Applications of Differential Equations Test Review

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) How long would it take \$8000 to grow to \$24,000 at 5% compounded continuously? Round your answer to the nearest tenth of a year. 1) ________

A) 22.6 years B) 22.0 years C) 22.2 years D) 31.2 years

2) The decay equation for a radioactive substance is known to be $y = y_0e^{-0.062t}$, with t in days. About how long will it take for the amount of substance to decay to 37% of its original value? 2) ________

A) 16.0 days B) 58.2 days C) 16.2 days D) 7.0 days

3) An artifact is discovered at a certain site. If it has 62 % of the carbon-14 it originally contained, what is the approximate age of the artifact, to the nearest year? (Carbon-14 decays at the rate of .0125% annually.) 3) ________

A) 1661 years old B) 3824 years old
C) 3040 years old D) 4960 years old

4) The radioactive decay of a substance can be modeled by the differential equation $\frac{dy}{dt} = -0.101y$, where t is measured in years. Find the half-life of the substance. Round your answer to the nearest hundredth year. 4) ________

A) 6.86 years B) 9.33 years C) 7.48 years D) 61.53 years

5) A certain population is growing at a continuous rate so that the population doubles every 11 years. How long does it take for the population to triple? 5) ________

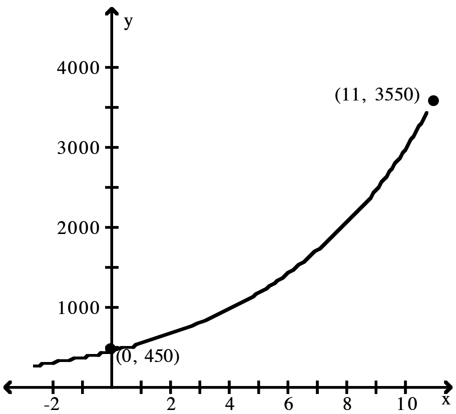
A) 18.4 years B) 17.4 years C) 17.9 years D) 16.5 years

6) A bacterial culture has an initial population of 10,000. If its population declines to 5000 in 4 hours, what will it be at the end of 6 hours? Assume that the population decreases according to the exponential model. 6) ________

A) 3536 B) 2500 C) 1768 D) 5743

Find the exponential function $y = y_0e^{kt}$ whose graph passes through the two given points.

7) 7) ________



A) $y = 3550\ e^{0.19t}$ B) $y = 450e^{0.19t}$

C) $y = 450e^{0.17t}$ D) $y = 450t^{0.19}$

Use Newton's Law of Cooling to solve the problem.

8) A dish of lasagna baked at 350°F is taken out of the oven into a kitchen that is 71°F. After 8 minutes, the temperature of the lasagna is 281.9°F. When will its temperature be 225°F? Round your answer to the nearest minute. 8) ________

A) 12 minutes after it was taken out of the oven

B) 17 minutes after it was taken out of the oven

C) 24 minutes after it was taken out of the oven

D) 21 minutes after it was taken out of the oven

9) A cup of coffee with temperature 104°F is placed in a freezer with temperature 0°F. After 8 minutes, the temperature of the coffee is 53.1°F. What will its temperature be 11 minutes after it is placed in the freezer? Round your answer to the nearest degree. 9) ________

A) 41°F B) 32°F C) 35°F D) 38°F

Solve the problem.

10) In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. In a certain reaction, the change of the substance satisfies the differential equation 10) ________

$$\frac{dy}{dt} = -0.58y$$

where y is measured in grams and t is measured in hours. If there are 100 grams of the substance when t = 0, how many grams will be left after the first hour?

A) 99.42 g B) 58.0 g C) 56.0 g D) 85.5 g

11) The resisting force on a moving object such as a car coasting to a stop is proportional to its velocity and is thus equal to $-kv$ for some positive constant k. Using the law Force = Mass × Acceleration, the velocity of an object slowed by air resistance satisfies the differential equation $m\frac{dv}{dt} = -kv$. Solving this equation gives $v = v0e^{-(k/m)t}$, where v_0 is the velocity of the object at time $t = 0$. 11) ________

A 72-kg cyclist on a 6-kg bicycle starts coasting on level ground at 10 m/sec. The value of k is about 3.9 kg/sec. About how far will the cyclist coast before reaching a complete stop?

A) ≈ 780 meters B) ≈ 185 meters C) ≈ 196 meters D) ≈ 200 meters

12) The resisting force on a moving object such as a car coasting to a stop is proportional to its velocity and is thus equal to $-kv$ for some positive constant k. Using the law Force = Mass × Acceleration, the velocity of an object slowed by air resistance satisfies the differential equation $m\frac{dv}{dt} = -kv$. Solving this equation gives $v = v0e^{-(k/m)t}$, where v_0 is the velocity of the object at time $t = 0$. 12) ________

A 58-kg cyclist on a 6-kg bicycle starts coasting on level ground at 7 m/sec. The value of k is about 3.8 kg/sec. How long will it take the cyclist's speed to drop to 2 m/sec?

A) ≈ 80.2 seconds B) ≈ 19.1 seconds

C) ≈ 21.1 seconds D) ≈ 72.7 seconds

13) A population of rabbits is given by the formula 13) ________

$$P(t) = \frac{800}{1 + e^{5.1 - 0.32t}},$$

where t is the number of months after a few rabbits are released. This function is the solution of a logistic differential equation. Identify k and the carrying capacity.

A) carrying capacity = 800, $k = 0.0004$ B) carrying capacity = $e^{5.1}$, $k = 0.0004$

C) carrying capacity = 800, $k = 0.32$ D) carrying capacity = $e^{5.1}$, $k = 0.32$

14) The table shows the population of a certain city for selected years between 1950 and 2003. 14) ________

Years after 1950	Population
0	7891
20	103,087
30	191,064
40	241,552
50	265, 058
55	269,116

By finding a logistic regression equation to model the data, determine what the population of the city will approach in the long run.

A) 278, 715 B) 266,078 C) 254,180 D) 271,976

15) The table shows the population of a certain city for selected years between 1950 and 2003. 15) _______

Years after 1950	Population
0	7891
20	103,087
30	191,064
40	241,552
50	265, 058
55	269,116

By finding a logistic regression equation to model the data, determine when the population of the city will first exceed 271,000.

A) In 2010 B) In 2012 C) In 2015 D) In 2018

Use Euler's method to solve the initial value problem.

16) $\frac{dy}{dx} = x - 3$ and $y = 4$ when $x = 1$ 16) _______

Use Euler's method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

A) 3.43 B) 3.68 C) 3.5 D) 3.48

Solve the problem.

17) Suppose that electricity is draining from a capacitor at a rate proportional to the voltage V across its terminals and that, if t is measured in seconds, 17) _______

$$\frac{dV}{dt} = -\frac{1}{40}V$$

How long will it take the voltage to drop to 20% of its original value?

A) 81.1 seconds B) 64.4 seconds C) 70.2 seconds D) 74.0 seconds

Use Euler's method to solve the initial value problem.

18) $\frac{dy}{dx} = x - 4y$ and $y = 4$ when $x = 2$ 18) _______

Use Euler's method with increments of $\Delta x = -0.1$ to approximate the value of y when $x = 1.7$.

A) 10.218 B) 10.138 C) 10.308 D) 10.348

1) B
2) A
3) B
4) A
5) B
6) A
7) B
8) B
9) A
10) C
11) D
12) C
13) A
14) D
15) C
16) A
17) B
18) B