Chapter 1
Prerequisites for Calculus

■ Section 1.1 Lines (pp. 1–9)

Quick Review 1.1

1. $y = -2 + 4(3 - 3) = -2 + 4(0) = -2 + 0 = -2$

2. $$\begin{aligned} 3 &= 3 - 2(x + 1) \\ 3 &= 3 - 2x - 2 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

3. $m = \dfrac{2 - 3}{5 - 4} = \dfrac{-1}{1} = -1$

4. $m = \dfrac{2 - (-3)}{3 - (-1)} = \dfrac{5}{4}$

5. **(a)** $3(2) - 4\left(\dfrac{1}{4}\right) \stackrel{?}{=} 5$

   $6 - 1 = 5$ Yes

   (b) $3(3) - 4(-1) \stackrel{?}{=} 5$

   $13 \neq 5$ No

6. **(a)** $7 \stackrel{?}{=} -2(-1) + 5$

   $7 = 2 + 5$ Yes

   (b) $1 = -2(-2) + 5$

   $1 \neq 9$ No

7. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 1)^2 + (1 - 0)^2} \\ &= \sqrt{2} \end{aligned}$$

8. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - 2)^2 + \left(-\frac{1}{3} - 1\right)^2} \\ &= \sqrt{(-1)^2 + \left(-\frac{4}{3}\right)^2} \\ &= \sqrt{1 + \frac{16}{9}} \\ &= \sqrt{\frac{25}{9}} \\ &= \frac{5}{3} \end{aligned}$$

9. $$\begin{aligned} 4x - 3y &= 7 \\ -3y &= -4x + 7 \\ y &= \frac{4}{3}x - \frac{7}{3} \end{aligned}$$

10. $$\begin{aligned} -2x + 5y &= -3 \\ 5y &= 2x - 3 \\ y &= \frac{2}{5}x - \frac{3}{5} \end{aligned}$$

Section 1.1 Exercises

1. $\Delta x = -1 - 1 = -2$

   $\Delta y = -1 - 2 = -3$

2. $\Delta x = -1 - (-3) = 2$

   $\Delta y = -2 - 2 = -4$

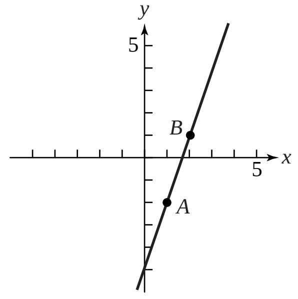
3. $\Delta x = -8 - (-3) = -5$

   $\Delta y = 1 - 1 = 0$

4. $\Delta x = 0 - 0 = 0$

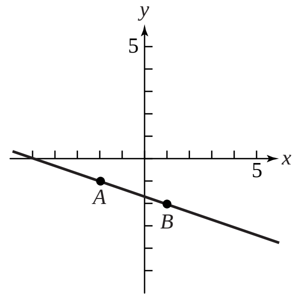
   $\Delta y = -2 - 4 = -6$

5. **(a, c)**



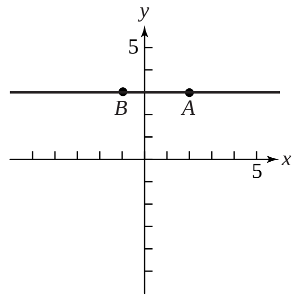
   (b) $m = \dfrac{1 - (-2)}{2 - 1} = \dfrac{3}{1} = 3$

6. **(a, c)**



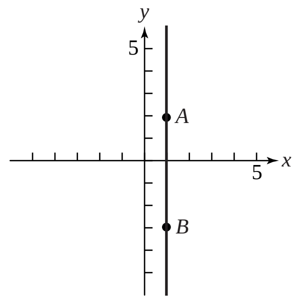
   (b) $m = \dfrac{-2 - (-1)}{1 - (-2)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

7. **(a, c)**



   (b) $m = \dfrac{3 - 3}{-1 - 2} = \dfrac{0}{-3} = 0$

8. **(a, c)**



8. continued

(b) $m = \frac{-3-2}{1-1} = \frac{-5}{0}$ (undefined)
This line has no slope.

9. (a) $x = 2$
(b) $y = 3$

10. (a) $x = -1$
(b) $y = \frac{4}{3}$

11. (a) $x = 0$
(b) $y = -\sqrt{2}$

12. (a) $x = -\pi$
(b) $y = 0$

13. $y = 1(x-1) + 1$

14. $y = -1[x - (-1)] + 1$
$y = -1(x+1) + 1$

15. $y = 2(x-0) + 3$

16. $y = -2[x - (-4)] + 0$
$y = -2(x+4) + 0$

17. $m = \frac{3-0}{2-0} = \frac{3}{2}$

$y = \frac{3}{2}(x-0) + 0$

$y = \frac{3}{2}x$

$2y = 3x$

$3x - 2y = 0$

18. $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

$y = 0(x-1) + 1$

$y = 1$

19. $m = \frac{-2-0}{-2-(-2)} = \frac{-2}{0}$ (undefined)

Vertical line: $x = -2$

20. $m = \frac{-2-1}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$

$y = -\frac{3}{4}[x - (-2)] + 1$

$4y = -3(x+2) + 4$

$4y = -3x - 2$

$3x + 4y = -2$

21. $y = 3x - 2$

22. $y = -1x + 2$ or $y = -x + 2$

23. $y = -\frac{1}{2}x - 3$

24. $y = \frac{1}{3}x - 1$

25. The line contains (0, 0) and (10, 25).

$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$

$y = \frac{5}{2}x$

26. The line contains (0, 0) and (5, 2).

$m = \frac{2-0}{5-0} = \frac{2}{5}$

$y = \frac{2}{5}x$

27. $3x + 4y = 12$

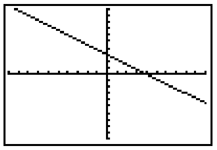
$4y = -3x + 12$

$y = -\frac{3}{4}x + 3$

(a) Slope: $-\frac{3}{4}$

(b) y-intercept: 3

(c)



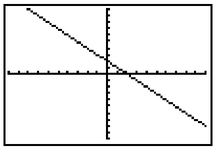
$[-10, 10]$ by $[-10, 10]$

28. $x + y = 2$
$y = -x + 2$

(a) Slope: -1

(b) y-intercept: 2

(c)



$[-10, 10]$ by $[-10, 10]$

29. $\frac{x}{3} + \frac{y}{4} = 1$

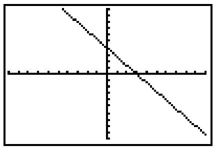
$\frac{y}{4} = -\frac{x}{3} + 1$

$y = -\frac{4}{3}x + 4$

(a) Slope: $-\frac{4}{3}$

(b) y-intercept: 4

(c)



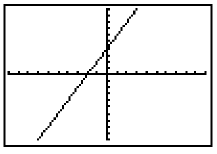
$[-10, 10]$ by $[-10, 10]$

30. $y = 2x + 4$

(a) Slope: 2

(b) y-intercept: 4

(c)



$[-10, 10]$ by $[-10, 10]$

31. (a) The desired line has slope -1 and passes through $(0, 0)$: $y = -1(x - 0) + 0$ or $y = -x$.

(b) The desired line has slope $\frac{-1}{-1} = 1$ and passes through $(0, 0)$: $y = 1(x - 0) + 0$ or $y = x$.

32. (a) The given equation is equivalent to $y = -2x + 4$. The desired line has slope -2 and passes through $(-2, 2)$: $y = -2(x + 2) + 2$ or $y = -2x - 2$.

(b) The desired line has slope $\frac{-1}{-2} = \frac{1}{2}$ and passes through $(-2, 2)$: $y = \frac{1}{2}(x + 2) + 2$ or $y = \frac{1}{2}x + 3$.

33. (a) The given line is vertical, so we seek a vertical line through $(-2, 4)$: $x = -2$.

(b) We seek a horizontal line through $(-2, 4)$: $y = 4$.

34. (a) The given line is horizontal, so we seek a horizontal line through $\left(-1, \frac{1}{2}\right)$: $y = \frac{1}{2}$.

(b) We seek a vertical line through $\left(-1, \frac{1}{2}\right)$: $x = -1$.

35. $m = \frac{9 - 2}{3 - 1} = \frac{7}{2}$

$f(x) = \frac{7}{2}(x - 1) + 2 = \frac{7}{2}x - \frac{3}{2}$

Check: $f(5) = \frac{7}{2}(5) - \frac{3}{2} = 16$, as expected.

Since $f(x) = \frac{7}{2}x - \frac{3}{2}$, we have $m = \frac{7}{2}$ and $b = -\frac{3}{2}$.

36. $m = \frac{-4 - (-1)}{4 - 2} = \frac{-3}{2} = -\frac{3}{2}$

$f(x) = -\frac{3}{2}(x - 2) + (-1) = -\frac{3}{2}x + 2$

Check: $f(6) = -\frac{3}{2}(6) + 2 = -7$, as expected.

Since $f(x) = -\frac{3}{2}x + 2$, we have $m = -\frac{3}{2}$ and $b = 2$.

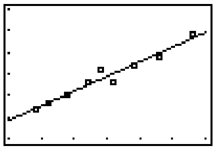
37.
$$\begin{aligned} -\frac{2}{3} &= \frac{y - 3}{4 - (-2)} \\ -\frac{2}{3}(6) &= y - 3 \\ -4 &= y - 3 \\ -1 &= y \end{aligned}$$

38.
$$\begin{aligned} 2 &= \frac{2 - (-2)}{x - (-8)} \\ 2(x + 8) &= 4 \\ x + 8 &= 2 \\ x &= -6 \end{aligned}$$

39. (a) $y = 0.680x + 9.013$

(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.

(c)



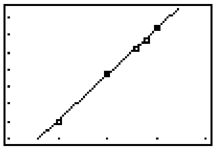
[15, 45] by [15, 45]

(d) When $x = 30$, $y \approx 0.680(30) + 9.013 = 29.413$. She weighs about 29 pounds.

40. (a) $y = 1{,}060.4233x - 2{,}077{,}548.669$

(b) The slope is 1,060.4233. It represents the approximate rate of increase in earnings in dollars per year.

(c)



[1975, 1995] by [20,000, 35,000]

(d) When $x = 2000$, $y \approx 1{,}060.4233(2000) - 2{,}077{,}548.669 \approx 43{,}298$. In 2000, the construction workers' average annual compensation will be about \$43,298.

41. $y = 1 \cdot (x - 3) + 4$
$y = x - 3 + 4$
$y = x + 1$
This is the same as the equation obtained in Example 5.

42. (a) When $y = 0$, we have $\frac{x}{c} = 1$, so $x = c$.

When $x = 0$, we have $\frac{y}{d} = 1$, so $y = d$.

(b) When $y = 0$, we have $\frac{x}{c} = 2$, so $x = 2c$.

When $x = 0$, we have $\frac{y}{d} = 2$, so $y = 2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

43. (a) The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1. The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

44. (a) $m \approx \frac{68 - 69.5}{0.4 - 0} = \frac{-1.5}{0.4} = -3.75$ degrees/inch

(b) $m \approx \frac{10 - 68}{4 - 0.4} = \frac{-58}{3.6} \approx -16.1$ degrees/inch

(c) $m \approx \frac{5 - 10}{4.7 - 4} = \frac{-5}{0.7} = -7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation
Poorest insulator: Gypsum wallboard
The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

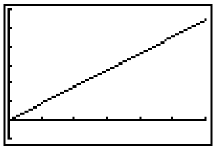
45. Slope: $k = \dfrac{\Delta p}{\Delta d} = \dfrac{10.94 - 1}{100 - 0} = \dfrac{9.94}{100}$
$= 0.0994$ atmospheres per meter

At 50 meters, the pressure is

$p = 0.0994(50) + 1 = 5.97$ atmospheres.

46. **(a)** $d(t) = 45t$

(b)



[0, 6] by [−50, 300]

(c) The slope is 45, which is the speed in miles per hour.

(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

47. **(a)** $y = 5632x - 11{,}080{,}280$

(b) The rate at which the median price is increasing in dollars per year

(c) $y = 2732x - 5{,}362{,}360$

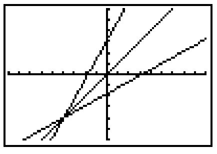
(d) The median price is increasing at a rate of about $5632 per year in the Northeast, and about $2732 per year in the Midwest. It is increasing more rapidly in the Northeast.

48. **(a)** Suppose x°F is the same as x°C.

$$x = \frac{9}{5}x + 32$$
$$\left(1 - \frac{9}{5}\right)x = 32$$
$$-\frac{4}{5}x = 32$$
$$x = -40$$

Yes, -40°F is the same as -40°C.

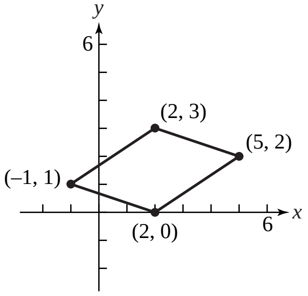
(b)

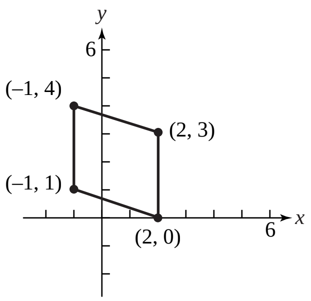


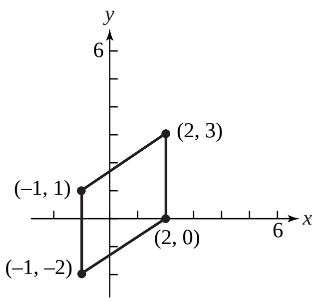
[−90, 90] by [−60, 60]

It is related because all three lines pass through the point $(-40, -40)$ where the Fahrenheit and Celsius temperatures are the same.

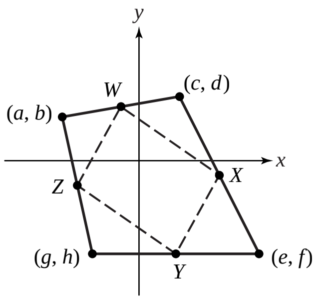
49. The coordinates of the three missing vertices are (5, 2), (−1, 4) and (−1, −2), as shown below.







50.



Suppose that the vertices of the given quadrilateral are (a, b), (c, d), (e, f), and (g, h). Then the midpoints of the consecutive sides are $W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

$$WX: \frac{\frac{d+f}{2} - \frac{b+d}{2}}{\frac{c+e}{2} - \frac{a+c}{2}} = \frac{f-b}{e-a}$$
$$XY: \frac{\frac{f+h}{2} - \frac{d+f}{2}}{\frac{e+g}{2} - \frac{c+e}{2}} = \frac{h-d}{g-c}$$
$$ZY: \frac{\frac{f+h}{2} - \frac{h+b}{2}}{\frac{e+g}{2} - \frac{g+a}{2}} = \frac{f-b}{e-a}$$
$$WZ: \frac{\frac{h+b}{2} - \frac{b+d}{2}}{\frac{g+a}{2} - \frac{a+c}{2}} = \frac{h-d}{g-c}$$

Opposite sides have the same slope and are parallel.

51. The radius through (3, 4) has slope $\frac{4-0}{3-0} = \frac{4}{3}$.

The tangent line is tangent to this radius, so its slope is $\frac{-1}{4/3} = -\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$y = -\frac{3}{4}(x-3) + 4$

$y = -\frac{3}{4}x + \frac{9}{4} + 4$

$y = -\frac{3}{4}x + \frac{25}{4}$

52. **(a)** The equation for line L can be written as

$y = -\frac{A}{B}x + \frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-A/B} = \frac{B}{A}$ and passes through (a, b), so its equation is $y = \frac{B}{A}(x-a) + b$.

(b) Substituting $\frac{B}{A}(x-a) + b$ for y in the equation for line L gives:

$$\begin{aligned}
Ax + B\left[\frac{B}{A}(x-a) + b\right] &= C \\
A^2x + B^2(x-a) + ABb &= AC \\
(A^2 + B^2)x &= B^2a + AC - ABb \\
x &= \frac{B^2a + AC - ABb}{A^2 + B^2}
\end{aligned}$$

Substituting the expression for x in the equation for line L gives:

$$\begin{aligned}
&A\left(\frac{B^2a + AC - ABb}{A^2 + B^2}\right) + By = C \\
&By = \frac{-A(B^2a + AC - ABb)}{A^2 + B^2} + \frac{C(A^2 + B^2)}{A^2 + B^2} \\
&By = \frac{-AB^2a - A^2C + A^2Bb + A^2C + B^2C}{A^2 + B^2} \\
&By = \frac{A^2Bb + B^2C - AB^2a}{A^2 + B^2} \\
&y = \frac{A^2b + BC - ABa}{A^2 + B^2}
\end{aligned}$$

The coordinates of Q are $\left(\frac{B^2a + AC - ABb}{A^2 + B^2}, \frac{A^2b + BC - ABa}{A^2 + B^2}\right)$.

