6.4-6.6 Review Name______________

1. Suppose the decay equation for a radioactive element is known to be $y = y_0 e^{-1.2t}$ where t is measured in years.

A) What is the half-life of the element?

$$\text{half-life} = \frac{\ln 2}{|k|} = \frac{\ln 2}{1.2} = \boxed{.578 \text{ yrs.}}$$

B) How many years will it take for 75% of a sample of this element to decay?

$\frac{y}{y_0} = .25$ 25% remaining

$.25 = e^{-1.2t}$

$\boxed{t = 1.155 \text{ yrs.}}$

2. The table gives the percentage , P, of households with a VCR, as function of the year. Let $t = 0$ represent 1978, $t = 1$ represent 1979, and so forth.

Year	P(%)
1978	0.3
1979	0.5
1980	1.1
1981	1.8
1982	3.1
1983	5.5
1984	10.6
1985	20.8
1986	36.0
1987	48.7
1988	58.0
1989	64.6
1990	71.9
1991	71.9

A) Find the logistic regression equation for the data.

$$\boxed{P = \frac{73.745}{1 + 535.581e^{-.769t}}}$$

B) Find the carrying capacity predicted by the regression equation.

$\boxed{73.745\%}$

C) Find when the rate of growth predicted by the regression equation changes from increasing to decreasing. Estimate the percentage of VCR's at this time. During what year does this happen?

percentage of VCRS → $\boxed{36.8725\%}$

$$36.8725 = \frac{73.745}{1 + 535.581e^{-.769}}$$

$t = 8.173$ yrs. $1978 + 8.173 = 1986.173$

$\boxed{1986}$

3. The temperature of a cake was $350°F$ at the instant it came out of the oven. After 10 minutes, its temperature is $200°F$. If room temperature is $70°F$, how much longer will it take for the cake to cool to $90°F$?

$T = T_m + Ce^{kt}$

$T = 70 + Ce^{kt}$

$350 = 70 + C$

$C = 280$

$T(0) = 350 \to C$

$T(10) = 200 \to k$

$T(?) = 90$

$T = 70 + 280e^{kt}$

$200 = 70 + 280e^{10k}$

$k = -.077 \to A$

$T = 70 + 280e^{-.077t}$

$90 = 70 + 280e^{-.077t}$

$t = 34.396$

$\boxed{24.396 \text{ min. longer}}$

4. Determine a logistic equation given the carrying capacity is 150, P(0)=15 and P(2)=30. Use your equation to predict the number of years it will take the population to reach 100.

$m = 150 \to A$, $\to k$

$P = \dfrac{m}{1 + Ae^{-kt}}$

$P = \dfrac{150}{1 + Ae^{-kt}}$

$15 = \dfrac{150}{1 + A}$

$A = 9$

$P = \dfrac{150}{1 + 9e^{-kt}}$

$30 = \dfrac{150}{1 + 9e^{-2k}}$

$30 + 270e^{-2k} = 150$

$k = .405$

$\boxed{P = \dfrac{150}{1 + 9e^{-.405t}}}$

$100 = \dfrac{150}{1 + 9e^{-.405t}}$

$\boxed{t = 7.129 \text{ yrs.}}$

5. A population of wild horses is represented by the logistic differential equation

$\dfrac{dP}{dt} = 0.08P - 0.00004P^2$, where t is measured in years.

$\dfrac{dP}{dt} = .00004P(2000 - P)$

A) Find k and the carrying capacity for the population.

$.00004 = k$

$\boxed{m = 2000}$

$\cdot 2000$

$\boxed{k = .08}$

B) The initial population is 10 horses. Find a formula for the population in terms of t.

$P(0) = 10$

$P = \dfrac{2000}{1 + Ae^{-.08t}}$

$10 = \dfrac{2000}{1 + A}$

$A = 199$

$\boxed{P = \dfrac{2000}{1 + 199e^{-.08t}}}$

C) How long will it take for the horse population to reach 350?

$350 = \dfrac{2000}{1 + 199e^{-.08t}}$

$350 + 69650e^{-.08t} = 2000$

$\boxed{t = 46.784 \text{ yrs.}}$