)^2(40) - 2t = 4000\pi - 2t$

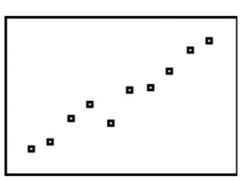


63. Since $V = 4000\pi - 2t$, we know that $\pi r^2 h = 4000\pi - 2t$. In this case, $r = 10'$, so
$100\pi h = 4000\pi - 2t,\ h = \dfrac{4000\pi - 2t}{100\pi} = 40 - \dfrac{t}{50\pi}$



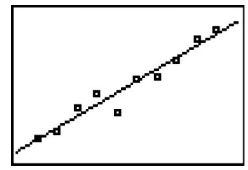
64. Since the depth of the tank is decreasing by 2 feet per hour, we know that the tank is losing a total volume of $V = \pi r^2 h = \pi(10)^2(2) = 200\pi$ cubic feet per hour. The volume of remaining oil in the tank is the amount of original oil subtracting the amount which has been drained, or $V = 4000\pi - 200\pi t$. This is a significantly higher loss than our solution in #68!

65. **(a)**



[4, 15] by [940, 1700]

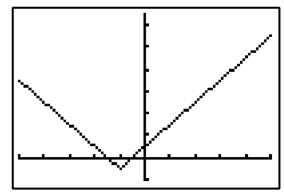
(b) The regression line is $y = 61.133x + 725.333$.



[4, 15] by [940, 1700]

(c) $61.133(20) + 725.333 \approx 1948$ (thousands of barrels)

66. **(a)**

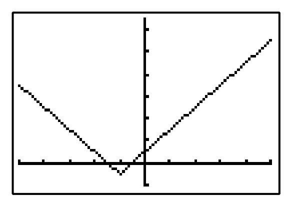


$[-10, 10]$ by $[-2, 18]$

(b) The linear model would eventually intersect the x-axis, which would represent a swimmer covering 100 meters in a time of 0.00. This is clearly impossible.

(c) Based on the data, 52 seconds represents the limit of women's capability in this race. The addition of future data could determine a different model.

(d) The regression curve is $y = (97.100)(0.9614^x)$.



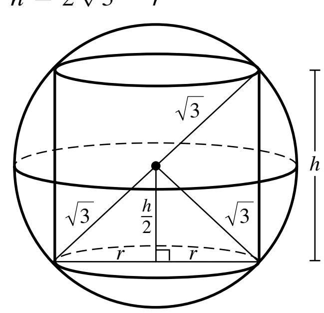
$[-10, 10]$ by $[-2, 18]$

(e) $(97.100)(0.9614^{108}) \approx 1.38$. Add 52 to find the projected winning time in 2008: $1.38 + 52 = 53.38$ seconds.

67. **(a)** $r^2 + \left(\dfrac{h}{2}\right)^2 = (\sqrt{3})^2$,

$\dfrac{h^2}{4} = 3 - r^2,\ h^2 = 12 - 4r^2,\ h = \sqrt{12 - 4r^2}$,

$h = 2\sqrt{3 - r^2}$

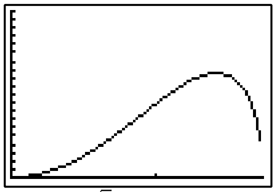


$h = 2\sqrt{3 - r^2}$

(b) $V = \pi r^2 h = (\pi r^2)(2\sqrt{3 - r^2}) = 2\pi r^2\sqrt{3 - r^2}$

(c) Since $\sqrt{3 - r^2} \ge 0,\ 3 - r^2 \ge 0$
$3 \ge r^2,\ -\sqrt{3} \le r \le \sqrt{3}$.
However, $r < 0$ are invalid values, so the domain is $[0, \sqrt{3}]$.

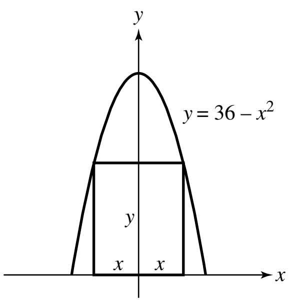
(d)



$[0, \sqrt{3}]$ by $[0, 20]$

(e) 12.57 in^3

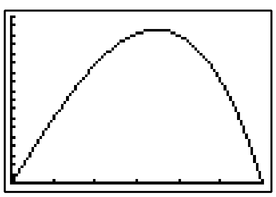
68.



(a) $A = 2xy = 2x(36 - x^2) = 72x - 2x^3$

(b) $36 - x^2 \geq 0, (6 - x)(6 + x) \geq 0, -6 \leq x \leq 6.$ However, $x < 0$ are invalid values, so the domain is $[0, 6]$.

(c)

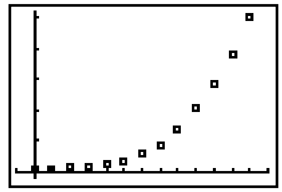


$[0, 6]$ by $[0, 180]$

(d) The maximum area occurs when $x \approx 3.46$, or an area of approximately 166.28 square units.

Chapter 1 Project

1.



$[-1, 13]$ by $[-100, 2600]$

2. The exponential regression produces $y \approx 21.956(1.511)^x$.

3. 2000: For $x = 13$, $y \approx 4690$
2001: For $x = 14$, $y \approx 7085$

4. The model, which is based on data from the early, high-growth period of Starbucks Coffee's company history, does not account for the effects of gradual market saturation by Starbucks and its competitors. The actual growth in the number of locations is slowing while the model increases more rapidly.

5. The logistic regression produces

$$y \approx \frac{4914.198}{1 + 269.459\, e^{-0.486x}}.$$

6. 2000: For $x = 13$, $y \approx 3048$
2001: For $x = 14$, $y \approx 3553$

These predictions are less than the actual numbers, but are not off by as much as the numbers derived from the exponential model were. For the year 2020 $(x = 33)$, the logistic model predicts about 4914 locations. (This prediction is probably too conservative.)