(c) Distance $= \sqrt{(x-a)^2 + (y-b)^2}$

$$\begin{aligned}
&= \sqrt{\left(\frac{B^2a + AC - ABb}{A^2 + B^2} - a\right)^2 + \left(\frac{A^2b + BC - ABa}{A^2 + B^2} - b\right)^2} \\
&= \sqrt{\left(\frac{B^2a + AC - ABb - a(A^2 + B^2)}{A^2 + B^2}\right)^2 + \left(\frac{A^2b + BC - ABa - b(A^2 + B^2)}{A^2 + B^2}\right)^2} \\
&= \sqrt{\left(\frac{AC - ABb - A^2a}{A^2 + B^2}\right)^2 + \left(\frac{BC - ABa - B^2b}{A^2 + B^2}\right)^2} \\
&= \sqrt{\left(\frac{A(C - Bb - Aa)}{A^2 + B^2}\right)^2 + \left(\frac{B(C - Aa - Bb)}{A^2 + B^2}\right)^2} \\
&= \sqrt{\frac{A^2(C - Aa - Bb)^2}{(A^2 + B^2)^2} + \frac{B^2(C - Aa - Bb)^2}{(A^2 + B^2)^2}} \\
&= \sqrt{\frac{(A^2 + B^2)(C - Aa - Bb)^2}{(A^2 + B^2)^2}} \\
&= \sqrt{\frac{(C - Aa - Bb)^2}{A^2 + B^2}} \\
&= \frac{|C - Aa - Bb|}{\sqrt{A^2 + B^2}} \\
&= \frac{|Aa + Bb - C|}{\sqrt{A^2 + B^2}}
\end{aligned}$$

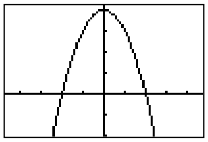
■ Section 1.2 Functions and Graphs
(pp. 9–19)

Exploration 1 Composing Functions

1. $y_3 = g \circ f, y_4 = f \circ g$

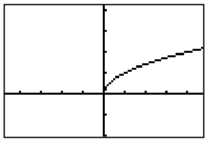
2. Domain of y_3: $[-2, 2]$ Range of y_3: $[0, 2]$

y_1:



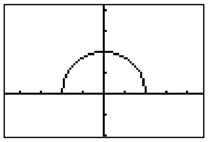
$[-4.7, 4.7]$ by $[-2, 4.2]$

y_2:



$[-4.7, 4.7]$ by $[-2, 4.2]$

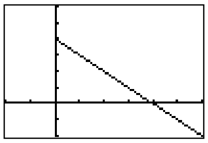
y_3:



$[-4.7, 4.7]$ by $[-2, 4.2]$

3. Domain of y_4: $[0, \infty)$; Range of y_4: $(-\infty, 4]$

y_4:



$[-2, 6]$ by $[-2, 6]$

4. $y_3 = y_2\,(y_1(x)) = \sqrt{y_1(x)} = \sqrt{4 - x^2}$
$y_4 = y_1(y_2(x)) = 4 - (y_2(x))^2 = 4 - (\sqrt{x})^2 = 4 - x, x \ge 0$

Quick Review 1.2

1. $3x - 1 \le 5x + 3$
$-2x \le 4$
$x \ge -2$
Solution: $[-2, \infty)$

2. $x(x - 2) > 0$
Solutions to $x(x - 2) = 0$: $x = 0, x = 2$
Test $x = -1$: $-1(-1 - 2) = 3 > 0$
$x(x - 2) > 0$ is true when $x < 0$.
Test $x = 1$: $1(1 - 2) = -1 < 0$
$x(x - 2) > 0$ is false when $0 < x < 2$.
Test $x = 3$: $3(3 - 2) = 3 > 0$
$x(x - 2) > 0$ is true when $x > 2$.
Solution set: $(-\infty, 0) \cup (2, \infty)$

3. $|x - 3| \le 4$
$-4 \le x - 3 \le 4$
$-1 \le x \le 7$
Solution set: $[-1, 7]$

4. $|x - 2| \ge 5$
$x - 2 \le -5$ or $x - 2 \ge 5$
$x \le -3$ or $x \ge 7$
Solution set: $(-\infty, -3] \cup [7, \infty)$

5. $x^2 < 16$
Solutions to $x^2 = 16$: $x = -4, x = 4$
Test $x = -6$ $(-6)^2 = 36 > 16$
$x^2 < 16$ is false when $x < -4$
Test $x = 0$: $0^2 = 0 < 16$
$x^2 < 16$ is true when $-4 < x < 4$
Test $x = 6$: $6^2 = 36 > 16$
$x^2 < 16$ is false when $x > 4$.
Solution set: $(-4, 4)$

6. $9 - x^2 \ge 0$
Solutions to $9 - x^2 = 0$: $x = -3, x = 3$
Test $x = -4$: $9 - (-4)^2 = 9 - 16 = -7 < 0$
$9 - x^2 \ge 0$ is false when $x < -3$.
Test $x = 0$: $9 - 0^2 = 9 > 0$
$9 - x^2 \ge 0$ is true when $-3 < x < 3$.
Test $x = 4$: $9 - 4^2 = 9 - 16 = -7 < 0$
$9 - x^2 \ge 0$ is false when $x > 3$.
Solution set: $[-3, 3]$

7. Translate the graph of f 2 units left and 3 units downward.

8. Translate the graph of f 5 units right and 2 units upward.

9. (a)
$$\begin{aligned} f(x) &= 4 \\ x^2 - 5 &= 4 \\ x^2 - 9 &= 0 \\ (x + 3)(x - 3) &= 0 \\ x &= -3 \text{ or } x = 3 \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= -6 \\ x^2 - 5 &= -6 \\ x^2 &= -1 \end{aligned}$$
No real solution

10. (a)
$$\begin{aligned} f(x) &= -5 \\ \frac{1}{x} &= -5 \\ x &= -\frac{1}{5} \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 0 \\ \frac{1}{x} &= 0 \end{aligned}$$
No solution

11. (a)
$$\begin{aligned} f(x) &= 4 \\ \sqrt{x + 7} &= 4 \\ x + 7 &= 16 \\ x &= 9 \end{aligned}$$
Check: $\sqrt{9 + 7} = \sqrt{16} = 4$; it checks.

(b)
$$\begin{aligned} f(x) &= 1 \\ \sqrt{x + 7} &= 1 \\ x + 7 &= 1 \\ x &= -6 \end{aligned}$$
Check: $\sqrt{-6 + 7} = 1$; it checks.

12. (a)
$$\begin{aligned} f(x) &= -2 \\ \sqrt[3]{x - 1} &= -2 \\ x - 1 &= -8 \\ x &= -7 \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= 3 \\ \sqrt[3]{x - 1} &= 3 \\ x - 1 &= 27 \\ x &= 28 \end{aligned}$$

Section 1.2 Exercises

1. Since $A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$, the formula is $A = \frac{\pi d^2}{4}$, where A represents area and d represents diameter.

2. Let h represent height and let s represent side length.

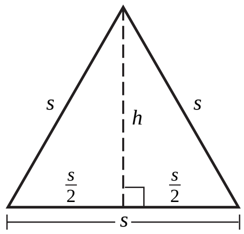
$h^2 + \left(\frac{s}{2}\right)^2 = s^2$

$h^2 = s^2 - \frac{1}{4}s^2$

$h^2 = \frac{3}{4}s^2$

$h = \pm\frac{\sqrt{3}}{2}s$

Since side length and height must be positive, the formula is $h = \frac{\sqrt{3}}{2}s$.



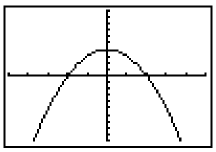
3. $S = 6e^2$, where S represents surface area and e represents edge length.

4. $V = \frac{4}{3}\pi r^3$, where V represents volume and r represents radius.

5. **(a)** $(-\infty, \infty)$ or all real numbers

(b) $(-\infty, 4]$

(c)



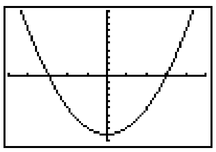
$[-5, 5]$ by $[-10, 10]$

(d) Symmetric about y-axis (even)

6. **(a)** $(-\infty, \infty)$ or all real numbers

(b) $[-9, \infty)$

(c)



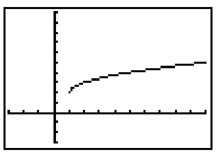
$[-5, 5]$ by $[-10, 10]$

(d) Symmetric about the y-axis (even)

7. **(a)** Since we require $x - 1 \geq 0$, the domain is $[1, \infty)$.

(b) $[2, \infty)$

(c)



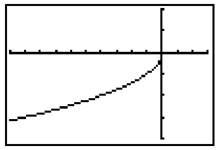
$[-3, 10]$ by $[-3, 10]$

(d) None

8. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$.

(b) $(-\infty, 0]$

(c)



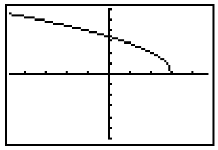
$[-10, 3]$ by $[-4, 2]$

(d) None

9. **(a)** Since we require $3 - x \geq 0$, the domain is $(-\infty, 3]$.

(b) $[0, \infty)$

(c)



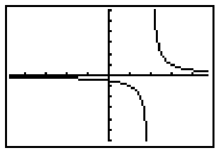
$[-4.7, 4.7]$ by $[-6, 6]$

(d) None

10. **(a)** Since we require $x - 2 \neq 0$, the domain is $(-\infty, 2) \cup (2, \infty)$.

(b) Since $\frac{1}{x-2}$ can assume any value except 0, the range is $(-\infty, 0) \cup (0, \infty)$.

(c)



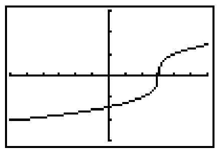
$[-4.7, 4.7]$ by $[-6, 6]$

(d) None

11. **(a)** $(-\infty, \infty)$ or all real numbers

(b) $(-\infty, \infty)$ or all real numbers

(c)



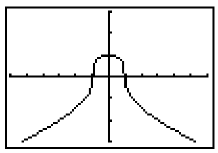
$[-6, 6]$ by $[-3, 3]$

(d) None

12. **(a)** $(-\infty, \infty)$ or all real numbers

(b) The maximum function value is attained at the point $(0, 1)$, so the range is $(-\infty, 1]$.

(c)



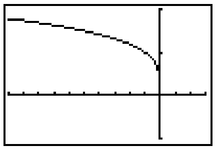
$[-6, 6]$ by $[-3, 3]$

(d) Symmetric about the y-axis (even)

13. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$.

(b) $[0, \infty)$

(c)



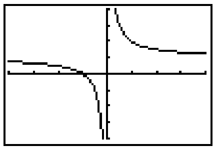
$[-10, 3]$ by $[-1, 2]$

(d) None

14. **(a)** Since we require $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

(b) Note that $\frac{1}{x}$ can assume any value except 0, so $1 + \frac{1}{x}$ can assume any value except 1. The range is $(-\infty, 1) \cup (1, \infty)$.

(c)



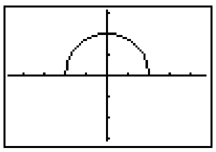
$[-4, 4]$ by $[-4, 4]$

(d) None

15. **(a)** Since we require $4 - x^2 \geq 0$, the domain is $[-2, 2]$.

(b) Since $4 - x^2$ will be between 0 and 4, inclusive (for x in the domain), its square root is between 0 and 2, inclusive. The range is $[0, 2]$.

(c)



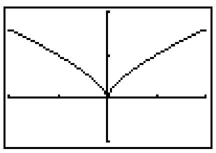
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(d) Symmetric about the y-axis (even)

16. **(a)** This function is equivalent to $y = \sqrt[3]{x^2}$, so its domain is all real numbers.

(b) $[0, \infty)$

(c)



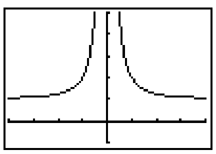
$[-2, 2]$ by $[-1, 2]$

(d) Symmetric about the y-axis (even)

17. **(a)** Since we require $x^2 \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$

(b) Since $\frac{1}{x^2} > 0$ for all x, the range is $(1, \infty)$.

(c)



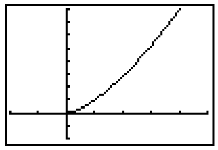
$[-4, 4]$ by $[-1, 5]$

(d) Symmetric about the y-axis (even)

18. **(a)** This function is equivalent to $y = \sqrt{x^3}$, so its domain is $[0, \infty)$.

(b) $[0, \infty)$

(c)



$[-2, 5]$ by $[-2, 8]$

(d) None

19. Even, since the function is an even power of x.

20. Neither, since the function is a sum of even and odd powers of x.

21. Neither, since the function is a sum of even and odd powers of x $(x^1 + 2x^0)$.

22. Even, since the function is a sum of even powers of x $(x^2 - 3x^0)$.

23. Even, since the function involves only even powers of x.

24. Odd, since the function is a sum of odd powers of x.

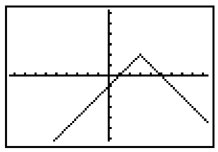
25. Odd, since the function is a quotient of an odd function (x^3) and an even function $(x^2 - 1)$.

26. Neither, since, (for example), $y(-2) = 4^{1/3}$ and $y(2) = 0$.

27. Neither, since, (for example), $y(-1)$ is defined and $y(1)$ is undefined.

28. Even, since the function involves only even powers of x.

29. **(a)**



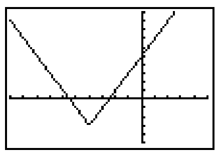
$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Note that $f(x) = -|x - 3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

(b) $(-\infty, \infty)$

(c) $(-\infty, 2]$

30. **(a)** The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward.

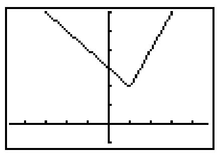


$[-10, 5]$ by $[-5, 10]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[-3, \infty)$

31. **(a)**

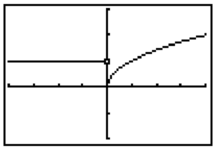


$[-4.7, 4.7]$ by $[-1, 6]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[2, \infty)$

32. **(a)**

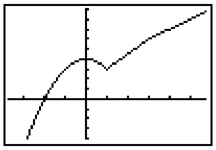


$[-4, 4]$ by $[-2, 3]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

33. **(a)**

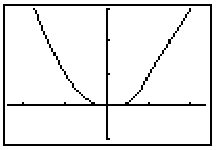


$[-3.7, 5.7]$ by $[-4, 9]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $(-\infty, \infty)$ or all real numbers

34. **(a)**



$[-2.35, 2.35]$ by $[-1, 3]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

37. No

38. Yes

39. Yes

40. No

41. Line through (0, 0) and (1, 1): $y = x$
Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$$

42. $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

43. Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \dfrac{0 - 1}{5 - 2} = \dfrac{-1}{3} = -\dfrac{1}{3}$,

so $y = -\dfrac{1}{3}(x - 2) + 1 = -\dfrac{1}{3}x + \dfrac{5}{3}$

$$f(x) = \begin{cases} -x + 2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

44. Line through $(-1, 0)$ and $(0, -3)$:

$m = \dfrac{-3 - 0}{0 - (-1)} = \dfrac{-3}{1} = -3$, so $y = -3x - 3$

Line through (0, 3) and $(2, -1)$:

$m = \dfrac{-1 - 3}{2 - 0} = \dfrac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ -2x + 3, & 0 < x \le 2 \end{cases}$$

45. Line through $(-1, 1)$ and (0, 0): $y = -x$

Line through (0, 1) and (1, 1): $y = 1$

Line through (1, 1) and (3, 0):

$m = \dfrac{0 - 1}{3 - 1} = \dfrac{-1}{2} = -\dfrac{1}{2}$,

so $y = -\dfrac{1}{2}(x - 1) + 1 = -\dfrac{1}{2}x + \dfrac{3}{2}$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$$

46. Line through $(-2, -1)$ and (0, 0): $y = \dfrac{1}{2}x$

Line through (0, 2) and (1, 0): $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x + 2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

47. Line through $\left(\dfrac{T}{2}, 0\right)$ and $(T, 1)$:

$m = \dfrac{1 - 0}{T - (T/2)} = \dfrac{2}{T}$, so $y = \dfrac{2}{T}\left(x - \dfrac{T}{2}\right) + 0 = \dfrac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

48. $f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$

49. **(a)** $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(b) $g(f(x)) = (x + 5)^2 - 3$
$= (x^2 + 10x + 25) - 3$
$= x^2 + 10x + 22$

(c) $f(g(0)) = 0^2 + 2 = 2$

(d) $g(f(0)) = 0^2 + 10 \cdot 0 + 22 = 22$

(e) $g(g(-2)) = [(-2)^2 - 3]^2 - 3 = 1^2 - 3 = -2$

(f) $f(f(x)) = (x + 5) + 5 = x + 10$

50. **(a)** $f(g(x)) = (x - 1) + 1 = x$

(b) $g(f(x)) = (x + 1) - 1 = x$

(c) $f(g(x)) = 0$

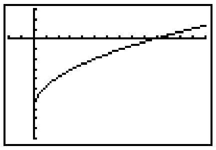
(d) $g(f(0)) = 0$

(e) $g(g(-2)) = (-2 - 1) - 1 = -3 - 1 = -4$

(f) $f(f(x)) = (x + 1) + 1 = x + 2$

51. **(a)** Enter $y_1 = f(x) = x - 7$, $y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$,
and $y_4 = (g \circ f)(x) = y_2(y_1(x))$

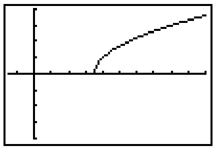
$f \circ g$:



$[-10, 70]$ by $[-10, 3]$

Domain: $[0, \infty)$
Range: $[-7, \infty)$

$g \circ f$:



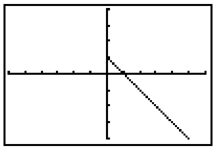
$[-3, 20]$ by $[-4, 4]$

Domain: $[7, \infty)$
Range: $[0, \infty)$

(b) $(f \circ g)(x) = \sqrt{x} - 7$ $\quad$ $(g \circ f)(x) = \sqrt{x - 7}$

52. **(a)** Enter $y_1 = f(x) = 1 - x^2$, $y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$,
and $y_4 = (g \circ f)(x) = y_2(y_1(x))$

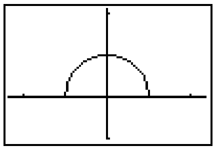
$f \circ g$:



$[-6, 6]$ by $[-4, 4]$

Domain: $[0, \infty)$
Range: $(-\infty, 1]$

$g \circ f$:



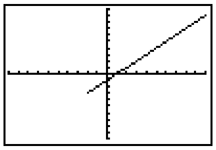
$[-2.35, 2.35]$ by $[-1, 2.1]$

Domain: $[-1, 1]$
Range: $[0, 1]$

(b) $(f \circ g)(x) = 1 - (\sqrt{x})^2 = 1 - x,\ x \ge 0$
$(g \circ f)(x) = \sqrt{1 - x^2}$

53. **(a)** Enter $y_1 = f(x) = x^2 - 3$, $y_2 = g(x) = \sqrt{x + 2}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$,
and $y_4 = (g \circ f)(x) = y_2(y_1(x))$.

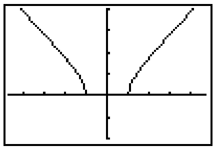
$f \circ g$:



$[-10, 10]$ by $[-10, 10]$

Domain: $[-2, \infty)$
Range: $[-3, \infty)$

$g \circ f$:



$[-4.7, 4.7]$ by $[-2, 4]$

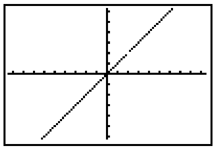
Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[0, \infty)$

(b)
$$\begin{aligned}(f \circ g)(x) &= (\sqrt{x + 2})^2 - 3 \\ &= (x + 2) - 3,\ x \ge -2 \\ &= x - 1,\ x \ge -2\end{aligned}$$
$$(g \circ f)(x) = \sqrt{(x^2 - 3) + 2} = \sqrt{x^2 - 1}$$

54. **(a)** Enter $y_1(x) = f(x) = \dfrac{2x - 1}{x + 3}$, $y_2 = \dfrac{3x + 1}{2 - x}$,

$y_3 = (f \circ g)(x) = y_1(y_2(x))$,
and $y_4 = (g \circ f)(x) = y_2(y_1(x))$.
Use a "decimal window" such as the one shown.

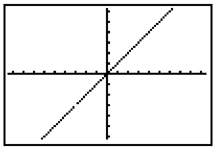
$f \circ g$:



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 2) \cup (2, \infty)$

$g \circ f$:

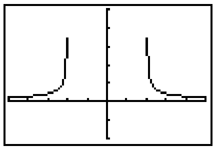


$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Domain: $(-\infty, -3) \cup (-3, \infty)$
Range: $(-\infty, -3) \cup (-3, \infty)$

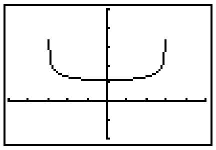
(b)
$$\begin{aligned}(f \circ g)(x) &= \frac{2\left(\frac{3x + 1}{2 - x}\right) - 1}{\frac{3x + 1}{2 - x} + 3} \\ &= \frac{2(3x + 1) - (2 - x)}{(3x + 1) + 3(2 - x)},\ x \ne 2 \\ &= \frac{7x}{7},\ x \ne 2 \\ &= x,\ x \ne 2\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= \frac{3\left(\frac{2x - 1}{x + 3}\right) + 1}{2 - \frac{2x - 1}{x + 3}} \\ &= \frac{3(2x - 1) + (x + 3)}{2(x + 3) - (2x - 1)},\ x \ne -3 \\ &= \frac{7x}{7},\ x \ne -3 \\ &= x,\ x \ne -3\end{aligned}$$

55. 

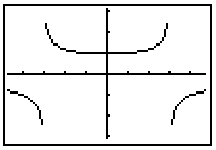
$[-5, 5]$ by $[-2, 5]$

We require $x^2 - 4 \geq 0$ (so that the square root is defined) and $x^2 - 4 \neq 0$ (to avoid division by zero), so the domain is $(-\infty, -2) \cup (2, \infty)$. For values of x in the domain, $x^2 - 4$ $\left(\text{and hence } \sqrt{x^2 - 4} \text{ and } \frac{1}{\sqrt{x^2 - 4}}\right)$ can attain any positive value, so the range is $(0, \infty)$. (Note that grapher failure may cause the range to appear as a finite interval on a grapher.

56. 

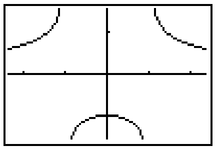
$[-5, 5]$ by $[-2, 5]$

We require $9 - x^2 \geq 0$ (so that the fourth root is defined) and $9 - x^2 \neq 0$ (to avoid division by zero), so the domain is $(-3, 3)$. For values of x in the domain, $9 - x^2$ can attain any value in $(0, 9]$. Therefore, $\sqrt[4]{9 - x^2}$ can attain any value in $(0, \sqrt{3}]$, and $\frac{2}{\sqrt[4]{9 - x^2}}$ can attain any value in $\left[\frac{2}{\sqrt{3}}, \infty\right)$. The range is $\left[\frac{2}{\sqrt{3}}, \infty\right)$ or approximately $[1.15, \infty)$. (Note that grapher failure may cause the range to appear as a finite interval on a grapher.)

57. 

$[-4.7, 4.7]$ by $[-3.1, 3.1]$

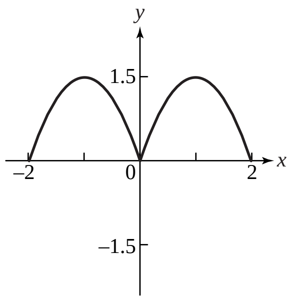
We require $9 - x^2 \neq 0$, so the domain is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$. For values of x in the domain, $9 - x^2$ can attain any value in $(-\infty, 0) \cup (0, 9]$, so $\sqrt[3]{9 - x^2}$ can attain any value in $(-\infty, 0) \cup (0, \sqrt[3]{9}]$. Therefore, $\frac{2}{\sqrt[3]{9 - x^2}}$ can attain any value in $(-\infty, 0) \cup \left[\frac{2}{\sqrt[3]{9}}, \infty\right)$. The range is $(-\infty, 0) \cup \left[\frac{2}{\sqrt[3]{9}}, \infty\right)$ or approximately $(-\infty, 0) \cup [0.96, \infty)$. (Note that grapher failure can cause the intervals in the range to appear as finite intervals on a grapher.)

58. 

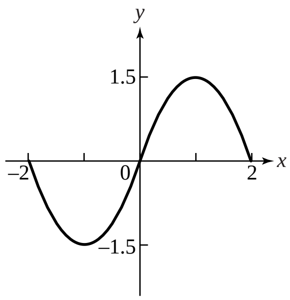
$[-2.35, 2.35]$ by $[-1.55, 1.55]$

We require $x^2 - 1 \neq 0$, so the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$. For values of x in the domain, $x^2 - 1$ can attain any value in $[-1, 0) \cup (0, \infty)$, so $\sqrt[3]{x^2 - 1}$ can also attain any value in $[-1, 0) \cup (0, \infty)$. Therefore, $\frac{1}{\sqrt[3]{x - 1}}$ can attain any value in $(-\infty, -1] \cup (0, \infty)$. The range is $(-\infty, -1] \cup (0, \infty)$. (Note that grapher failure can cause the intervals in the range to appear as finite intervals on a grapher.)

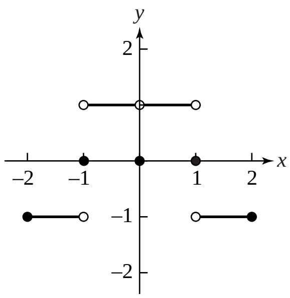
59. **(a)**



(b)

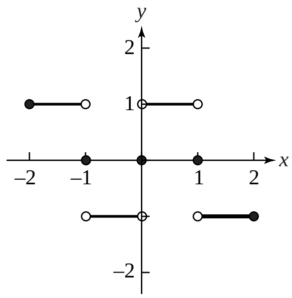


60. **(a)**

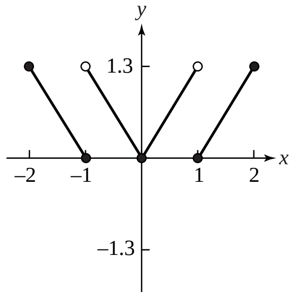


60. continued

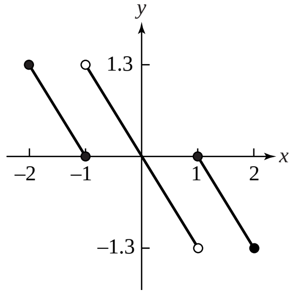
(b)



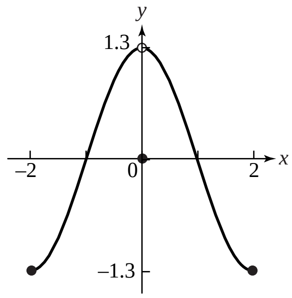
61. (a)



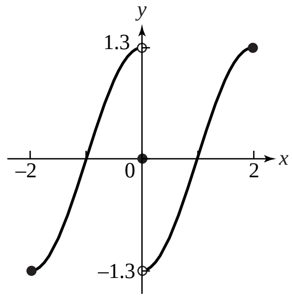
(b)



62. (a)



(b)



63. (a) Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \dfrac{1}{g(x)} = x$, we know that $\dfrac{1}{g(x)} = x - 1$, so $g(x) = \dfrac{1}{x - 1}$.

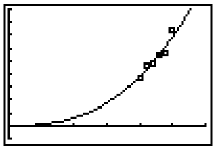
(c) Since $(f \circ g)(x) = f\left(\dfrac{1}{x}\right) = x$, $f(x) = \dfrac{1}{x}$.

(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$.
The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
$\dfrac{1}{x-1}$	$1 + \dfrac{1}{x}$	$x, x \neq -1$
$\dfrac{1}{x}$	$\dfrac{1}{x}$	$x, x \neq 0$
$\sqrt{x}$	x^2	$\lvert x \rvert, x \geq 0$

64. (a) Note that the data in the table begins at $x = 20$. (We do not include the initial investment in the data.) The power regression equation is $y = 27.1094x^{2.651044}$.

(b)



[0, 30] by [−20,000, 180,000]

(c) When $x = 30$, $y \approx 223{,}374$. According to the power regression equation, the investment will grow to approximately \$223,374.

(d) The linear regression equation is $y = 12{,}577.97x - 177{,}275.52$. When $x = 30$, $y \approx 200{,}064$. According to the linear regression equation, the investment will grow to approximately \$200,064.

65. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

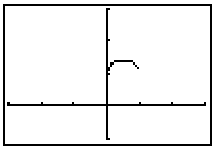
(c)
$$
\begin{aligned}
h &= \sqrt{16 - r^2} \\
&= \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2} \\
&= \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)} \\
&= \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}} \\
&= \sqrt{\frac{16\pi x}{4\pi^2} - \frac{x^2}{4\pi^2}} \\
&= \frac{\sqrt{16\pi x - x^2}}{2\pi}
\end{aligned}
$$

(d)
$$
\begin{aligned}
V &= \frac{1}{3}\pi r^2 h \\
&= \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi} \\
&= \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}
\end{aligned}
$$

66. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

(b)
$$\begin{aligned} C(0) &= \$1{,}200{,}000 \\ C(500) &\approx \$1{,}175{,}812 \\ C(1000) &\approx \$1{,}186{,}512 \\ C(1500) &= \$1{,}212{,}000 \\ C(2000) &\approx \$1{,}243{,}732 \\ C(2500) &\approx \$1{,}278{,}479 \\ C(3000) &\approx \$1{,}314{,}870 \end{aligned}$$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point P.

67. (a)



$[-3, 3]$ by $[-1, 3]$

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain $[0, 1]$, the same as the domain of $y_1 + y_2$ found in part (b).

Domain of $\dfrac{y_1}{y_2}$: $[0, 1)$

Domain of $\dfrac{y_2}{y_1}$: $(0, 1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.

The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

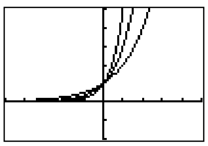
68. (a) Yes. Since $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (f \cdot g)(x)$, the function $(f \cdot g)(x)$ will also be even.

(b) The product will be even, since
$$\begin{aligned} (f \cdot g)(-x) &= f(-x) \cdot g(-x) \\ &= (-f(x)) \cdot (-g(x)) \\ &= f(x) \cdot g(x) \\ &= (f \cdot g)(x). \end{aligned}$$

■ Section 1.3 Exponential Functions (pp. 20–26)

Exploration 1 Exponential Functions

1.



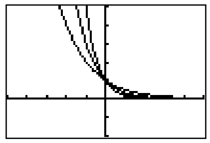
$[-5, 5]$ by $[-2, 5]$

2. $x > 0$

3. $x < 0$

4. $x = 0$

5.



$[-5, 5]$ by $[-2, 5]$

6. $2^{-x} < 3^{-x} < 5^{-x}$ for $x < 0$; $2^{-x} > 3^{-x} > 5^{-x}$ for $x > 0$; $2^{-x} = 3^{-x} = 5^{-x}$ for $x = 0$.

Quick Review 1.3

1. Using a calculator, $5^{2/3} \approx 2.924$.

2. Using a calculator, $3^{\sqrt{2}} \approx 4.729$.

3. Using a calculator, $3^{-1.5} \approx 0.192$.

4. $x^3 = 17$
$x = \sqrt[3]{17}$
$x \approx 2.5713$

5. $x^5 = 24$
$x = \sqrt[5]{24}$
$x \approx 1.8882$

6. $x^{10} = 1.4567$
$x = \pm\sqrt[10]{1.4567}$
$x \approx \pm 1.0383$

7. $500(1.0475)^5 \approx \$630.58$

8. $1000(1.063)^3 \approx \$1201.16$

9.
$$\begin{aligned} \frac{(x^{-3}y^2)^2}{(x^4y^3)^3} &= \frac{x^{-6}y^4}{x^{12}y^9} \\ &= x^{-6-12}y^{4-9} \\ &= x^{-18}y^{-5} \\ &= \frac{1}{x^{18}y^5} \end{aligned}$$

10.
$$\begin{aligned} \left(\frac{a^3b^{-2}}{c^4}\right)^2\left(\frac{a^4c^{-2}}{b^3}\right)^{-1} &= \frac{a^6b^{-4}}{c^8} \cdot \frac{b^3}{a^4c^{-2}} \\ &= \frac{a^6}{b^4c^8} \cdot \frac{b^3c^2}{a^4} \\ &= a^{6-4}b^{-4+3}c^{-8+2} \\ &= a^2b^{-1}c^{-6} = \frac{a^2}{bc^6} \end{aligned}$$

Section 1.3 Exercises

1. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

2. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\frac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

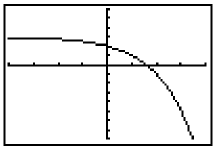
3. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 2. (e)

4. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^x$, is the reflection about the x-axis of the graph in Exercise 1. (c)

5. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

6. The graph of $y = 1.5^x - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

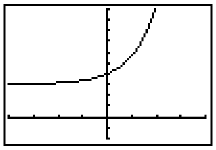
7.



[−4, 4] by [−8, 6]

Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$
x-intercept: ≈ 1.585
y-intercept: 2

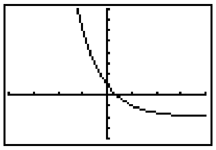
8.



[−4, 4] by [−2, 10]

Domain: $(-\infty, \infty)$
Range: $(3, \infty)$
x-intercept: None
y-intercept: 4

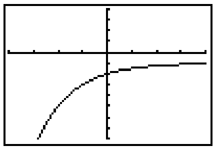
9.



[−4, 4] by [−4, 8]

Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
x-intercept: ≈ 0.405
y-intercept: 1

10.



[−4, 4] by [−8, 4]

Domain: $(-\infty, \infty)$
Range: $(-\infty, -1)$
x-intercept: None
y-intercept: -2

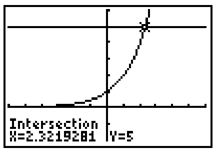
11. $9^{2x} = (3^2)^{2x} = 3^{4x}$

12. $16^{3x} = (2^4)^{3x} = 2^{12x}$

13. $\left(\frac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

14. $\left(\frac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

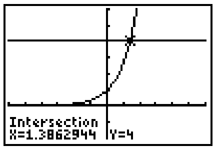
15.



[−6, 6] by [−2, 6]

$x \approx 2.3219$

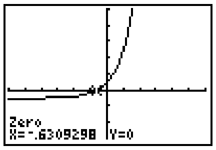
16.



[−6, 6] by [−2, 6]

$x \approx 1.3863$

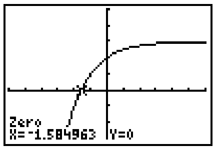
17.



[−6, 6] by [−3, 5]

$x \approx -0.6309$

18.



[−6, 6] by [−3, 5]

$x \approx -1.5850$

19.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

20.

x	y	Δy
1	1	
		−3
2	−2	
		−3
3	−5	
		−3
4	−8	

21.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

22.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.794	

23. Let t be the number of years. Solving $500{,}000(1.0375)^t = 1{,}000{,}000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

24. **(a)** The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
Population in 1915: $P(25) \approx 12{,}315$
Population in 1940: $P(50) \approx 24{,}265$

(b) Solving $P(t) = 50{,}000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

25. **(a)** $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38.1145$. There will be 1 gram remaining after about 38.1145 days.

26. Let t be the number of years. Solving $2300(1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

27. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

28. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve

$A\left(1 + \frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to

$\left(1 + \frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that

$t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years 2 months.)

29. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

30. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

31. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve

$A\left(1 + \frac{0.0575}{365}\right)^{365t} = 3A$, which is equivalent to

$\left(1 + \frac{0.0575}{365}\right)^{365t} = 3$. Solving graphically, we find that

$t \approx 19.108$. It will take about 19.108 years.

32. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

33. After t hours, the population is $P(t) = 2^{t/0.5}$ or, equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

34. **(a)** Each year, the number of cases is $100\% - 20\% = 80\%$ of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take about 10.319 years.

(b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

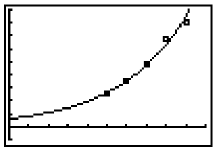
35. Since $\Delta x = 1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.

36. **(a)** When $t = 0$, $B = 100e^0 = 100$. There were 100 bacteria present initially.

(b) When $t = 6$, $B = 100e^{0.693(6)} \approx 6394.351$.
After 6 hours, there are about 6394 bacteria.

(c) Solving $100e^{0.693t} = 200$ graphically, we find that $t \approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

37. **(a)** Let $x = 0$ represent 1900, $x = 1$ represent 1901, and so on. The regression equation is $P(x) = 6.033(1.030)^x$.

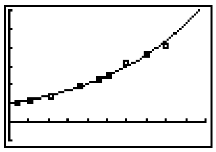


[0, 100] by [−10, 90]

(b) The regression equation gives an estimate of $P(0) \approx 6.03$ million, which is not very close to the actual population.

(c) Since the equation is of the form $P(x) = P(0) \cdot 1.030^x$, the annual rate of growth is about 3%.

38. **(a)** The regression equation is $P(x) = 4.831(1.019)^x$.



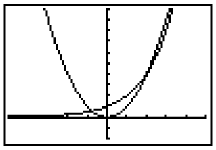
[0, 100] by [−5, 30]

(b) $P(90) \approx 26.3$ million

(c) Since the equation is of the form $P(x) = P(0) \cdot 1.019^x$, the annual rate of growth is approximately 1.9%.

39. $5422(1.018)^{19} \approx 7609.7$ million

40. (a)



$[-5, 5]$ by $[-2, 10]$

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $Y1$	change in $Y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

It happens by the time $x = 4$.

(c) Solving graphically, $x \approx -0.7667$, $x = 2$, $x = 4$.

(d) The solution set is approximately $(-0.7667, 2) \cup (4, \infty)$.

41. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.
Dividing, we have

$$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$$
$$a^2 = 9$$
$$a = \pm 3$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

42. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$. Dividing, we have

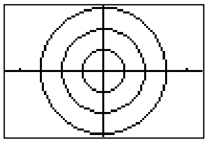
$$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$$
$$a^2 = 0.25$$
$$a = \pm 0.5$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

■ Section 1.4 Parametric Equations (pp. 26–31)

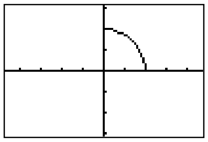
Exploration 1 Parametrizing Circles

1. Each is a circle with radius $|a|$. As $|a|$ increases, the radius of the circle increases.



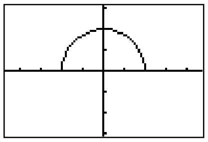
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

2. $0 \le t \le \frac{\pi}{2}$:



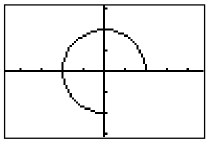
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$0 \le t \le \pi$:



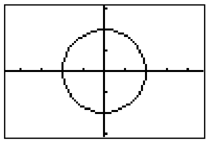
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$0 \le t \le \frac{3\pi}{2}$:



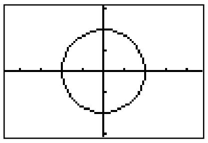
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$2\pi \le t \le 4\pi$:



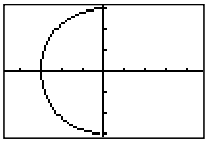
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$0 \le t \le 4\pi$:

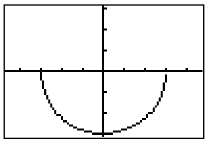


$[-4.7, 4.7]$ by $[-3.1, 3.1]$

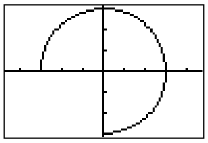
Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete circle. If $d = 2\pi$, you get the complete circle. If $d > 2\pi$, you get the complete circle but portions of the circle will be traced out more than once. For example, if $d = 4\pi$ the entire circle is traced twice.

3. 

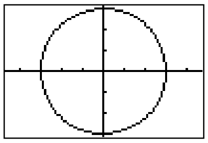
$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$
initial point: (0, 3)
terminal point: (0, −3)



$\pi \le t \le 2\pi$
initial point: (−3, 0)
terminal point: (3, 0)



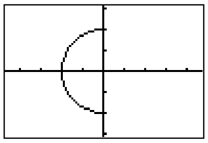
$\frac{3\pi}{2} \le t \le 3\pi$
initial point: (0, −3)
terminal point: (−3, 0)



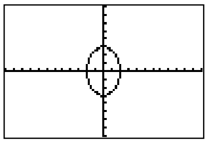
$\pi \le t \le 5\pi$
initial point: (−3, 0)
terminal point: (−3, 0)

4. For $0 \le t \le 2\pi$ the complete circle is traced once clockwise beginning and ending at (2, 0).
For $\pi \le t \le 3\pi$ the complete circle is traced once clockwise beginning and ending at (−2, 0).

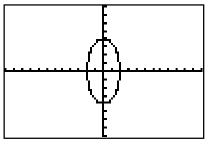
For $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ the half circle below is traced clockwise starting at (0, −2) and ending at (0, 2).



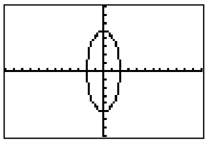
Exploration 2 Parametrizing Ellipses

1. $a = 2$, $b = 3$: 

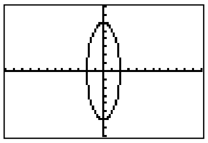
[−12, 12] by [−8, 8]

$a = 2$, $b = 4$: 

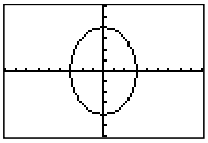
[−12, 12] by [−8, 8]

$a = 2$, $b = 5$: 

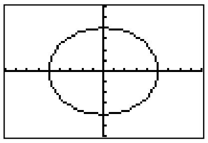
[−12, 12] by [−8, 8]

$a = 2$, $b = 6$: 

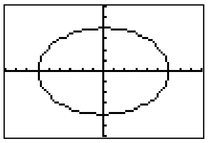
[−12, 12] by [−8, 8]

2. $a = 3$, $b = 4$: 

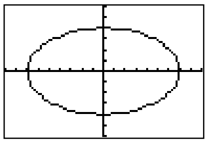
[−9, 9] by [−6, 6]

$a = 5$, $b = 4$: 

[−9, 9] by [−6, 6]

$a = 6$, $b = 4$: 

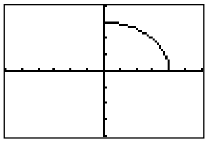
[−9, 9] by [−6, 6]

$a = 7$, $b = 4$: 

[−9, 9] by [−6, 6]

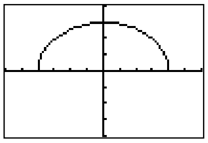
3. If $|a| > |b|$, then the major axis is on the x-axis and the minor on the y-axis. If $|a| < |b|$, then the major axis is on the y-axis and the minor on the x-axis.

4. $0 \le t \le \frac{\pi}{2}$:



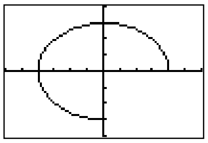
$[-6, 6]$ by $[-4, 4]$

$0 \le t \le \pi$:



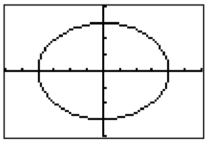
$[-6, 6]$ by $[-4, 4]$

$0 \le t \le \frac{3\pi}{2}$:



$[-6, 6]$ by $[-4, 4]$

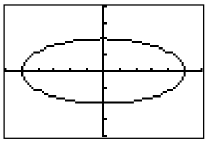
$0 \le t \le 4\pi$:



$[-6, 6]$ by $[-4, 4]$

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete ellipse. If $d = 2\pi$, you get the complete ellipse. If $d > 2\pi$, you get the complete ellipse but portions of the ellipse will be traced out more than once. For example, if $d = 4\pi$ the entire ellipse is traced twice.

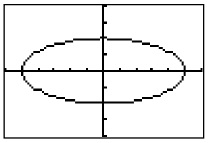
5. $0 \le t \le 2\pi$:



$[-6, 6]$ by $[-4, 4]$

initial point: $(5, 0)$
terminal point: $(5, 0)$

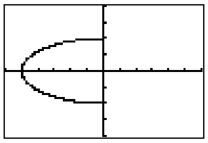
$\pi \le t \le 3\pi$:



$[-6, 6]$ by $[-4, 4]$

initial point: $(-5, 0)$
terminal point: $(-5, 0)$

$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$:



$[-6, 6]$ by $[-4, 4]$

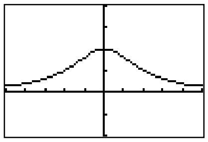
initial point: $(0, -2)$
terminal point: $(0, 2)$

Each curve is traced clockwise from the initial point to the terminal point.

Exploration 3 Graphing the Witch of Agnesi

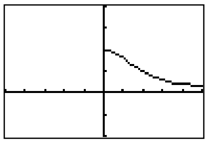
1. We used the parameter interval $[0, \pi]$ because our graphing calculator ignored the fact that the curve is not defined when $t = 0$ or π. The curve is traced from right to left across the screen. x ranges from $-\infty$ to ∞.

2. $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$:



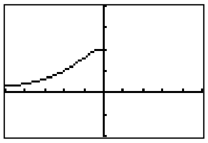
$[-5, 5]$ by $[-2, 4]$

$0 < t \le \frac{\pi}{2}$:



$[-5, 5]$ by $[-2, 4]$

$\frac{\pi}{2} \le t < \pi$:



$[-5, 5]$ by $[-2, 4]$

For $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$, the entire graph described in part 1 is drawn. The left branch is drawn from right to left across the screen starting at the point $(0, 2)$. Then the right branch is drawn from right to left across the screen stopping at the point $(0, 2)$. If you leave out $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

For $0 < t \le \frac{\pi}{2}$, the right branch is drawn from right to left across the screen stopping at the point $(0, 2)$. If you leave out $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

For $\frac{\pi}{2} \le t < \pi$, the left branch is drawn from right to left across the screen starting at the point $(0, 2)$. If you leave out $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

3. If you replace $x = 2 \cot t$ by $x = -2 \cot t$, the same graph is drawn except it is traced from left to right across the screen. If you replace $x = 2 \cot t$ by $x = 2 \cot (\pi - t)$, the same graph is drawn except it is traced from left to right across the screen.

Quick Review 1.4

1. $m = \frac{3 - 8}{4 - 1} = \frac{-5}{3} = -\frac{5}{3}$

$y = -\frac{5}{3}(x - 1) + 8$

$y = -\frac{5}{3}x + \frac{29}{3}$

2. $y = -4$

3. $x = 2$

4. When $y = 0$, we have $\frac{x^2}{9} = 1$, so the x-intercepts are -3 and 3. When $x = 0$, we have $\frac{y^2}{16} = 1$, so the y-intercepts are -4 and 4.

5. When $y = 0$, we have $\frac{x^2}{16} = 1$, so the x-intercepts are -4 and 4. When $x = 0$, we have $-\frac{y^2}{9} = 1$, which has no real solution, so there are no y-intercepts.

6. When $y = 0$, we have $0 = x + 1$, so the x-intercept is -1. When $x = 0$, we have $2y^2 = 1$, so the y-intercepts are $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

7. (a)
$$2(1)^2(1) + 1^2 \stackrel{?}{=} 3$$
$$3 = 3 \quad \text{Yes}$$

(b)
$$2(-1)^2(-1) + (-1)^2 \stackrel{?}{=} 3$$
$$-2 + 1 \stackrel{?}{=} 3$$
$$-1 \neq 3 \quad \text{No}$$

(c)
$$2\left(\frac{1}{2}\right)^2(-2) + (-2)^2 \stackrel{?}{=} 3$$
$$-1 + 4 \stackrel{?}{=} 3$$
$$3 = 3 \quad \text{Yes}$$

8. (a)
$$9(1)^2 - 18(1) + 4(3)^2 = 27$$
$$9 - 18 + 36 \stackrel{?}{=} 27$$
$$27 = 27 \quad \text{Yes}$$

(b)
$$9(1)^2 - 18(1) + 4(-3)^2 \stackrel{?}{=} 27$$
$$9 - 18 + 36 \stackrel{?}{=} 27$$
$$27 = 27 \quad \text{Yes}$$

(c)
$$9(-1)^2 - 18(-1) + 4(3)^2 \stackrel{?}{=} 27$$
$$9 + 18 + 36 \stackrel{?}{=} 27$$
$$63 \neq 27 \quad \text{No}$$

9. (a)
$$\begin{aligned} 2x + 3t &= -5 \\ 3t &= -2x - 5 \\ t &= \frac{-2x - 5}{3} \end{aligned}$$

(b)
$$\begin{aligned} 3y - 2t &= -1 \\ -2t &= -3y - 1 \\ 2t &= 3y + 1 \\ t &= \frac{3y + 1}{2} \end{aligned}$$

10. (a) The equation is true for $a \geq 0$.

(b) The equation is equivalent to "$\sqrt{a^2} = a$ or $\sqrt{a^2} = -a$." Since $\sqrt{a^2} = a$ is true for $a \geq 0$ and $\sqrt{a^2} = -a$ is true for $a \leq 0$, at least one of the two equations is true for all real values of a. Therefore, the given equation $\sqrt{a^2} = \pm a$ is true for all real values of a.

(c) The equation is true for all real values of a.

Section 1.4 Exercises

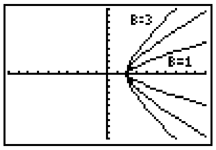
1. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \leq t \leq 2\pi$

2. Graph (a). Window: $[-2, 2]$ by $[-2, 2]$, $0 \leq t \leq 2\pi$

3. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \leq t \leq 2\pi$

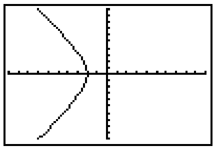
4. Graph (b). Window: $[-15, 15]$ by $[-15, 15]$, $0 \leq t \leq 2\pi$

5. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt." The graphs for $a = 2$ and $b = 1$, 2, and 3 are shown.



$[-10, 10]$ by $[-10, 10]$

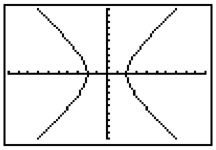
(b)



$[-10, 10]$ by $[-10, 10]$

This appears to be the left half of the same hyberbola.

(c)



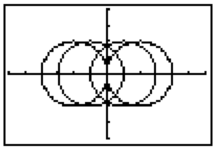
$[-10, 10]$ by $[-10, 10]$

One must be careful because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes of the hyperbola) in its graph. The extraneous lines can be avoided by using the grapher's dot mode instead of connected mode.

(d) Note that $\sec^2 t - \tan^2 t = 1$ by a standard trigonometric identity. Substituting $\frac{x}{a}$ for sec t and $\frac{y}{b}$ for tan t gives $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph. Substituting $\frac{y}{b}$ for sec t and $\frac{x}{a}$ for tan t in the identity $\sec^2 t - \tan^2 t = 1$ gives $\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1$.

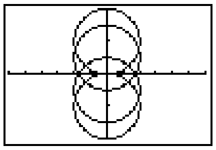
6. (a)



$[-6, 6]$ by $[-4, 4]$

The graph is a circle of radius 2 centered at $(h, 0)$. As h changes, the graph shifts horizontally.

(b)



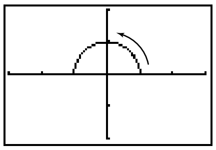
$[-6, 6]$ by $[-4, 4]$

The graph is a circle of radius 2 centered at $(0, k)$. At k changes, the graph shifts vertically.

(c) Since the circle is to be centered at $(2, -3)$, we use $h = 2$ and $k = -3$. Since a radius of 5 is desired, we need to change the coefficients of $\cos t$ and $\sin t$ to 5.
$x = 5\cos t + 2,\ y = 5\sin t - 3,\ 0 \le t \le 2\pi$

(d) $x = 5\cos t - 3,\ y = 2\sin t + 4,\ 0 \le t \le 2\pi$

7. (a)

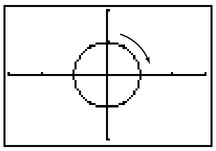


$[-3, 3]$ by $[-2, 2]$

Initial point: $(1, 0)$
Terminal point: $(-1, 0)$

(b) $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$
The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

8. (a)

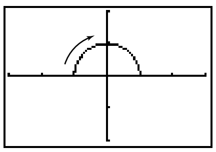


$[-3, 3]$ by $[-2, 2]$

Initial and terminal point: $(0, 1)$

(b) $x^2 + y^2 = \sin^2(2\pi t) + \cos^2(2\pi t) = 1$
The parametrized curve traces all of the circle defined by $x^2 + y^2 = 1$.

9. (a)

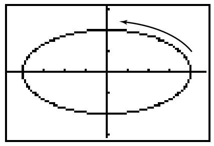


$[-3, 3]$ by $[-2, 2]$

Initial point: $(-1, 0)$
Terminal point: $(0, 1)$

(b) $x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$
The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

10. (a)



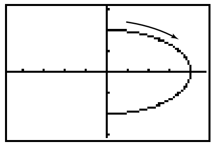
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Initial and terminal point: $(4, 0)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = \cos^2 t + \sin^2 t = 1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

11. (a)



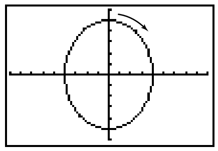
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Initial point: $(0, 2)$
Terminal point: $(0, -2)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = \sin^2 t + \cos^2 t = 1$

The parametrized curve traces the right half of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ (or all of the curve defined by $x = 2\sqrt{4 - y^2}$).

12. (a)



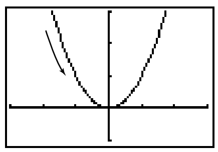
$[-9, 9]$ by $[-6, 6]$

Initial and terminal point: $(0, 5)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t = 1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$.

13. (a)

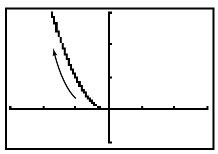


$[-3, 3]$ by $[-1, 3]$

No initial or terminal point.

(b) $y = 9t^2 = (3t)^2 = x^2$
The parametrized curve traces all of the parabola defined by $y = x^2$.

14. (a)

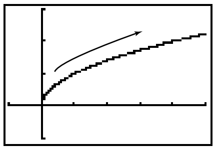


$[-3, 3]$ by $[-1, 3]$

Initial point: $(0, 0)$
Terminal point: None

(b) $y = t = (-\sqrt{t})^2 = x^2$
The parametrized curve traces the left half of the parabola defined by $y = x^2$ (or all of the curve defined by $x = -\sqrt{y}$).

15. (a)

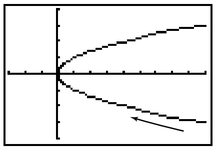


$[-1, 5]$ by $[-1, 3]$

Initial point: $(0, 0)$
Terminal point: None

(b) $y = \sqrt{t} = \sqrt{x}$
The parametrized curve traces all of the curve defined by $y = \sqrt{x}$ (or the upper half of the parabola defined by $x = y^2$).

16. (a)

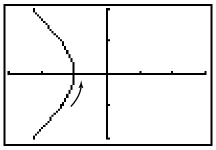


$[-3, 9]$ by $[-4, 4]$

No initial or terminal point.

(b) $x = \sec^2 t - 1 = \tan^2 t = y^2$
The parametrized curve traces all of the parabola defined by $x = y^2$.

17. (a)

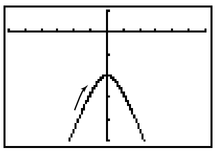


$[-3, 3]$ by $[-2, 2]$

No initial or terminal point. Note that it may be necessary to use a t-interval such as $[-1.57, 1.57]$ or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $x^2 - y^2 = \sec^2 t - \tan^2 t = 1$
The parametrized curve traces the left branch of the hyperbola defined by $x^2 - y^2 = 1$ (or all of the curve defined by $x = -\sqrt{y^2 + 1}$).

18. (a)

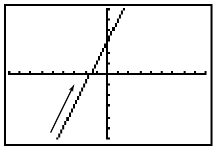


$[-6, 6]$ by $[-5, 1]$

No initial or terminal point. Note that it may be necessary to use a t-interval such as $[-1.57, 1.57]$ or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $\left(\frac{y}{2}\right)^2 - x^2 = \sec^2 t - \tan^2 t = 1$
The parametrized curve traces the lower branch of the hyperbola defined by $\left(\frac{y}{2}\right)^2 - x^2 = 1$ (or all of the curve defined by $y = -2\sqrt{x^2 + 1}$).

19. (a)

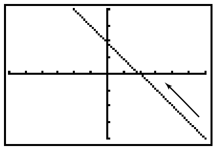


$[-9, 9]$ by $[-6, 6]$

No initial or terminal point.

(b) $y = 4t - 7 = 2(2t - 5) + 3 = 2x + 3$
The parametrized curve traces all of the line defined by $y = 2x + 3$.

20. (a)

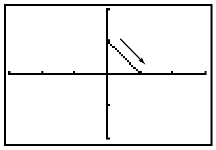


$[-6, 6]$ by $[-4, 4]$

No initial or terminal point.

(b) $y = 1 + t = 2 - (1 - t) = 2 - x = -x + 2$
The parametrized curve traces all of the line defined by $y = -x + 2$.

21. (a)

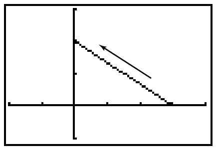


$[-3, 3]$ by $[-2, 2]$

Initial point: $(0, 1)$
Terminal point: $(1, 0)$

(b) $y = 1 - t = 1 - x = -x + 1$
The Cartesian equation is $y = -x + 1$. The portion traced by the parametrized curve is the segment from $(0, 1)$ to $(1, 0)$.

22. (a)

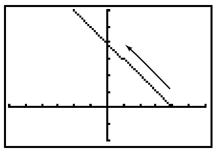


$[-2, 4]$ by $[-1, 3]$

Initial point: $(3, 0)$
Terminal point: $(0, 2)$

(b) $y = 2t = (2t - 2) + 2 = -\frac{2}{3}(3 - 3t) + 2 = -\frac{2}{3}x + 2$
The Cartesian equation is $y = -\frac{2}{3}x + 2$. The portion traced by the curve is the segment from $(3, 0)$ to $(0, 2)$.

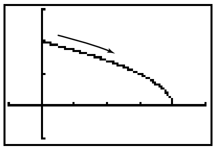
23. (a)



$[-6, 6]$ by $[-2, 6]$

Initial point: $(4, 0)$
Terminal point: None

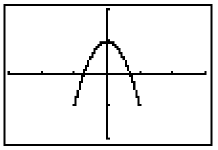
(b) $y = \sqrt{t} = 4 - (4 - \sqrt{t}) = 4 - x = -x + 4$
The parametrized curve traces the portion of the line defined by $y = -x + 4$ to the left of $(4, 0)$, that is, for $x \leq 4$.

24. (a) 

$[-1, 5]$ by $[-1, 3]$

Initial point: (0, 2)
Terminal point: (4, 0)

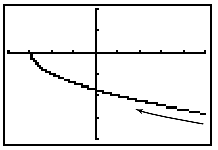
(b) $y = \sqrt{4 - t^2} = \sqrt{4 - x}$
The parametrized curve traces the right portion of the curve defined by $y = \sqrt{4 - x}$, that is, for $x \geq 0$.

25. (a) 

$[-3, 3]$ by $[-2, 2]$

No initial or terminal point, since the t-interval has no beginning or end. The curve is traced and retraced in both directions.

(b)
$$\begin{aligned} y &= \cos 2t \\ &= \cos^2 t - \sin^2 t \\ &= 1 - 2\sin^2 t \\ &= 1 - 2x^2 \\ &= -2x^2 + 1 \end{aligned}$$
The parametrized curve traces the portion of the parabola defined by $y = -2x^2 + 1$ corresponding to $-1 \leq x \leq 1$.

26. (a) 

$[-4, 5]$ by $[-4, 2]$

Initial point: None
Terminal point: $(-3, 0)$

(b) $x = t^2 - 3 = y^2 - 3$
The parametrized curve traces the lower half of the parabola defined by $x = y^2 - 3$ (or all of the curve defined by $y = -\sqrt{x + 3}$).

27. Using $(-1, -3)$ we create the parametric equations $x = -1 + at$ and $y = -3 + bt$, representing a line which goes through $(-1, -3)$ at $t = 0$. We determine a and b so that the line goes through (4, 1) when $t = 1$.
Since $4 = -1 + a$, $a = 5$.
Since $1 = -3 + b$, $b = 4$.
Therefore, one possible parametrization is $x = -1 + 5t$, $y = -3 + 4t$, $0 \leq t \leq 1$.

28. Using $(-1, 3)$ we create the parametric equations $x = -1 + at$ and $y = 3 + bt$, representing a line which goes through $(-1, 3)$ at $t = 0$. We determine a and b so that the line goes through $(3, -2)$ at $t = 1$.
Since $3 = -1 + a$, $a = 4$.
Since $-2 = 3 + b$, $b = -5$.
Therefore, one possible parametrization is $x = -1 + 4t$, $y = 3 - 5t$, $0 \leq t \leq 1$.

29. The lower half of the parabola is given by $x = y^2 + 1$ for $y \leq 0$. Substituting t for y, we obtain one possible parametrization: $x = t^2 + 1$, $y = t$, $t \leq 0$.

30. The vertex of the parabola is at $(-1, -1)$, so the left half of the parabola is given by $y = x^2 + 2x$ for $x \leq -1$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t$, $t \leq -1$.

31. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at (2, 3) for $t = 0$ and passes through $(-1, -1)$ at $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$.

Since slope $= \dfrac{\Delta x}{\Delta t} = \dfrac{-1 - 2}{1 - 0} = -3$,

$x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where

$g(0) = 3$ and $g(1) = -1$.

Since slope $= \dfrac{\Delta y}{\Delta t} = \dfrac{-1 - 3}{1 - 0} = -4$,

$y = g(t) = -4t + 3 = 3 - 4t$.

One possible parametrization is:

$x = 2 - 3t$, $y = 3 - 4t$, $t \geq 0$.

32. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(-1, 2)$ for $t = 0$ and passes through (0, 0) at $t = 1$. Then $x = f(t)$, where $f(0) = -1$ and $f(1) = 0$.

Since slope $= \dfrac{\Delta x}{\Delta t} = \dfrac{0 - (-1)}{1 - 0} = 1$,

$x = f(t) = 1t + (-1) = -1 + t$.

Also, $y = g(t)$, where $g(0) = 2$ and $g(1) = 0$.

Since slope $= \dfrac{\Delta y}{\Delta t} = \dfrac{0 - 2}{1 - 0} = -2$,

$y = g(t) = -2t + 2 = 2 - 2t$.

One possible parametrization is:

$x = -1 + t$, $y = 2 - 2t$, $t \geq 0$.

33. The graph is in Quadrant I when $0 < y < 2$, which corresponds to $1 < t < 3$. To confirm, note that $x(1) = 2$ and $x(3) = 0$.

34. The graph is in Quadrant II when $2 < y \leq 4$, which corresponds to $3 < t \leq 5$. To confirm, note that $x(3) = 0$ and $x(5) = -2$.

35. The graph is in Quadrant III when $-6 \leq y < -4$, which corresponds to $-5 \leq t < -3$. To confirm, note that $x(-5) = -2$ and $x(-3) = 0$.

36. The graph is in Quadrant IV when $-4 < y < 0$, which corresponds to $-3 < t < 1$. To confirm, note that $x(-3) = 0$ and $x(1) = 2$.

37. The graph of $y = x^2 + 2x + 2$ lies in Quadrant I for all $x > 0$. Substituting t for x, we obtain one possible parametrization:
$x = t$, $y = t^2 + 2t + 2$, $t > 0$.

38. The graph of $y = \sqrt{x + 3}$ lies in Quadrant I for all $x \geq 0$. Substituting t for x, we obtain one possible parametrization:
$x = t$, $y = \sqrt{t + 3}$, $t > 0$.

39. Possible answers:

(a) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 2\pi$

(b) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 2\pi$

(c) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 4\pi$

(d) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 4\pi$

40. Possible answers:

(a) $x = -a \cos t$, $y = b \sin t$, $0 \le t \le 2\pi$

(b) $x = -a \cos t$, $y = -b \sin t$, $0 \le t \le 2\pi$

(c) $x = -a \cos t$, $y = b \sin t$, $0 \le t \le 4\pi$

(d) $x = -a \cos t$, $y = -b \sin t$, $0 \le t \le 4\pi$

41. Note that $m\angle OAQ = t$, since alternate interior angles formed by a transversal of parallel lines are congruent.

Therefore, $\tan t = \frac{OQ}{AQ} = \frac{2}{x}$, so $x = \frac{2}{\tan t} = 2 \cot t$.

Now, by equation (iii), we know that

$$\begin{aligned} AB &= \frac{(AQ)^2}{AO} \\ &= \left(\frac{AQ}{AO}\right)(AQ) \\ &= (\cos t)(x) \\ &= (\cos t)(2 \cot t) \\ &= \frac{2\cos^2 t}{\sin t}. \end{aligned}$$

Then equation (ii) gives

$$y = 2 - AB \sin t = 2 - \frac{2\cos^2 t}{\sin t} \cdot \sin t = 2 - 2\cos^2 t$$
$$= 2\ \sin^2 t.$$

The parametric equations are:

$x = 2 \cot t$, $y = 2\ \sin^2 t$, $0 < t < \pi$

Note: Equation (iii) may not be immediately obvious, but it may be justified as follows. Sketch segment QB. Then $\angle OBQ$ is a right angle, so $\triangle ABQ \sim \triangle AQO$, which gives $\frac{AB}{AQ} = \frac{AQ}{AO}$.

42. (a) If $x_2 = x_1$ then the line is a vertical line and the first parametric equation gives $x = x_1$, while the second will give all real values for y since it cannot be the case that $y_2 = y_1$ as well.

Otherwise, solving the first equation for t gives $t = (x - x_1)/(x_2 - x_1)$.
Substituting that into the second equation gives $y = y_1 + [(y_2 - y_1)/(x_2 - x_1)](x - x_1)$ which is the point-slope form of the equation for the line through (x_1, y_1) and (x_2, y_2).

Note that the first equation will cause x to take on all real values, because $(x_2 - x_1)$ is not zero. Therefore, all of the points on the line will be traced out.

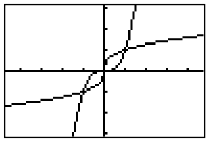
(b) Use the equations for x and y given in part (a), with $0 \le t \le 1$.

■ Section 1.5 Functions and Logarithms (pp. 32–40)

Exploration 1 Testing for Inverses Graphically

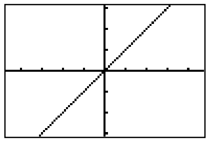
1. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



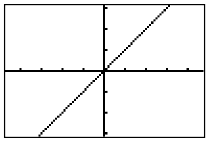
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

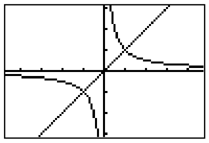
(c) $g \circ f$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

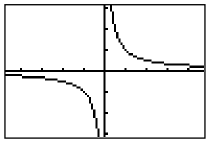
2. It appears that $f \circ g = g \circ f = g$, suggesting that f may be the identity function.

(a) f and g:



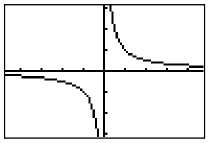
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

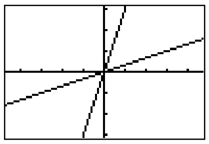
(c) $g \circ f$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

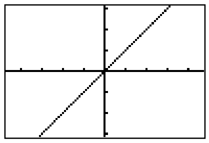
3. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

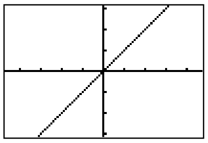
(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

3. continued

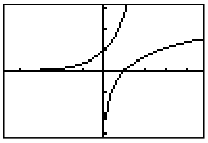
(c) $g \circ f$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

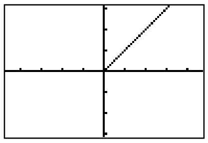
4. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverse of each other. (Notice that the domain of $f \circ g$ is $(0, \infty)$ and the domain of $g \circ f$ is $(-\infty, \infty)$.)

(a) f and g:



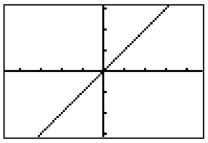
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $f \circ g$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

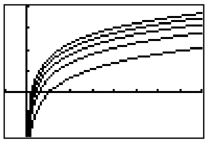
(c) $g \circ f$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

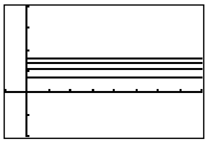
Exploration 2 Supporting the Product Rule

1. They appear to be vertical translations of each other.



$[-1, 8]$ by $[-2, 4]$

2. This graph suggests that each difference $(y_3 = y_1 - y_2)$ is a constant.



$[-1, 8]$ by $[-2, 4]$

3. $y_3 = y_1 - y_2 = \ln(ax) - \ln x = \ln a + \ln x - \ln x = \ln a$
Thus, the difference $y_3 = y_1 - y_2$ is the constant $\ln a$.

Quick Review 1.5

1. $(f \circ g)(1) = f(g(1)) = f(2) = 1$

2. $(g \circ f)(-7) = g(f(-7)) = g(-2) = 5$

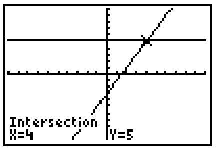
3. $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt[3]{(x^2 + 1) - 1}$
$= \sqrt[3]{x^2} = x^{2/3}$

4. $(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x - 1})$
$= (\sqrt[3]{x - 1})^2 + 1$
$= (x - 1)^{2/3} + 1$

5. Substituting t for x, one possible answer is:
$x = t,\ y = \dfrac{1}{t - 1},\ t \geq 2.$

6. Substituting t for x, one possible answer is:
$x = t,\ y = t,\ t < -3.$

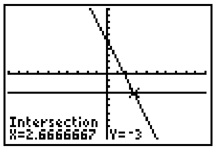
7.



$[-10, 10]$ by $[-10, 10]$

$(4, 5)$

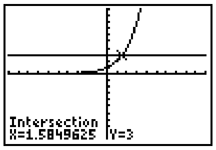
8.



$[-10, 10]$ by $[-10, 10]$

$\left(\dfrac{8}{3}, -3\right) \approx (2.67, -3)$

9. (a)

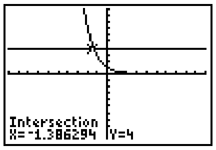


$[-10, 10]$ by $[-10, 10]$

$(1.58, 3)$

(b) No points of intersection, since $2^x > 0$ for all values of x.

10. (a)



$[-10, 10]$ by $[-10, 10]$

$(-1.39, 4)$

(b) No points of intersection, since $e^{-x} > 0$ for all values of x.

Section 1.5 Exercises

1. No, since (for example) the horizontal line $y = 2$ intersects the graph twice.

2. Yes, since each horizontal line intersects the graph only once.

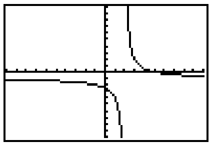
3. Yes, since each horizontal line intersects the graph at most once.

4. No, since (for example) the horizontal line $y = 0.5$ intersects the graph twice.

5. Yes, since each horizontal line intersects the graph only once.

6. No, since (for example) the horizontal line $y = 2$ intersects the graph at more than one point.

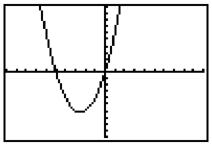
7.



$[-10, 10]$ by $[-10, 10]$

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

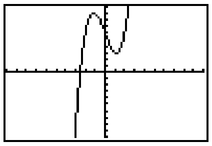
8.



$[-10, 10]$ by $[-10, 10]$

No, the function is not one-to-one since (for example) the horizontal line $y = 0$ intersects the graph twice, so it does not have an inverse function.

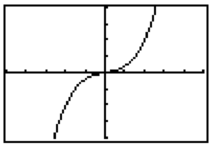
9.



$[-10, 10]$ by $[-10, 10]$

No, the function is not one-to-one since (for example) the horizontal line $y = 5$ intersects the graph more than once, so it does not have an inverse function.

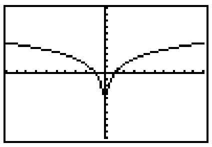
10.



$[-5, 5]$ by $[-20, 20]$

Yes, the function is one-to-one since each horizontal line intersects the graph only once, so it has an inverse function.

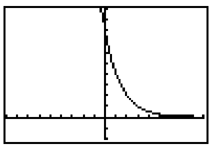
11.



$[-10, 10]$ by $[-10, 10]$

No, the function is not one-to-one since each horizontal line intersects the graph twice, so it does not have an inverse function.

12.



$[-9, 9]$ by $[-2, 10]$

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

13. $y = 2x + 3$

$y - 3 = 2x$

$\frac{y-3}{2} = x$

Interchange x and y.

$\frac{x-3}{2} = y$

$f^{-1}(x) = \frac{x-3}{2}.$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{x-3}{2}\right)$$
$$= 2\left(\frac{x-3}{2}\right) + 3$$
$$= (x - 3) + 3$$
$$= x$$

$$(f^{-1} \circ f)(x) = f^{-1}(2x + 3)$$
$$= \frac{(2x + 3) - 3}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

14. $y = 5 - 4x$

$4x = 5 - y$

$x = \frac{5-y}{4}$

Interchange x and y.

$y = \frac{5-x}{4}$

$f^{-1}(x) = \frac{5-x}{4}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{5-x}{4}\right) = 5 - 4\left(\frac{5-x}{4}\right)$$
$$= 5 - (5 - x)$$
$$= x$$

$$(f^{-1} \circ f)(x) = f^{-1}(5 - 4x)$$
$$= \frac{5 - (5 - 4x)}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

15. $y = x^3 - 1$

$y + 1 = x^3$

$(y + 1)^{1/3} = x$

Interchange x and y.

$(x + 1)^{1/3} = y$

$f^{-1}(x) = (x + 1)^{1/3}$ or $\sqrt[3]{x+1}$

Verify.

$$(f \circ f^{-1})(x) = f(\sqrt[3]{x+1})$$
$$= (\sqrt[3]{x+1})^3 - 1 = (x + 1) - 1 = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(x^3 - 1)$$
$$= \sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{x^3} = x$$

16. $y = x^2 + 1, x \geq 0$

$y - 1 = x^2, x \geq 0$

$\sqrt{y-1} = x$

Interchange x and y.

$\sqrt{x-1} = y$

$f^{-1}(x) = \sqrt{x-1}$ or $(x-1)^{1/2}$

Verify. For $x \geq 1$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1) + 1 = x$$

For $x > 0$, (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2+1) = \sqrt{(x^2+1) - 1} = \sqrt{x^2} = |x| = x$$

17. $y = x^2, x \leq 0$

$x = -\sqrt{y}$

Interchange x and y.

$y = -\sqrt{x}$

$f^{-1}(x) = -\sqrt{x}$ or $-x^{1/2}$

Verify.

For $x \geq 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

For $x \leq 0$, (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^2) = -\sqrt{x^2} = -|x| = x$

18. $y = x^{2/3}, x \geq 0$

$y^{3/2} = (x^{2/3})^{3/2}, x \geq 0$

$y^{3/2} = x$

Interchange x and y.

$x^{3/2} = y$

$f^{-1}(x) = x^{3/2}$

Verify.

For $x \geq 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(x^{3/2}) = (x^{3/2})^{2/3} = x$

for $x \geq 0$, (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^{2/3}) = (x^{2/3})^{3/2} = |x| = x$

19. $y = -(x-2)^2, x \leq 2$

$(x-2)^2 = -y, x \leq 2$

$x - 2 = -\sqrt{-y}$

$x = 2 - \sqrt{-y}$

Interchange x and y.

$y = 2 - \sqrt{-x}$

$f^{-1}(x) = 2 - \sqrt{-x}$ or $2 - (-x)^{1/2}$

Verify.

For $x \leq 0$ (the domain of f^{-1})

$$(f \circ f^{-1})(x) = f(2 - \sqrt{-x}) = -[(2 - \sqrt{-x}) - 2]^2 = -(-\sqrt{-x})^2 = -|x| = x$$

For $x \leq 2$ (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(-(x-2)^2) = 2 - \sqrt{(x-2)^2} = 2 - |x-2| = 2 + (x-2) = x$$

20. $y = (x^2 + 2x + 1), x \geq -1$

$y = (x+1)^2, x \geq -1$

$\sqrt{y} = x + 1$

$\sqrt{y} - 1 = x$

Interchange x and y.

$\sqrt{x} - 1 = y$

$f^{-1}(x) = \sqrt{x} - 1$ or $x^{1/2} - 1$

Verify.

For $x \geq 0$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f(\sqrt{x} - 1) = [(\sqrt{x}-1)^2 + 2(\sqrt{x}-1) + 1] = (\sqrt{x})^2 - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1 = (\sqrt{x})^2 = x$$

For $x \geq -1$ (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2 + 2x + 1) = \sqrt{x^2 + 2x + 1} - 1 = \sqrt{(x+1)^2} - 1 = |x+1| - 1 = (x+1) - 1 = x$$

21. $y = \dfrac{1}{x^2}, x > 0$

$x^2 = \dfrac{1}{y}, x > 0$

$x = \sqrt{\dfrac{1}{y}} = \dfrac{1}{\sqrt{y}}$

Interchange x and y.

$y = \dfrac{1}{\sqrt{x}}$

$f^{-1}(x) = \dfrac{1}{\sqrt{x}}$ or $\dfrac{1}{x^{1/2}}$

Verify.

For $x > 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{(1/\sqrt{x})^2} = x$

For $x > 0$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}\left(\dfrac{1}{x^2}\right) = \dfrac{1}{\sqrt{1/x^2}} = \sqrt{x^2} = |x| = x$

22. $y = \dfrac{1}{x^3}$

$x^3 = \dfrac{1}{y}$

$x = \sqrt[3]{\dfrac{1}{y}} = \dfrac{1}{\sqrt[3]{y}}$

Interchange x and y.

$y = \dfrac{1}{\sqrt[3]{x}}$

$f^{-1}(x) = \dfrac{1}{\sqrt[3]{x}}$ or $\dfrac{1}{x^{1/3}}$

Verify.

$(f \circ f^{-1})(x) = f\left(\dfrac{1}{\sqrt[3]{x}}\right) = \dfrac{1}{(1/\sqrt[3]{x})^3} = x$

$(f^{-1} \circ f)(x) = f^{-1}\left(\dfrac{1}{x^3}\right) = \dfrac{1}{\sqrt[3]{1/x^3}} = x$

23. $y = \dfrac{2x+1}{x+3}$

$xy + 3y = 2x + 1$

$xy - 2x = 1 - 3y$

$(y - 2)x = 1 - 3y$

$x = \dfrac{1-3y}{y-2}$

Interchange x and y.

$y = \dfrac{1-3x}{x-2}$

$f^{-1}(x) = \dfrac{1-3x}{x-2}$

Verify.

$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{1-3x}{x-2}\right)\\ &= \frac{2\left(\frac{1-3x}{x-2}\right)+1}{\frac{1-3x}{x-2}+3}\\ &= \frac{2(1-3x)+(x-2)}{(1-3x)+3(x-2)}\\ &= \frac{-5x}{-5} = x\end{aligned}$$

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}\left(\frac{2x+1}{x+3}\right)\\ &= \frac{1-3\left(\frac{2x+1}{x+3}\right)}{\frac{2x+1}{x+3}-2}\\ &= \frac{(x+3)-3(2x+1)}{(2x+1)-2(x+3)}\\ &= \frac{-5x}{-5} = x\end{aligned}$$

24. $y = \dfrac{x+3}{x-2}$

$xy - 2y = x + 3$

$xy - x = 2y + 3$

$x(y - 1) = 2y + 3$

$x = \dfrac{2y+3}{y-1}$

Interchange x and y.

$y = \dfrac{2x+3}{x-1}$

$f^{-1}(x) = \dfrac{2x+3}{x-1}$

Verify.

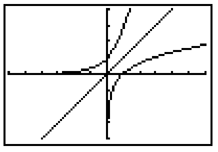
$$\begin{aligned}(f \circ f^{-1})(x) &= f\left(\frac{2x+3}{x-1}\right)\\ &= \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}\\ &= \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)}\\ &= \frac{5x}{5} = x\end{aligned}$$

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}\left(\frac{x+3}{x-2}\right)\\ &= \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}\\ &= \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)}\\ &= \frac{5x}{5} = x\end{aligned}$$

25. Graph of f: $x_1 = t, y_1 = e^t$

Graph of f^{-1}: $x_2 = e^t, y_2 = t$

Graph of $y = x$: $x_3 = t, y_3 = t$

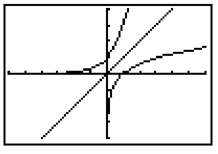


$[-6, 6]$ by $[-4, 4]$

26. Graph of f: $x_1 = t, y_1 = 3^t$

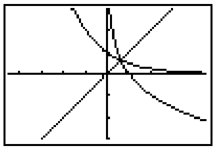
Graph of f^{-1}: $x_2 = 3^t, y_2 = t$

Graph of $y = x$: $x_3 = t, y_3 = t$



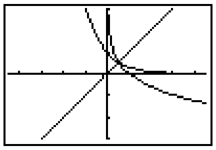
$[-6, 6]$ by $[-4, 4]$

27. Graph of f: $x_1 = t, y_1 = 2^{-t}$
Graph of f^{-1}: $x_2 = 2^{-t}, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



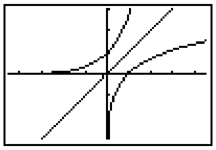
$[-4.5, 4.5]$ by $[-3, 3]$

28. Graph of f: $x_1 = t, y_1 = 3^{-t}$
Graph of f^{-1}: $x_2 = 3^{-t}, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



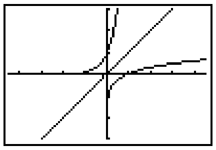
$[-4.5, 4.5]$ by $[-3, 3]$

29. Graph of f: $x_1 = t, y_1 = \ln t$
Graph of f^{-1}: $x_2 = \ln t, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



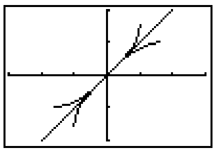
$[-4.5, 4.5]$ by $[-3, 3]$

30. Graph of f: $x_1 = t, y_1 = \log t$
Graph of f^{-1}: $x_2 = \log t, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



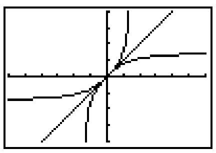
$[-4.5, 4.5]$ by $[-3, 3]$

31. Graph of f: $x_1 = t, y_1 = \sin^{-1} t$
Graph of f^{-1}: $x_2 = \sin^{-1} t, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



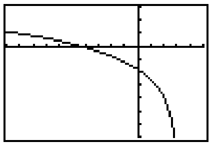
$[-3, 3]$ by $[-2, 2]$

32. Graph of f: $x_1 = t, y_1 = \tan^{-1} t$
Graph of f^{-1}: $x_2 = \tan^{-1} t, y_2 = t$
Graph of $y = x$: $x_3 = t, y_3 = t$



$[-6, 6]$ by $[-4, 4]$

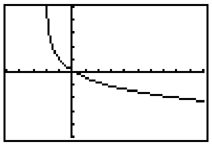
33.



$[-10, 5]$ by $[-7, 3]$

Domain: $(\infty, 3)$
Range: $(-\infty, \infty)$

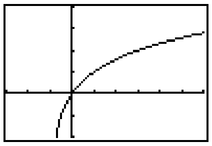
34.



$[-5, 10]$ by $[-5, 5]$

Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$

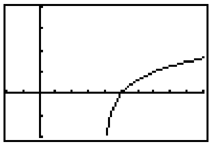
35.



$[-3, 6]$ by $[-2, 4]$

Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$

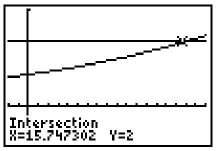
36.



$[-2, 10]$ by $[-2, 4]$

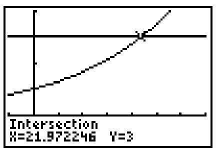
Domain: $(4, \infty)$
Range: $(-\infty, \infty)$

37. $(1.045)^t = 2$
$\ln(1.045)^t = \ln 2$
$t \ln 1.045 = \ln 2$
$t = \dfrac{\ln 2}{\ln 1.045} \approx 15.75$
Graphical support:



$[-2, 18]$ by $[-1, 3]$

38. $e^{0.05t} = 3$
$\ln e^{0.05t} = \ln 3$
$0.05t = \ln 3$
$t = \dfrac{\ln 3}{0.05} = 20 \ln 3 \approx 21.97$
Graphical support:



$[-5, 35]$ by $[-1, 4]$

39. $e^x + e^{-x} = 3$

$e^x - 3 + e^{-x} = 0$

$e^x(e^x - 3 + e^{-x}) = e^x(0)$

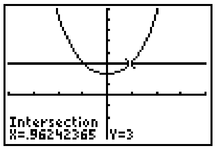
$(e^x)^2 - 3e^x + 1 = 0$

$e^x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$e^x = \dfrac{3 \pm \sqrt{5}}{2}$

$x = \ln\left(\dfrac{3 \pm \sqrt{5}}{2}\right) \approx -0.96 \text{ or } 0.96$

Graphical support:



$[-4, 4]$ by $[-4, 8]$

40. $2^x + 2^{-x} = 5$

$2^x - 5 + 2^{-x} = 0$

$2^x(2^x - 5 + 2^{-x}) = 2^x(0)$

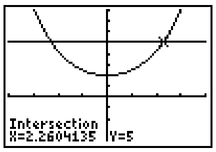
$(2^x)^2 - 5(2^x) + 1 = 0$

$2^x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

$2^x = \dfrac{5 \pm \sqrt{21}}{2}$

$x = \log_2\left(\dfrac{5 \pm \sqrt{21}}{2}\right) \approx -2.26 \text{ or } 2.26$

Graphical support:



$[-4, 4]$ by $[-4, 8]$

41. $\ln y = 2t + 4$

$e^{\ln y} = e^{2t + 4}$

$y = e^{2t + 4}$

42. $\ln(y - 1) - \ln 2 = x + \ln x$

$\ln(y - 1) = x + \ln x + \ln 2$

$e^{\ln(y-1)} = e^{x + \ln x + \ln 2}$

$y - 1 = e^x(x)(2)$

$y = 2xe^x + 1$

43. $y = \dfrac{100}{1 + 2^{-x}}$

$1 + 2^{-x} = \dfrac{100}{y}$

$2^{-x} = \dfrac{100}{y} - 1$

$\log_2(2^{-x}) = \log_2\left(\dfrac{100}{y} - 1\right)$

$-x = \log_2\left(\dfrac{100}{y} - 1\right)$

$x = -\log_2\left(\dfrac{100}{y} - 1\right)$

$= -\log_2\left(\dfrac{100 - y}{y}\right)$

$= \log_2\left(\dfrac{y}{100 - y}\right)$

Interchange x and y.

$y = \log_2\left(\dfrac{x}{100 - x}\right)$

$f^{-1}(x) = \log_2\left(\dfrac{x}{100 - x}\right)$

Verify.

$(f \circ f^{-1})(x) = f\left(\log_2 \dfrac{x}{100 - x}\right)$

$= \dfrac{100}{1 + 2^{-\log_2\left(\frac{x}{100 - x}\right)}}$

$= \dfrac{100}{1 + 2^{\log_2\left(\frac{100 - x}{x}\right)}}$

$= \dfrac{100}{1 + \dfrac{100 - x}{x}}$

$= \dfrac{100x}{x + (100 - x)} = \dfrac{100x}{100} = x$

$(f^{-1} \circ f)(x) = f^{-1}\left(\dfrac{100}{1 + 2^{-x}}\right)$

$= \log_2\left(\dfrac{\dfrac{100}{1 + 2^{-x}}}{100 - \dfrac{100}{1 + 2^{-x}}}\right)$

$= \log_2\left(\dfrac{100}{100(1 + 2^{-x}) - 100}\right)$

$= \log_2\left(\dfrac{1}{2^{-x}}\right) = \log_2(2^x) = x$

44. $y = \dfrac{50}{1 + 1.1^{-x}}$

$1 + 1.1^{-x} = \dfrac{50}{y}$

$1.1^{-x} = \dfrac{50}{y} - 1$

$\log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\dfrac{50}{y} - 1\right)$

$-x = \log_{1.1}\left(\dfrac{50}{y} - 1\right)$

$x = -\log_{1.1}\left(\dfrac{50}{y} - 1\right) = -\log_{1.1}\left(\dfrac{50 - y}{y}\right) = \log_{1.1}\left(\dfrac{y}{50 - y}\right)$

Interchange x and y:

$y = \log_{1.1}\left(\dfrac{x}{50 - x}\right)$

$f^{-1}(x) = \log_{1.1}\left(\dfrac{x}{50 - x}\right)$

Verify.

$$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50 - x}\right)\right)$$

$$= \frac{50}{1 + 1.1^{-\log_{1.1}\left(\frac{x}{50 - x}\right)}}$$

$$= \frac{50}{1 + 1.1^{\log_{1.1}\left(\frac{50 - x}{x}\right)}}$$

$$= \frac{50}{1 + \frac{50 - x}{x}}$$

$$= \frac{50x}{x + (50 - x)} = \frac{50x}{50} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1 + 1.1^{-x}}\right)$$

$$= \log_{1.1}\left(\frac{\frac{50}{1 + 1.1^{-x}}}{50 - \frac{50}{1 + 1.1^{-x}}}\right)$$

$$= \log_{1.1}\left(\frac{50}{50(1 + 1.1^{-x}) - 50}\right)$$

$$= \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^{x}) = x$$

45. **(a)**
$$\begin{aligned} f(f(x)) &= \sqrt{1 - (f(x))^2} \\ &= \sqrt{1 - (1 - x^2)} \\ &= \sqrt{x^2} = |x| = x, \text{ since } x \geq 0 \end{aligned}$$

(b) $f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$ for all $x \neq 0$

46. **(a)** Amount $= 8\left(\dfrac{1}{2}\right)^{t/12}$

(b) $8\left(\dfrac{1}{2}\right)^{t/12} = 1$

$\left(\dfrac{1}{2}\right)^{t/12} = \dfrac{1}{8}$

$\left(\dfrac{1}{2}\right)^{t/12} = \left(\dfrac{1}{2}\right)^{3}$

$\dfrac{t}{12} = 3$

$t = 36$

There will be 1 gram remaining after 36 hours.

47. $500(1.0475)^t = 1000$

$1.0475^t = 2$

$\ln(1.0475^t) = \ln 2$

$t \ln 1.0475 = \ln 2$

$t = \dfrac{\ln 2}{\ln 1.0475} \approx 14.936$

It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

48. $375{,}000(1.0225)^t = 1{,}000{,}000$

$1.0225^t = \dfrac{8}{3}$

$\ln(1.0225^t) = \ln\left(\dfrac{8}{3}\right)$

$t \ln 1.0225 = \ln\left(\dfrac{8}{3}\right)$

$t = \dfrac{\ln(8/3)}{\ln 1.0225} \approx 44.081$

It will take about 44.081 years.

49. **(a)** $y = -2539.852 + 636.896 \ln x$

(b) When $x = 75$, $y \approx 209.94$. About 209.94 million metric tons were produced.

(c)
$$\begin{aligned} -2539.852 + 636.896 \ln x &= 400 \\ 636.896 \ln x &= 2939.852 \\ \ln x &= \frac{2939.852}{636.896} \\ x &= e^{\frac{2939.852}{636.896}} \approx 101.08 \end{aligned}$$

According to the regression equation, Saudi Arabian oil production will reach 400 million metric tons when $x \approx 101.08$, in about 2001.

50. **(a)** $y = -590.969 + 152.817 \ln x$

(b) When $x = 85$, $y \approx 87.94$.
About 87.94 million metric tons were produced.

(c) $-590.969 + 152.817 \ln x = 120$

$$152.817 \ln x = 710.969$$
$$\ln x = \frac{710.969}{152.817}$$
$$x = e^{\frac{710.969}{152.817}} \approx 104.84$$

According to the regression equation, oil production will reach 120 million metric tons when $x \approx 104.84$, in about 2005.

51. **(a)** Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$ so $mx_1 = mx_2$. Since $m \neq 0$, this gives $x_1 = x_2$.

(b) $y = mx + b$

$y - b = mx$

$\frac{y-b}{m} = x$

Interchange x and y.

$\frac{x-b}{m} = y$

$f^{-1}(x) = \frac{x-b}{m}$

The slopes are reciprocals.

(c) If the original functions both have slope m, each of the inverse functions will have slope $\frac{1}{m}$. The graphs of the inverses will be parallel lines with nonzero slope.

(d) If the original functions have slopes m and $-\frac{1}{m}$, respectively, then the inverse functions will have slopes $\frac{1}{m}$ and $-m$, respectively. Since each of $\frac{1}{m}$ and $-m$ is the negative reciprocal of the other, the graphs of the inverses will be perpendicular lines with nonzero slopes.

52. **(a)** y_2 is a vertical shift (upward) of y_1, although it's difficult to see that near the vertical asymptote at $x = 0$. One might use "trace" or "table" to verify this.

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $e^{y_2 - y_1} = a$, $\ln(e^{y_2 - y_1}) = \ln a$,
$y_2 - y_1 = \ln a$, $y_1 = y_2 - \ln a = \ln x - \ln a$

53. If the graph of $f(x)$ passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.
Alternate answer: If $g(x_1) = g(x_2)$ then $-f(x_1) = -f(x_2)$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

54. Suppose that $g(x_1) = g(x_2)$. Then $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$,
$f(x_1) = f(x_2)$, and x_1 and x_2 since f is one-to-one.

55. **(a)** The expression $a(b^{c-x}) + d$ is defined for all values of x, so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.

(b) The expression $a \log_b(x - c) + d$ is defined when $x - c > 0$, so the domain is (c, ∞).
Since $a \log_b(x - c) + d$ attains every real value for some value of x, the range is $(-\infty, \infty)$.

56. **(a)** Suppose $f(x_1) = f(x_2)$. Then:

$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$

$(ax_1 + b)(cx_2 + d) = (ax_2 + b)(cx_1 + d)$

$acx_1x_2 + adx_1 + bcx_2 + bd$
$= acx_1x_2 + adx_2 + bcx_1 + bd$

$adx_1 + bcx_2 = adx_2 + bcx_1$

$(ad - bc)x_1 = (ad - bc)x_2$

Since $ad - bc \neq 0$, this means that $x_1 = x_2$.

(b) $y = \frac{ax + b}{cx + d}$

$cxy + dy = ax + b$

$(cy - a)x = -dy + b$

$x = \frac{-dy + b}{cy - a}$

Interchange x and y:

$y = \frac{-dx + b}{cx - a}$

$f^{-1}(x) = \frac{-dx + b}{cx - a}$

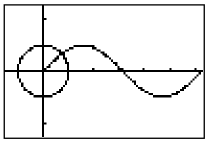
(c) As $x \to \pm\infty$, $f(x) = \frac{ax + b}{cx + d} \to \frac{a}{c}$, so the horizontal asymptote is $y = \frac{a}{c}$ $(c \neq 0)$. Since $f(x)$ is undefined at $x = -\frac{d}{c}$, the vertical asymptote is $x = -\frac{d}{c}$.

(d) As $x \to \pm\infty$, $f^{-1}(x) = \frac{-dx + b}{cx - a} \to -\frac{d}{c}$, so the horizontal asymptote is $y = -\frac{d}{c}$ $(c \neq 0)$. Since $f^{-1}(x)$ is undefined at $x = \frac{a}{c}$, the vertical asymptote is $x = \frac{a}{c}$.
The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

■ Section 1.6 Trigonometric Functions (pp. 41–51)

Exploration 1 Unwrapping Trigonometric Functions

1. (x_1, y_1) is the circle of radius 1 centered at the origin (unit circle). (x_2, y_2) is one period of the graph of the sine function.

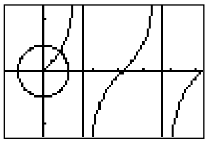


2. The y-values are the same in the interval $0 \le t \le 2\pi$.

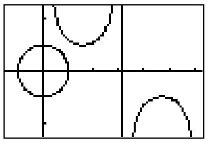
3. The y-values are the same in the interval $0 \le t \le 4\pi$.

4. The x_1-values and the y_2-values are the same in each interval.

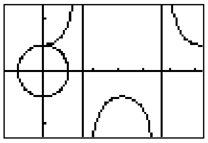
5. $y_2 = \tan t$:



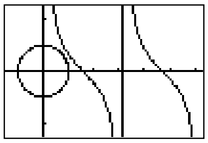
$y_2 = \csc t$:



$y_2 = \sec t$:



$y_2 = \cot t$:



For each value of t, the value of $y_2 = \tan t$ is equal to the ratio $\frac{y_1}{x_1}$.

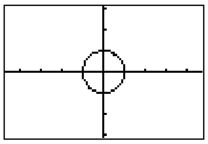
For each value of t, the value of $y_2 = \csc t$ is equal to the ratio $\frac{1}{y_1}$.

For each value of t, the value of $y_2 = \sec t$ is equal to the ratio $\frac{1}{x_1}$.

For each value of t, the value of $y_2 = \cot t$ is equal to the ratio $\frac{x_1}{y_1}$.

Exploration 2 Finding Sines and Cosines

1. The decimal viewing window $[-4.7, 4.7]$ by $[-3.1, 3.1]$ is square on the TI-82/83 and many other calculators. There are many other possibilities.



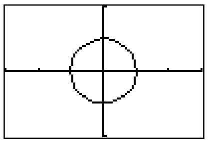
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

2. Using the Ask table setting for the independent variable on the TI-83 we obtain

T	X1T	Y1T
.5	.87758	.47943
1	.5403	.84147
2	-.4161	.9093
3.5	-.9365	-.3508
6.2	.99654	-.0831

X1T=cos(T)

3.



$[-3, 3]$ by $[-2, 2]$

Using trace, cos t and sin t are being computed for 0, 15, 30, … , 360 degrees.

Quick Review 1.6

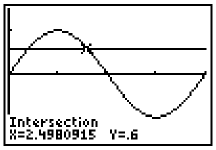
1. $\frac{\pi}{3} \cdot \frac{180°}{\pi} = 60°$

2. $-2.5 \cdot \frac{180°}{\pi} = \left(-\frac{450}{\pi}\right)^{\circ} \approx -143.24°$

3. $-40° \cdot \frac{\pi}{180°} = -\frac{2\pi}{9}$

4. $45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$

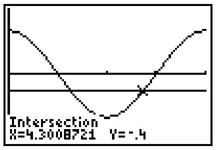
5.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 0.6435$, $x \approx 2.4981$

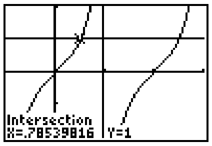
6.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 1.9823$, $x \approx 4.3009$

7.



$\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ by $[-2, 2]$

$x \approx 0.7854 \left(\text{or } \frac{\pi}{4}\right)$, $x \approx 3.9270 \left(\text{or } \frac{5\pi}{4}\right)$

8. $f(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = f(x)$
The graph is symmetric about the y-axis because if a point (a, b) is on the graph, then so is the point $(-a, b)$.

9. $f(-x) = (-x)^3 - 3(-x)$
$= -x^3 + 3x$
$= -(x^3 - 3x) = -f(x)$
The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

10. $x \ge 0$

Section 1.6 Exercises

1. Arc length $= \left(\frac{5\pi}{8}\right)(2) = \frac{5\pi}{4}$

2. Radius $= \dfrac{10}{175°\left(\frac{\pi}{180°}\right)} = \frac{72}{7\pi} \approx 3.274$

3. Angle $= \frac{7}{14} = \frac{1}{2}$ radian or about 28.65°

4. Angle $= \frac{3\pi/2}{6} = \frac{\pi}{4}$ radian or 45°

5. **(a)** The period of $y = \sec x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(b) The period of $y = \csc x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(c) The period of $y = \cot x$ is π, so the window should have length 2π.
One possible answer: $[0, 2\pi]$ by $[-3, 3]$

6. **(a)** The period of $y = \sin x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(b) The period of $y = \cos x$ is 2π, so the window should have length 4π.
One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(c) The period of $y = \tan x$ is π, so the window should have length 2π.
One possible answer: $[0, 2\pi]$ by $[-3, 3]$

7. Since $\frac{\pi}{6}$ is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $y = \sin^{-1} x$ and $\sin \frac{\pi}{6} = 0.5$, $\sin^{-1}(0.5) = \frac{\pi}{6}$ radian or $\frac{\pi}{6} \cdot \frac{180°}{\pi} = 30°$.

8. Since $-\frac{\pi}{4}$ is the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $y = \sin^{-1} x$ and $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ radian or $-\frac{\pi}{4} \cdot \frac{180°}{\pi} = -45°$.

9. Using a calculator, $\tan^{-1}(-5) \approx -1.3734$ radians or $-78.6901°$.

10. Using a calculator, $\cos^{-1}(0.7) \approx 0.7954$ radian or 45.5730°.

11. **(a)** Period $= \frac{2\pi}{2} = \pi$

(b) Amplitude $= 1.5$

(c) $[-2\pi, 2\pi]$ by $[-2, 2]$

12. **(a)** Period $= \frac{2\pi}{3}$

(b) Amplitude $= 2$

(c) $\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-4, 4]$

13. **(a)** Period $= \frac{2\pi}{2} = \pi$

(b) Amplitude $= 3$

(c) $[-2\pi, 2\pi]$ by $[-4, 4]$

14. **(a)** Period $= \frac{2\pi}{1/2} = 4\pi$

(b) Amplitude $= 5$

(c) $[-4\pi, 4\pi]$ by $[-10, 10]$

15. **(a)** Period $= \frac{2\pi}{\pi/3} = 6$

(b) Amplitude $= 4$

(c) $[-3, 3]$ by $[-5, 5]$

16. **(a)** Period $= \frac{2\pi}{\pi} = 2$

(b) Amplitude $= 1$

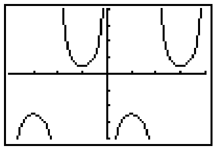
(c) $[-4, 4]$ by $[-2, 2]$

17. **(a)** Period $= \frac{2\pi}{3}$

(b) Domain: Since $\csc(3x + \pi) = \frac{1}{\sin(3x + \pi)}$, we require $3x + \pi \neq k\pi$, or $x \neq \frac{(k-1)\pi}{3}$. This requirement is equivalent to $x \neq \frac{k\pi}{3}$ for integers k.

(c) Since $|\csc(3x + \pi)| \geq 1$, the range excludes numbers between $-3 - 2 = -5$ and $3 - 2 = 1$. The range is $(-\infty, -5] \cup [1, \infty)$.

(d)



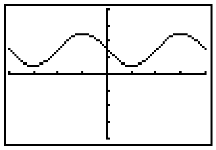
$\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-8, 8]$

18. **(a)** Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

(b) Domain: $(-\infty, \infty)$

(c) Since $|\sin(4x + \pi)| \leq 1$, the range extends from $-2 + 3 = 1$ to $2 + 3 = 5$. The range is $[1, 5]$.

(d)



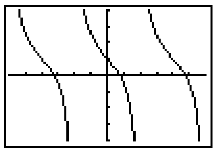
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

19. **(a)** Period $= \frac{\pi}{3}$

(b) Domain: We require $3x + \pi \neq \frac{k\pi}{2}$ for odd integers k. Therefore, $x \neq \frac{(k-2)\pi}{6}$ for odd integers k. This requirement is equivalent to $x \neq \frac{k\pi}{6}$ for odd integers k.

(c) Since the tangent function attains all real values, the range is $(-\infty, \infty)$.

(d)



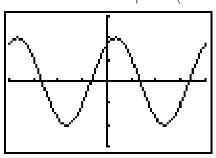
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

20. **(a)** Period $= \frac{2\pi}{2} = \pi$

(b) Domain: $(-\infty, \infty)$

(c) Range: Since $\left|\sin\left(2x + \frac{\pi}{3}\right)\right| \leq 1$, the range is $[-2, 2]$.

(d)



$[-\pi, \pi]$ by $[-3, 3]$

21. Note that $\sqrt{8^2 + 15^2} = 17$.

Since $\sin\theta = \frac{8}{17}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,
$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \frac{15}{17}$.

Therefore: $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8}{15}$,
$\cot\theta = \frac{1}{\tan\theta} = \frac{15}{8}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{17}{15}$, $\csc\theta = \frac{1}{\sin\theta} = \frac{17}{8}$

22. Note that $\sqrt{5^2 + 12^2} = 13$.

Since $\tan\theta = -\frac{5}{12} = \frac{-5/13}{12/13} = \frac{\sin\theta}{\cos\theta}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, we have $\sin\theta = -\frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. In summary:
$\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$,
$\cot\theta = \frac{1}{\tan\theta} = -\frac{12}{5}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$,
$\csc\theta = \frac{1}{\sin\theta} = -\frac{13}{5}$

23. Note that $r = \sqrt{(-3)^2 + 4^2} = 5$. Then:
$\sin\theta = \frac{y}{r} = \frac{4}{5}$, $\cos\theta = \frac{x}{r} = -\frac{3}{5}$, $\tan\theta = \frac{y}{x} = -\frac{4}{3}$,
$\cot\theta = \frac{x}{y} = -\frac{3}{4}$, $\sec\theta = \frac{r}{x} = -\frac{5}{3}$, $\csc\theta = \frac{r}{y} = \frac{5}{4}$

24. Note that $r = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$. Then:

$\sin\theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\cos\theta = \frac{x}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$,
$\tan\theta = \frac{y}{x} = \frac{2}{-2} = -1$, $\cot\theta = \frac{x}{y} = \frac{-2}{2} = -1$,
$\sec\theta = \frac{r}{x} = \frac{2\sqrt{2}}{-2} = -\sqrt{2}$, $\csc\theta = \frac{r}{y} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

25. The angle $\tan^{-1}(2.5) \approx 1.190$ is the solution to this equation in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Another solution in $0 \le x < 2\pi$ is $\tan^{-1}(2.5) + \pi \approx 4.332$. The solutions are $x \approx 1.190$ and $x \approx 4.332$.

26. The angle $\cos^{-1}(-0.7) \approx 2.346$ is the solution to this equation in the interval $0 \le x \le \pi$. Since the cosine function is even, the value $-\cos^{-1}(-0.7) \approx -2.346$ is also a solution, so any value of the form $\pm\cos^{-1}(-0.7) + 2k\pi$ is a solution, where k is an integer. In $2\pi \le x < 4\pi$ the solutions are $x = \cos^{-1}(-0.7) + 2\pi \approx 8.629$ and $x = -\cos^{-1}(-0.7) + 4\pi \approx 10.220$.

27. This equation is equivalent to $\sin x = \frac{1}{2}$, so the solutions in the interval $0 \le x < 2\pi$ are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

28. This equation is equivalent to $\cos x = -\frac{1}{3}$, so the solution in the interval $0 \le x \le \pi$ is $y = \cos^{-1}\left(-\frac{1}{3}\right) \approx 1.911$. Since the cosine function is even, the solutions in the interval $-\pi \le x < \pi$ are $x \approx -1.911$ and $x \approx 1.911$.

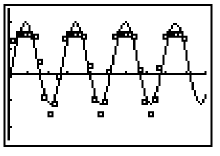
29. The solutions in the interval $0 \le x < 2\pi$ are $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. Since $y = \sin x$ has period 2π, the solutions are all of the form $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

30. The equation is equivalent to $\tan x = \frac{1}{-1} = -1$, so the solution in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$ is $x = \tan^{-1}(-1) = -\frac{\pi}{4}$. Since the period of $y = \tan x$ is π, all solutions are of the form $x = -\frac{\pi}{4} + k\pi$, where k is any integer. This is equivalent to $x = \frac{3\pi}{4} + k\pi$, where k is any integer.

31. Let $\theta = \cos^{-1}\left(\frac{7}{11}\right)$. Then $0 \le \theta \le \pi$ and $\cos\theta = \frac{7}{11}$, so
$\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right) = \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{7}{11}\right)^2}$
$= \frac{\sqrt{72}}{11} = \frac{6\sqrt{2}}{11} \approx 0.771$.

32. Let $\theta = \sin^{-1}\left(\frac{9}{13}\right)$. Then $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $\sin\theta = \frac{9}{13}$, so
$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{9}{13}\right)^2} = \frac{\sqrt{88}}{13}$. Therefore,
$\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{9/13}{\sqrt{88}/13} = \frac{9}{\sqrt{88}} \approx 0.959$.

33. **(a)** Using a graphing calculator with the sinusoidal regression feature, the equation is
$y = 1.543\sin(2468.635x - 0.494) + 0.438$.



[0, 0.01] by [−2.5, 2.5]

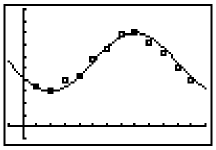
(b) The frequency is 2468.635 radians per second, which is equivalent to $\frac{2468.635}{2\pi} \approx 392.9$ cycles per second (Hz). The note is a "G."

34. **(a)** $b = \frac{2\pi}{12} = \frac{\pi}{6}$

(b) It's half of the difference, so $a = \frac{80 - 30}{2} = 25$.

(c) $k = \frac{80 + 30}{2} = 55$

(d) The function should have its minimum at $t = 2$ (when the temperature is 30°F) and its maximum at $t = 8$ (when the temperature is 80°F). The value of h is $\frac{2 + 8}{2} = 5$. Equation: $y = 25\sin\left[\frac{\pi}{6}(x - 5)\right] + 55$

(e) 

$[-1, 13]$ by $[-10, 100]$

35. **(a)** Amplitude $= 37$

(b) Period $= \dfrac{2\pi}{(2\pi/365)} = 365$

(c) Horizontal shift $= 101$

(d) Vertical shift $= 25$

36. **(a)** Highest: $25 + 37 = 62$°F
Lowest: $25 - 37 = -12$°F

(b) Average $= \dfrac{62 + (-12)}{2} = 25$°F

This average is the same as the vertical shift because the average of the highest and lowest values of $y = \sin x$ is 0.

37. **(a)** $\cot(-x) = \dfrac{\cos(-x)}{\sin(-x)} = \dfrac{\cos(x)}{-\sin(x)} = -\cot(x)$

(b) Assume that f is even and g is odd.

Then $\dfrac{f(-x)}{g(-x)} = \dfrac{f(x)}{-g(x)} = -\dfrac{f(x)}{g(x)}$ so $\dfrac{f}{g}$ is odd. The situation is similar for $\dfrac{g}{f}$.

38. **(a)** $\csc(-x) = \dfrac{1}{\sin(-x)} = \dfrac{1}{-\sin(x)} = -\csc(x)$

(b) Assume that f is odd.

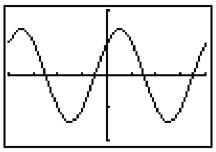
Then $\dfrac{1}{f(-x)} = \dfrac{1}{-f(x)} = -\dfrac{1}{f(x)}$ so $\dfrac{1}{f}$ is odd.

39. Assume that f is even and g is odd.
Then $f(-x)g(-x) = (f(x))(-g(x)) = -f(x)g(x)$ so (fg) is odd.

40. If (a, b) is the point on the unit circle corresponding to the angle θ, then $(-a, -b)$ is the point on the unit circle corresponding to the angle $(\theta + \pi)$ since it is exactly half way around the circle. This means that both $\tan(\theta)$ and $\tan(\theta + \pi)$ have the same value, $\dfrac{b}{a}$.

41. **(a)** Using a graphing calculator with the sinusoidal regression feature, the equation is
$y = 3.0014 \sin(0.9996x + 2.0012) + 2.9999$.

(b) $y = 3\sin(x + 2) + 3$

42. **(a)** 

$[-2\pi, 2\pi]$ by $[-2, 2]$

The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 (that is, $\sqrt{2}$)

Period $= 2\pi$

Horizontal shift ≈ -0.785 $\left(\text{that is, } -\dfrac{\pi}{4}\right)$

or 5.498 $\left(\text{that is, } \dfrac{7\pi}{4}\right)$

Vertical shift: 0

(c)
$$\begin{aligned}\sin\left(x + \frac{\pi}{4}\right) &= (\sin x)\left(\cos\frac{\pi}{4}\right) + (\cos x)\left(\sin\frac{\pi}{4}\right)\\ &= (\sin x)\left(\frac{1}{\sqrt{2}}\right) + (\cos x)\left(\frac{1}{\sqrt{2}}\right)\\ &= \frac{1}{\sqrt{2}}(\sin x + \cos x)\end{aligned}$$

Therefore, $\sin x + \cos x = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$.

43. **(a)** $\sqrt{2}\sin\left(ax + \dfrac{\pi}{4}\right)$

(b) See part (a).

(c) It works.

(d) $\sin\left(ax + \dfrac{\pi}{4}\right) = (\sin ax)\left(\cos\dfrac{\pi}{4}\right) + (\cos ax)\left(\sin\dfrac{\pi}{4}\right)$

$= (\sin ax)\left(\dfrac{1}{\sqrt{2}}\right) + (\cos ax)\left(\dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{1}{\sqrt{2}}(\sin ax + \cos ax)$

So, $\sin(ax) + \cos(ax) = \sqrt{2}\sin\left(ax + \dfrac{\pi}{4}\right)$.

44. **(a)** One possible answer:
$y = \sqrt{a^2 + b^2}\sin\left(x + \tan^{-1}\left(\dfrac{b}{a}\right)\right)$

(b) See part (a).

(c) It works.

(d) $\sin\left(x + \tan^{-1}\left(\dfrac{b}{a}\right)\right)$

$= \sin(x)\cos\left(\tan^{-1}\left(\dfrac{b}{a}\right)\right) + \cos(x)\sin\left(\tan^{-1}\left(\dfrac{b}{a}\right)\right)$

$= \sin(x)\left(\dfrac{a}{\sqrt{a^2 + b^2}}\right) + \cos(x)\left(\dfrac{b}{\sqrt{a^2 + b^2}}\right)$

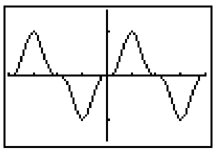
$= \dfrac{1}{\sqrt{a^2 + b^2}} \cdot (a\sin x + b\cos x)$

and multiplying through by the square root gives the desired result. Note that the substitutions

$\cos\left(\tan^{-1}\dfrac{b}{a}\right) = \dfrac{a}{\sqrt{a^2 + b^2}}$ and

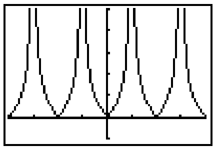
$\sin\left(\tan^{-1}\dfrac{b}{a}\right) = \dfrac{b}{\sqrt{a^2 + b^2}}$ depend on the requirement that a is positive. If a is negative, the formula does not work.

45. Since $\sin x$ has period 2π, $\sin^3(x + 2\pi) = \sin^3(x)$. This function has period 2π. A graph shows that no smaller number works for the period.



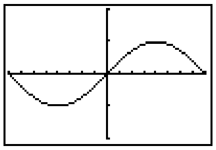
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

46. Since $\tan x$ has period π, $|\tan(x + \pi)| = |\tan x|$. This function has period π. A graph shows that no smaller number works for the period.



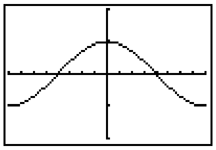
$[-2\pi, 2\pi]$ by $[-1, 5]$

47. The period is $\frac{2\pi}{60} = \frac{\pi}{30}$. One possible graph:



$\left[-\frac{\pi}{60}, \frac{\pi}{60}\right]$ by $[-2, 2]$

48. The period is $\frac{2\pi}{60\pi} = \frac{1}{30}$. One possible graph:



$\left[-\frac{1}{60}, \frac{1}{60}\right]$ by $[-2, 2]$

■ Chapter 1 Review Exercises (pp. 52–53)

1. $y = 3(x - 1) + (-6)$
$y = 3x - 9$

2. $y = -\frac{1}{2}(x + 1) + 2$
$y = -\frac{1}{2}x + \frac{3}{2}$

3. $x = 0$

4. $m = \frac{-2 - 6}{1 - (-3)} = \frac{-8}{4} = -2$
$y = -2(x + 3) + 6$
$y = -2x$

5. $y = 2$

6. $m = \frac{5 - 3}{-2 - 3} = \frac{2}{-5} = -\frac{2}{5}$
$y = -\frac{2}{5}(x - 3) + 3$
$y = -\frac{2}{5} + \frac{21}{5}$

7. $y = -3x + 3$

8. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2.
$y = 2(x - 3) + 1$
$y = 2x - 5$

9. Since $4x + 3y = 12$ is equivalent to $y = -\frac{4}{3}x + 4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$.
$y = -\frac{4}{3}(x - 4) - 12$
$y = -\frac{4}{3}x - \frac{20}{3}$

10. Since $3x - 5y = 1$ is equivalent to $y = \frac{3}{5}x - \frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$.
$y = -\frac{5}{3}(x + 2) - 3$
$y = -\frac{5}{3}x - \frac{19}{3}$

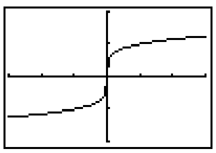
11. Since $\frac{1}{2}x + \frac{1}{3}y = 1$ is equivalent to $y = -\frac{3}{2}x + 3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$.
$y = \frac{2}{3}(x + 1) + 2$
$y = \frac{2}{3}x + \frac{8}{3}$

12. The line passes through $(0, -5)$ and $(3, 0)$
$m = \frac{0 - (-5)}{3 - 0} = \frac{5}{3}$
$y = \frac{5}{3}x - 5$

13. $m = \frac{2 - 4}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$
$f(x) = -\frac{1}{2}(x + 2) + 4$
$f(x) = -\frac{1}{2}x + 3$
Check: $f(4) = -\frac{1}{2}(4) + 3 = 1$, as expected.

14. The line passes through $(4, -2)$ and $(-3, 0)$.
$m = \frac{0 - (-2)}{-3 - 4} = \frac{2}{-7} = -\frac{2}{7}$
$y = -\frac{2}{7}(x - 4) - 2$
$y = -\frac{2}{7}x - \frac{6}{7}$

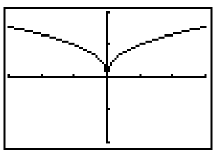
15.



$[-3, 3]$ by $[-2, 2]$

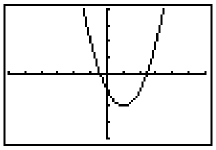
Symmetric about the origin.

16.



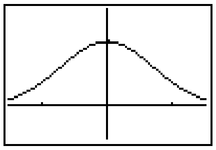
$[-3, 3]$ by $[-2, 2]$

Symmetric about the y-axis.

17. 

[−6, 6] by [−4, 4]

Neither

18. 

[−1.5, 1.5] by [−0.5, 1.5]

Symmetric about the y-axis.

19. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$
Even

20. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$
Odd

21. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$
Even

22. $y(-x) = \sec(-x)\tan(-x)$

$$= \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x}$$

$$= -\sec x \tan x = -y(x)$$

Odd

23. $y(-x) = \dfrac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \dfrac{x^4 + 1}{-x^3 + 2x} = -\dfrac{x^4 + 1}{x^3 - 2x} = -y(x)$
Odd

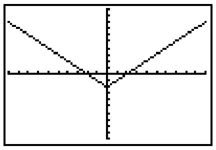
24. $y(-x) = 1 - \sin(-x) = 1 + \sin x$
Neither even nor odd

25. $y(-x) = -x + \cos(-x) = -x + \cos x$
Neither even nor odd

26. $y(-x) = \sqrt{(-x)^4 - 1} = \sqrt{x^4 - 1}$
Even

27. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

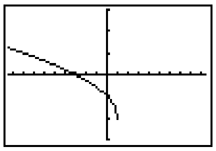
(b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

(c) 

[−10, 10] by [−10, 10]

28. **(a)** Since the square root requires $1 - x \ge 0$, the domain is $(-\infty, 1]$.

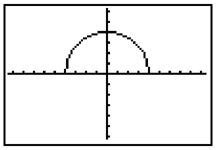
(b) Since $\sqrt{1 - x}$ attains all nonnegative values, the range is $[-2, \infty)$.

(c) 

[−9.4, 9.4] by [−3, 3]

29. **(a)** Since the square root requires $16 - x^2 \ge 0$, the domain is $[-4, 4]$.

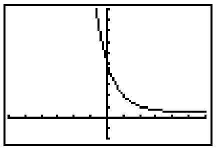
(b) For values of x in the domain, $0 \le 16 - x^2 \le 16$, so $0 \le \sqrt{16 - x^2} \le 4$. The range is $[0, 4]$.

(c) 

[−9.4, 9.4] by [−6.2, 6.2]

30. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

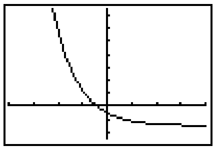
(b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

(c) 

[−6, 6] by [−4, 20]

31. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

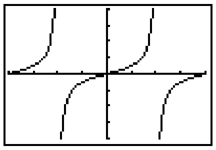
(b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

(c) 

[−4, 4] by [−5, 15]

32. **(a)** The function is equivalent to $y = \tan 2x$, so we require $2x \ne \dfrac{k\pi}{2}$ for odd integers k. The domain is given by $x \ne \dfrac{k\pi}{4}$ for odd integers k.

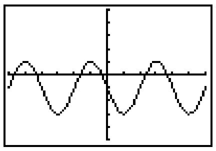
(b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

(c) 

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [−8, 8]

33. **(a)** The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The sine function attains values from -1 to 1, so $-2 \le 2\sin(3x + \pi) \le 2$, and hence $-3 \le 2\sin(3x + \pi) - 1 \le 1$. The range is $[-3, 1]$.

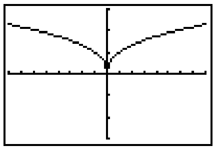
(c) 

$[-\pi, \pi]$ by [−5, 5]

34. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

(c)

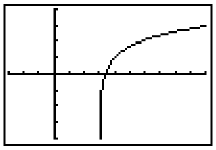


$[-8, 8]$ by $[-3, 3]$

35. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.

(b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

(c)

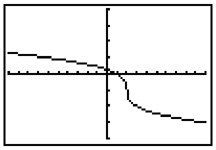


$[-3, 10]$ by $[-4, 4]$

36. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

(c)

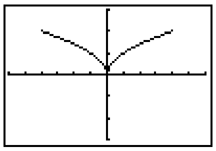


$[-10, 10]$ by $[-4, 4]$

37. (a) The function is defined for $-4 \le x \le 4$, so the domain is $[-4, 4]$.

(b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \le x \le 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

(c)

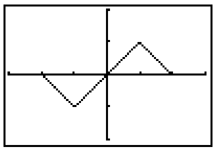


$[-6, 6]$ by $[-3, 3]$

38. (a) The function is defined for $-2 \le x \le 2$, so the domain is $[-2, 2]$.

(b) See the graph in part (c). The range is $[-1, 1]$.

(c)



$[-3, 3]$ by $[-2, 2]$

39. First piece: Line through (0, 1) and (1, 0)

$$m = \frac{0-1}{1-0} = \frac{-1}{1} = -1$$

$$y = -x + 1 \text{ or } 1 - x$$

Second piece:
Line through (1, 1) and (2, 0)

$$m = \frac{0-1}{2-1} = \frac{-1}{1} = -1$$

$$y = -(x-1) + 1$$

$$y = -x + 2 \text{ or } 2 - x$$

$$f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$$

40. First piece: Line through (0, 0) and (2, 5)

$$m = \frac{5-0}{2-0} = \frac{5}{2}$$

$$y = \frac{5}{2}x$$

Second piece: Line through (2, 5) and (4, 0)

$$m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2}$$

$$y = -\frac{5}{2}(x-2) + 5$$

$$y = -\frac{5}{2}x + 10 \text{ or } 10 - \frac{5x}{2}$$

$$f(x) = \begin{cases} \frac{5x}{2}, & 0 \le x < 2 \\ 10 - \frac{5x}{2}, & 2 \le x \le 4 \end{cases}$$

(Note: $x = 2$ can be included on either piece.)

41. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1/2 + 2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \ne 0$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{1/\sqrt{x+2} + 2}}$

$$= \frac{\sqrt[4]{x+2}}{\sqrt{1 + 2\sqrt{x+2}}}$$

42. (a)

$$\begin{aligned} (f \circ g)(-1) &= f(g(-1)) \\ &= f(\sqrt[3]{-1+1}) \\ &= f(0) = 2 - 0 = 2 \end{aligned}$$

(b) $(g \circ f)(2) = g(f(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$

(d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{x+1}) = \sqrt[3]{\sqrt[3]{x+1} + 1}$

43. (a) $(f \circ g)(x) = f(g(x))$

$$= f(\sqrt{x+2})$$
$$= 2 - (\sqrt{x+2})^2$$
$$= -x, x \geq -2$$

$(g \circ f)(x) = g(f(x))$

$$= g(2 - x^2)$$
$$= \sqrt{(2 - x^2) + 2} = \sqrt{4 - x^2}$$

(b) Domain of $f \circ g$: $[-2, \infty)$
Domain of $g \circ f$: $[-2, 2]$

(c) Range of $f \circ g$: $(-\infty, 2]$
Range of $g \circ f$: $[0, 2]$

44. (a) $(f \circ g)(x) = f(g(x))$

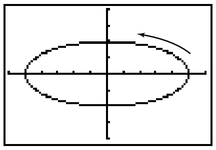
$$= f(\sqrt{1-x})$$
$$= \sqrt{\sqrt{1-x}}$$
$$= \sqrt[4]{1-x}$$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1 - \sqrt{x}}$

(b) Domain of $f \circ g$: $(-\infty, 1]$
Domain of $g \circ f$: $[0, 1]$

(c) Range of $f \circ g$: $[0, \infty)$
Range of $g \circ f$: $[0, 1]$

45. (a)

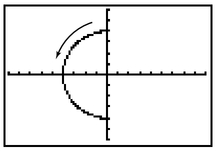


$[-6, 6]$ by $[-4, 4]$

Initial point: (5, 0)
Terminal point: (5, 0)
The ellipse is traced exactly once in a counterclockwise direction starting and ending at the point (5, 0).

(b) Substituting $\cos t = \frac{x}{5}$ and $\sin t = \frac{y}{2}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation
$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1.$
The entire ellipse is traced by the curve.

46. (a)

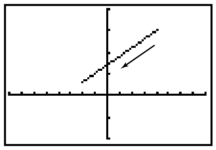


$[-9, 9]$ by $[-6, 6]$

Initial point: (0, 4)
Terminal point: None (since the endpoint $\frac{3\pi}{2}$ is not included in the t-interval)
The semicircle is traced in a counterclockwise direction starting at (0, 4) and extending to, but not including, (0, −4).

(b) Substituting $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{4}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$, or $x^2 + y^2 = 16$. The left half of the circle is traced by the parametrized curve.

47. (a)

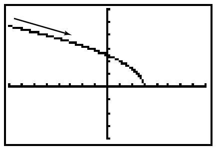


$[-8, 8]$ by $[-10, 20]$

Initial point: (4, 15)
Terminal point: (−2, 3)
The line segment is traced from right to left starting at (4, 15) and ending at (−2, 3).

(b) Substituting $t = 2 - x$ into $y = 11 - 2t$ gives the Cartesian equation $y = 11 - 2(2 - x)$, or $y = 2x + 7$. The part of the line from (4, 15) to (−2, 3) is traced by the parametrized curve.

48. (a)



$[-8, 8]$ by $[-4, 6]$

Initial point: None
Terminal point: (3, 0)
The curve is traced from left to right ending at the point (3, 0).

(b) Substituting $t = x - 1$ into $y = \sqrt{4 - 2t}$ gives the Cartesian equation $y = \sqrt{4 - 2(x - 1)}$, or $y = \sqrt{6 - 2x}$. The entire curve is traced by the parametrized curve.

49. (a) For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at $(-2, 5)$ for $t = 0$ and ends at $(4, 3)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -2$ and $f(1) = 4$. Since
slope $= \frac{\Delta x}{\Delta t} = \frac{4 - (-2)}{1 - 0} = 6,$
$x = f(t) = 6t - 2 = -2 + 6t.$
Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 3$. Since

slope $= \frac{\Delta y}{\Delta t} = \frac{3 - 5}{1 - 0} = -2,$

$y = g(t) = -2t + 5 = 5 - 2t.$

One possible parametrization is:
$x = -2 + 6t, y = 5 - 2t, 0 \leq t \leq 1$

50. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) passes through $(-3, -2)$ for $t = 0$ and $(4, -1)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -3$ and $f(1) = 4$. Since
$\text{slope} = \frac{\Delta x}{\Delta t} = \frac{4-(-3)}{1-0} = 7,$
$x = f(t) = 7t - 3 = -3 + 7t.$
Also, $y = g(t)$, where $g(0) = -2$ and $g(1) = -1$.
Since
$\text{slope} = \frac{\Delta y}{\Delta t} = \frac{-1-(-2)}{1-0} = 1$
$y = g(t) = t - 2 = -2 + t.$
One possible parametrization is:
$x = -3 + 7t,\ y = -2 + t,\ -\infty < t < \infty.$

51. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(2, 5)$ for $t = 0$ and passes through $(-1, 0)$ for $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$. Since
$\text{slope} = \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3,\ x = f(t) = -3t + 2 = 2 - 3t.$
Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 0$. Since
$\text{slope} = \frac{\Delta y}{\Delta t} = \frac{0-5}{1-0} = -5,\ y = g(t) = -5t + 5 = 5 - 5t.$
One possible parametrization is:
$x = 2 - 3t,\ y = 5 - 5t,\ t \ge 0.$

52. One possible parametrization is:
$x = t,\ y = t(t - 4),\ t \le 2.$

53. (a) $y = 2 - 3x$

$3x = 2 - y$

$x = \frac{2-y}{3}$

Interchange x and y.

$y = \frac{2-x}{3}$

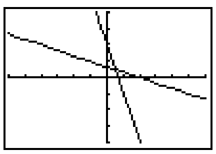
$f^{-1}(x) = \frac{2-x}{3}$

Verify.

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x))\\ &= f\left(\frac{2-x}{3}\right)\\ &= 2 - 3\left(\frac{2-x}{3}\right)\\ &= 2 - (2 - x) = x\end{aligned}$$

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(f(x))\\ &= f^{-1}(2 - 3x)\\ &= \frac{2-(2-3x)}{3}\\ &= \frac{3x}{3} = x\end{aligned}$$

(b)



$[-6, 6]$ by $[-4, 4]$

54. (a) $y = (x + 2)^2,\ x \ge -2$
$\sqrt{y} = x + 2$
$x = \sqrt{y} - 2$

Interchange x and y.

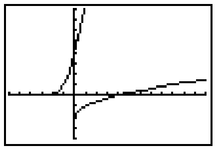
$y = \sqrt{x} - 2$
$f^{-1}(x) = \sqrt{x} - 2$
Verify.
For $x \ge 0$ (the domain of f^{-1})

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x))\\ &= f(\sqrt{x} - 2)\\ &= [(\sqrt{x} - 2) + 2]^2\\ &= (\sqrt{x})^2 = x\end{aligned}$$

For $x \ge -2$ (the domain of f),

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(f(x))\\ &= f^{-1}((x + 2)^2)\\ &= \sqrt{(x + 2)^2} - 2\\ &= |x + 2| - 2\\ &= (x + 2) - 2 = x\end{aligned}$$

(b)



$[-6, 12]$ by $[-4, 8]$

55. Using a calculator, $\sin^{-1}(0.6) \approx 0.6435$ radians or $36.8699°$.

56. Using a calculator, $\tan^{-1}(-2.3) \approx -1.1607$ radians or $-66.5014°$.

57. Since $\cos\theta = \frac{3}{7}$ and $0 \le \theta \le \pi$,

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{3}{7}\right)^2} = \sqrt{\frac{40}{49}} = \frac{\sqrt{40}}{7}.$$

Therefore,

$$\sin\theta = \frac{\sqrt{40}}{7},\ \cos\theta = \frac{3}{7},\ \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{40}}{3},$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{40}},\ \sec\theta = \frac{1}{\cos\theta} = \frac{7}{3},$$

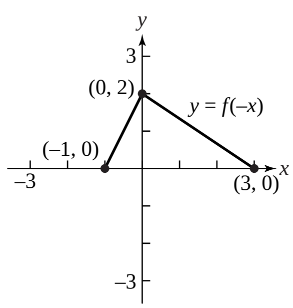
$$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{\sqrt{40}}$$

58. (a) Note that $\sin^{-1}(-0.2) \approx -0.2014$. In $[0, 2\pi)$, the solutions are $x = \pi - \sin^{-1}(-0.2) \approx 3.3430$ and $x = \sin^{-1}(-0.2) + 2\pi \approx 6.0818$.

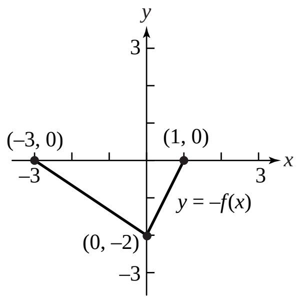
(b) Since the period of $\sin x$ is 2π, the solutions are $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer.

59. $e^{-0.2x} = 4$
$\ln e^{-0.2x} = \ln 4$
$-0.2x = \ln 4$
$x = \frac{\ln 4}{-0.2} = -5 \ln 4$

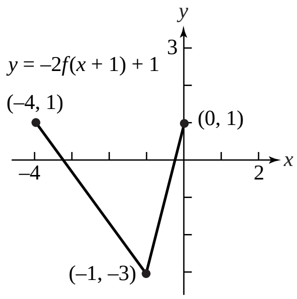
60. (a) The given graph is reflected about the y-axis.



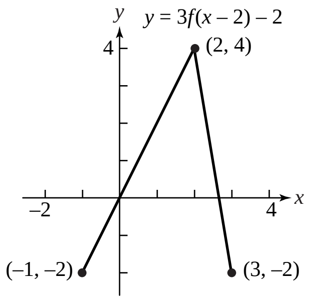
(b) The given graph is reflected about the x-axis.



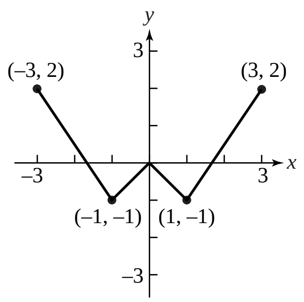
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



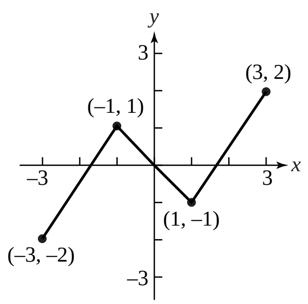
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



61. (a)



(b)



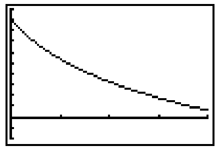
62. (a) $V = 100{,}000 - 10{,}000x,\ 0 \leq x \leq 10$

(b)
$$\begin{aligned} V &= 55{,}000 \\ 100{,}000 - 10{,}000x &= 55{,}000 \\ -10{,}000x &= -45{,}000 \\ x &= 4.5 \end{aligned}$$
The value is \$55,000 after 4.5 years.

63. (a) $f(0) = 90$ units

(b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units

(c)



[0, 4] by [−20, 100]

64. $1500(1.08)^t = 5000$

$$\begin{aligned} 1.08^t &= \frac{5000}{1500} = \frac{10}{3} \\ \ln (1.08)^t &= \ln \frac{10}{3} \\ t \ln 1.08 &= \ln \frac{10}{3} \\ t &= \frac{\ln (10/3)}{\ln 1.08} \\ t &\approx 15.6439 \end{aligned}$$

It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

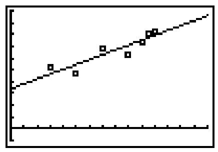
65. (a) $N(t) = 4 \cdot 2^t$

(b) 4 days: $4 \cdot 2^4 = 64$ guppies
1 week: $4 \cdot 2^7 = 512$ guppies

(c)
$$\begin{aligned} N(t) &= 2000 \\ 4 \cdot 2^t &= 2000 \\ 2^t &= 500 \\ \ln 2^t &= \ln 500 \\ t \ln 2 &= \ln 500 \\ t &= \frac{\ln 500}{\ln 2} \approx 8.9658 \end{aligned}$$
There will be 2000 guppies after 8.9658 days, or after nearly 9 days.

(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. (a) $y = 20.627x + 338.622$



[0, 30] by [−100, 1000]

(b) When $x = 30$, $y \approx 957.445$. According to the regression equation, about 957 degrees will be earned.

(c) The slope is 20.627. It represents the approximate annual increase in the number of doctorates earned by Hispanic Americans per year.

67. (a) $y = 14.60175 \cdot 1.00232^x$

(b) Solving $y = 25$ graphically, we obtain $x \approx 232$. According to the regression equation, the population will reach 25 million in the year 2132.

(c) 0.232%